Uniform 3-D networks and related sponge surfaces & polyhedra, as inspiration to innovative space structures

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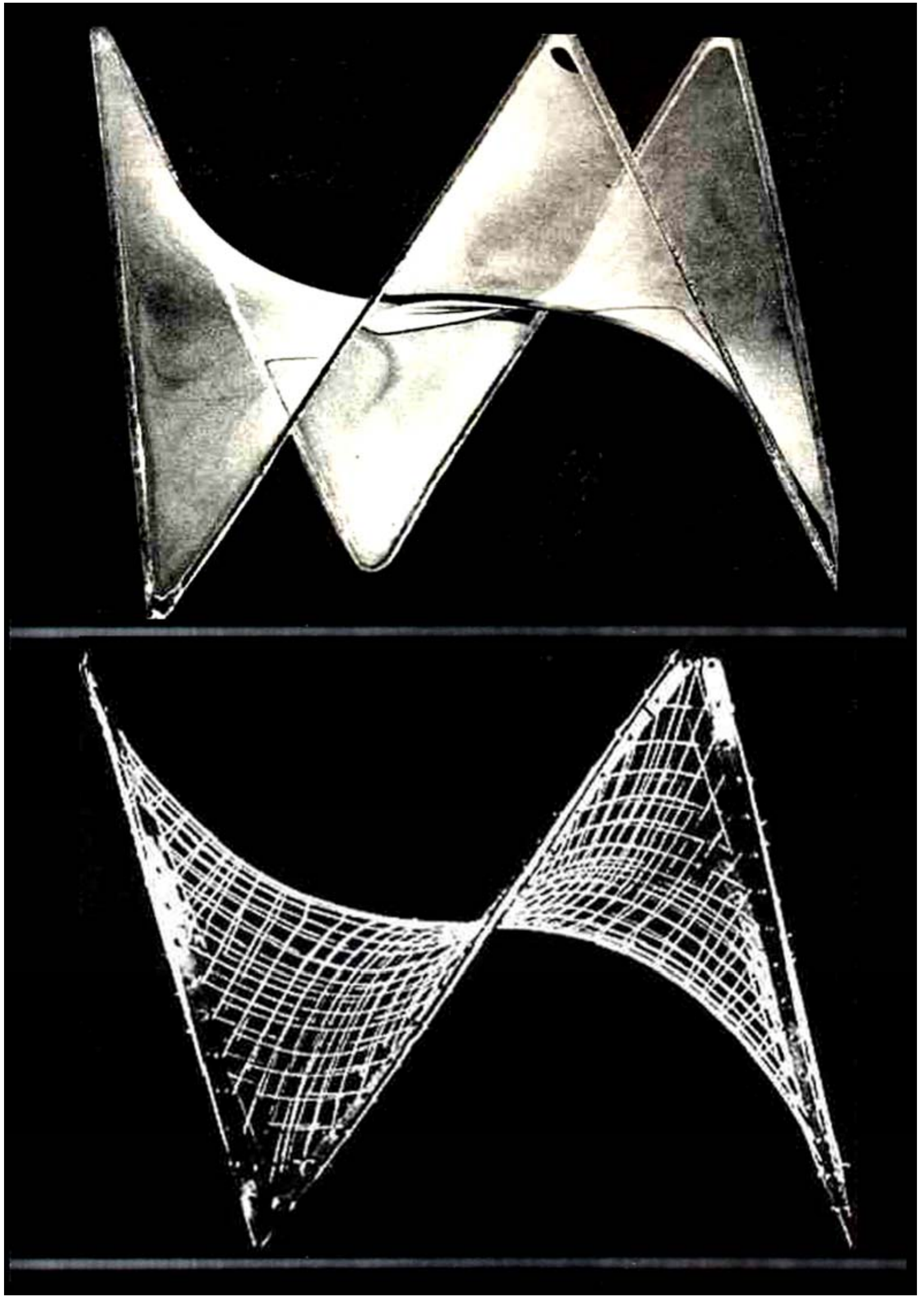
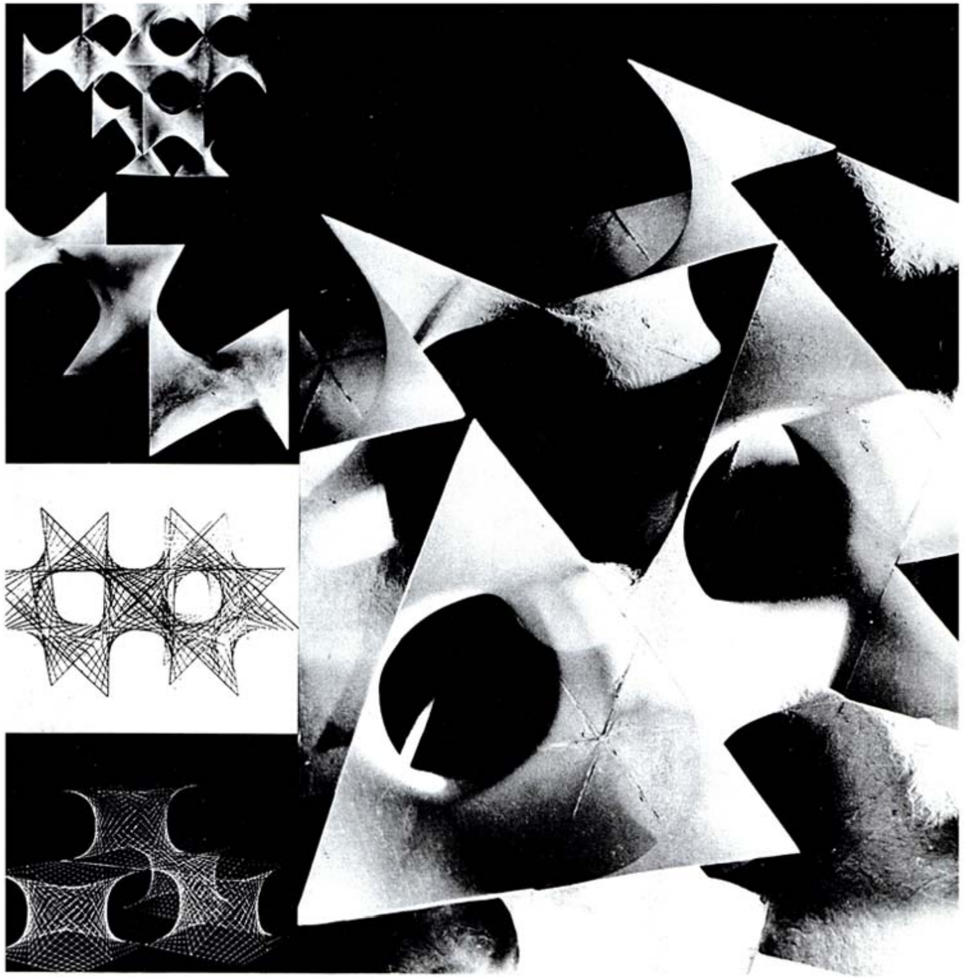
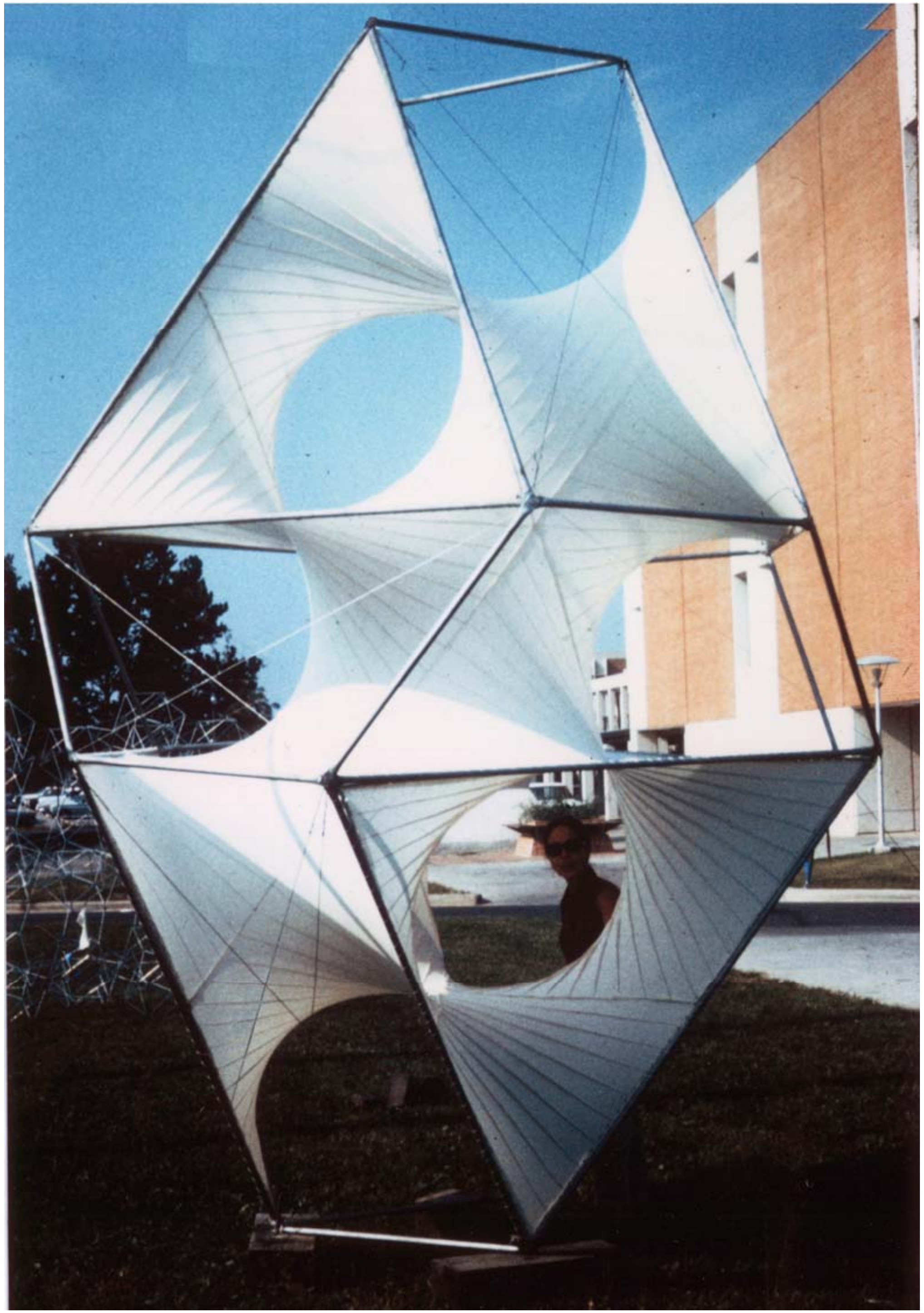
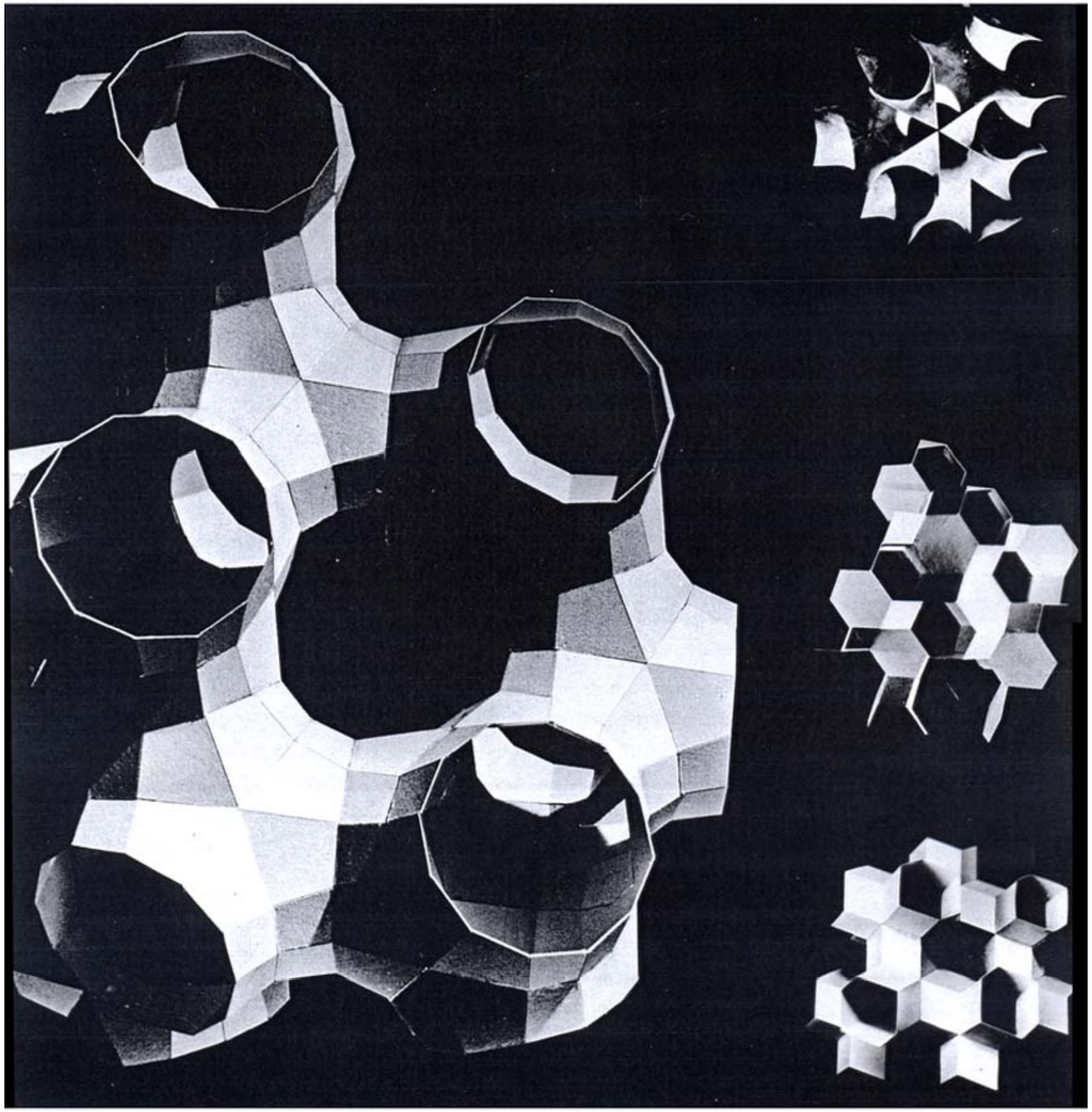


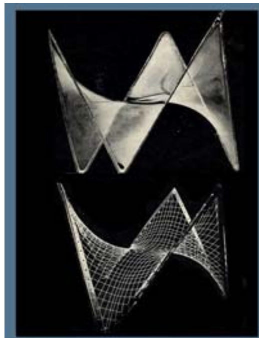


Periodic minimal and polyhyparic surfaces of genus 3 , subdividing space between two cubic lattices into two identical subspaces.

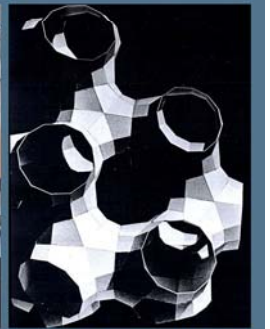




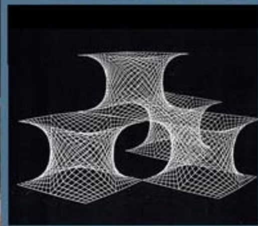
Periodic, uniform and non-uniform infinite sponge polyhedra related to a minimal (hyperbolic) surface of g =3 which subdivides space between two diamond lattices.

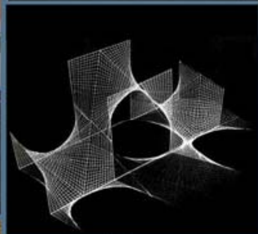


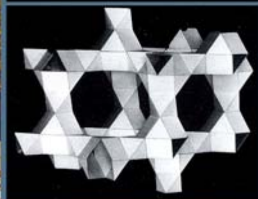
HEXAHYP-7 SHELL STRUCTURE
1969 - HAIFA ISRAEL
AQUAVILLE - PARIS BIENNALE
1969 - PARIS M.BURT



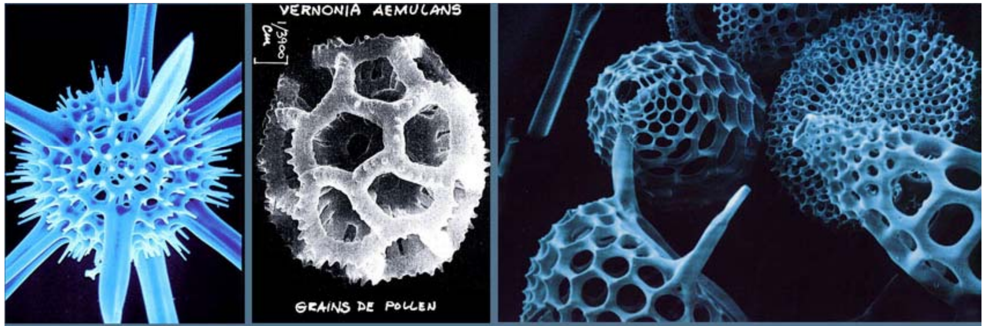
PREVIOUS RESEARCH EFFORTS ON THE THEME OF HYPERBOLIC SURFACES AND INFINITE POLYHEDRA AND APPLICATIONS TO LIGHT-WEIGHT STRUCTURES

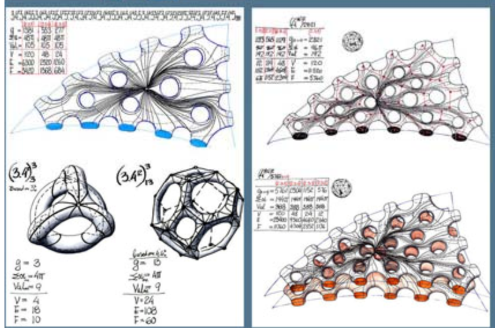




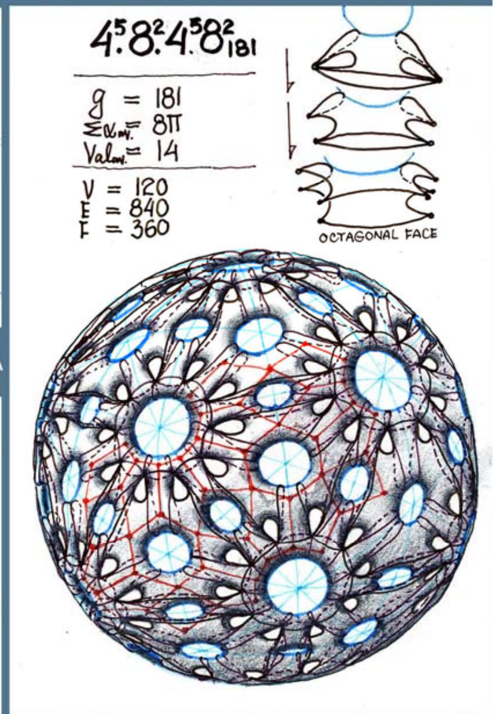


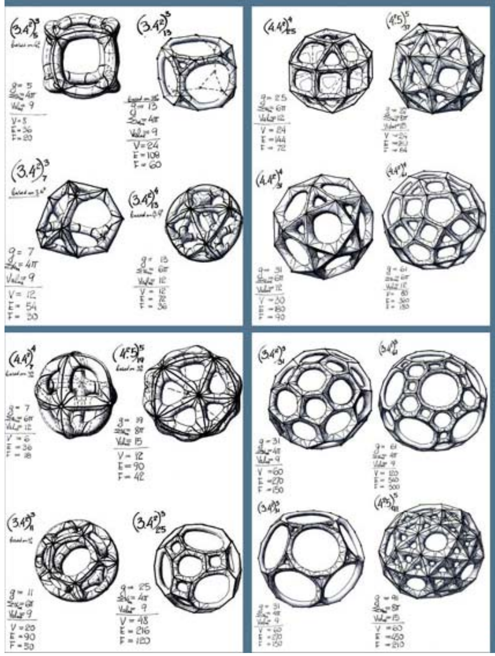
FLORIS - 1980, HYPERBOLIC MEMBRANES - INTERNATIONAL FLOWER EXHIBITION PAVILLIONS, HAIFA - ISRAEL.

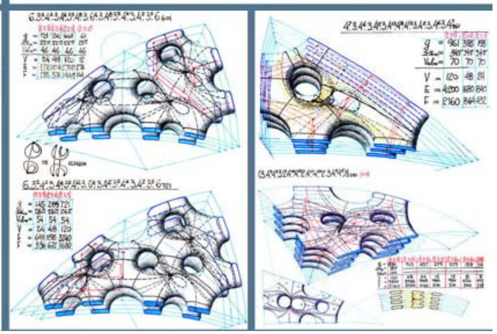


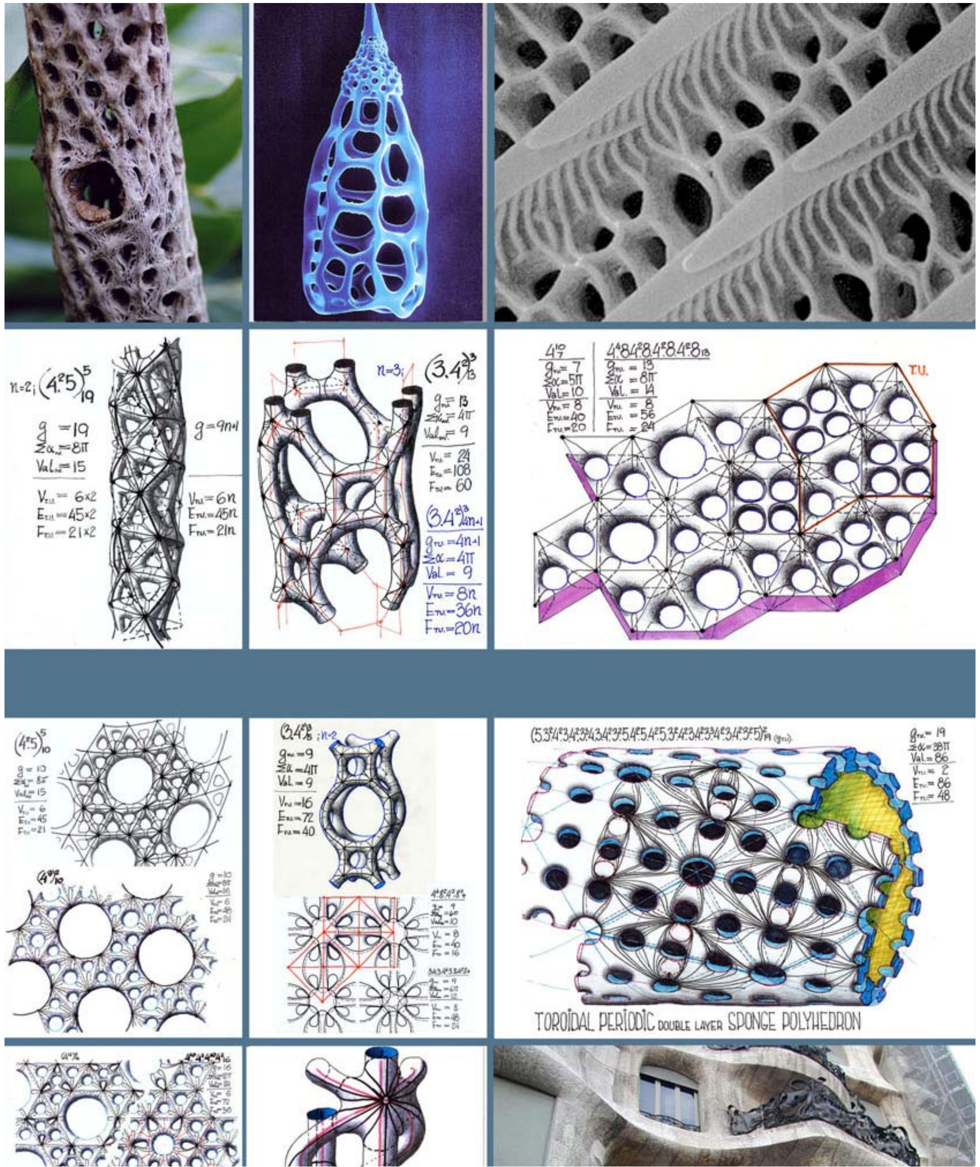


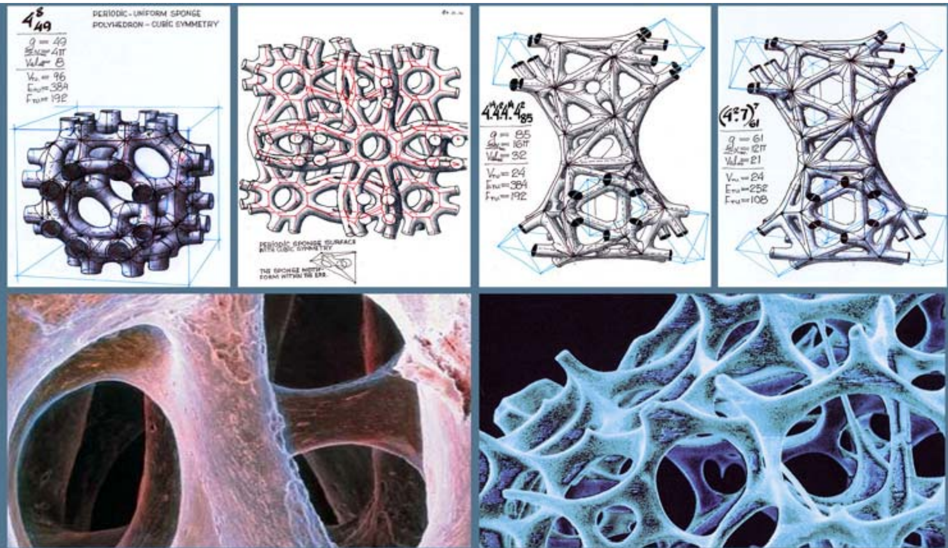
UNIFORM SPHERICAL SPONGE POLYHEDRA



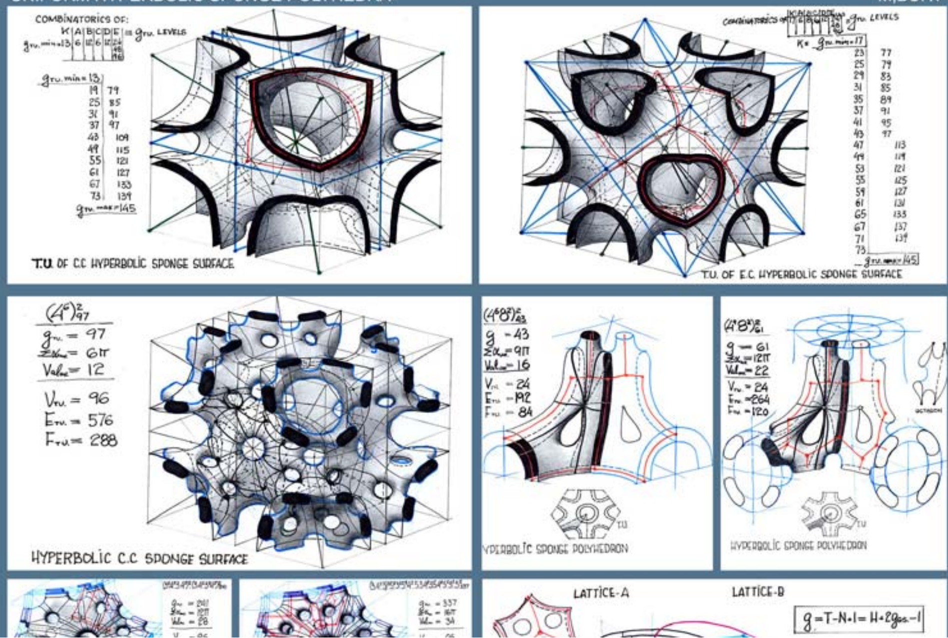





TOROIDAL PERIODIC DOUBLE LAYER SPONGE POLYHEDRON
T.U.



UNIFORM HYPERBOLIC SPONGE POLYHEDRA M,BURT



The subject of uniform 3D space networks is inseparable from that of the hyperbolical sponge surfaces and their tessellations as uniform sponge polyhedra.The statement by A.F, Wells, in his monumental work: **‘Structural Inorganic Chemistry’ (1962) that “The theory of these nets does not appear to be known”, challenged** the author into this recent research effort, resulting, so far, in more-than 250 distinct space networks, including some sets with an infinite number of members, each.

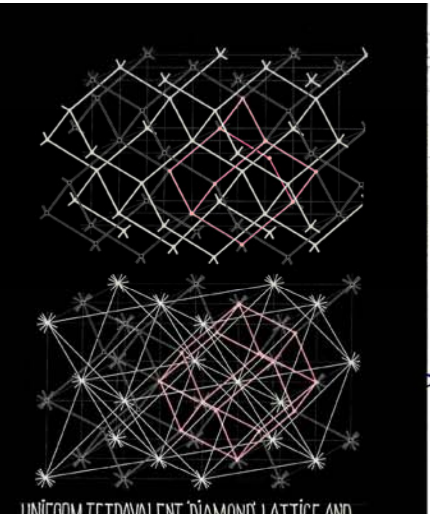
E

networks come in dual pairs. Each network is uniquely determined and is a reciprocal of its dual (complementary) companion.

very dual pair of networks is associated with a continuous perbolical sponge surface which subdivides the space between the , into two complementary sub-spaces.

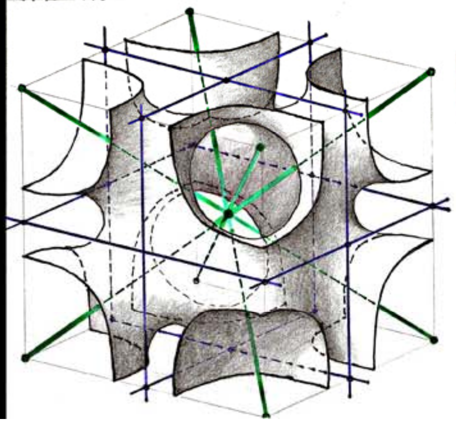
is trinity of the dual pair and the associated-reciprocal onge surface is the most conspicuous, all pervading ometric-topological phenomenon of our 3-D space,

sociated with its order and organization and more than anything else ermines the way we perceive and comprehend its structure

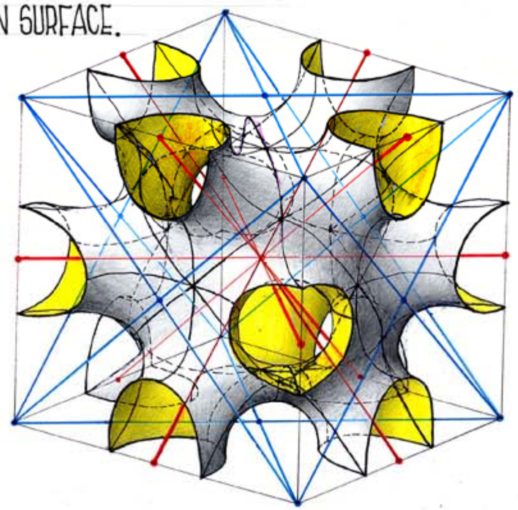


UNIFORM TETRAVALENT 'DIAMOND' LATTICE AND UNIFORM DODECAVALENT 'OCTET' LATTICE AND DUALS.

THE TRINITY: TWO DUAL-COMPLEMENTARY NETWORKS AND THE RECIPROCAL SURFACE-PARTITION, SBDIVIDING THE SPACE BETWEEN THE TWO.

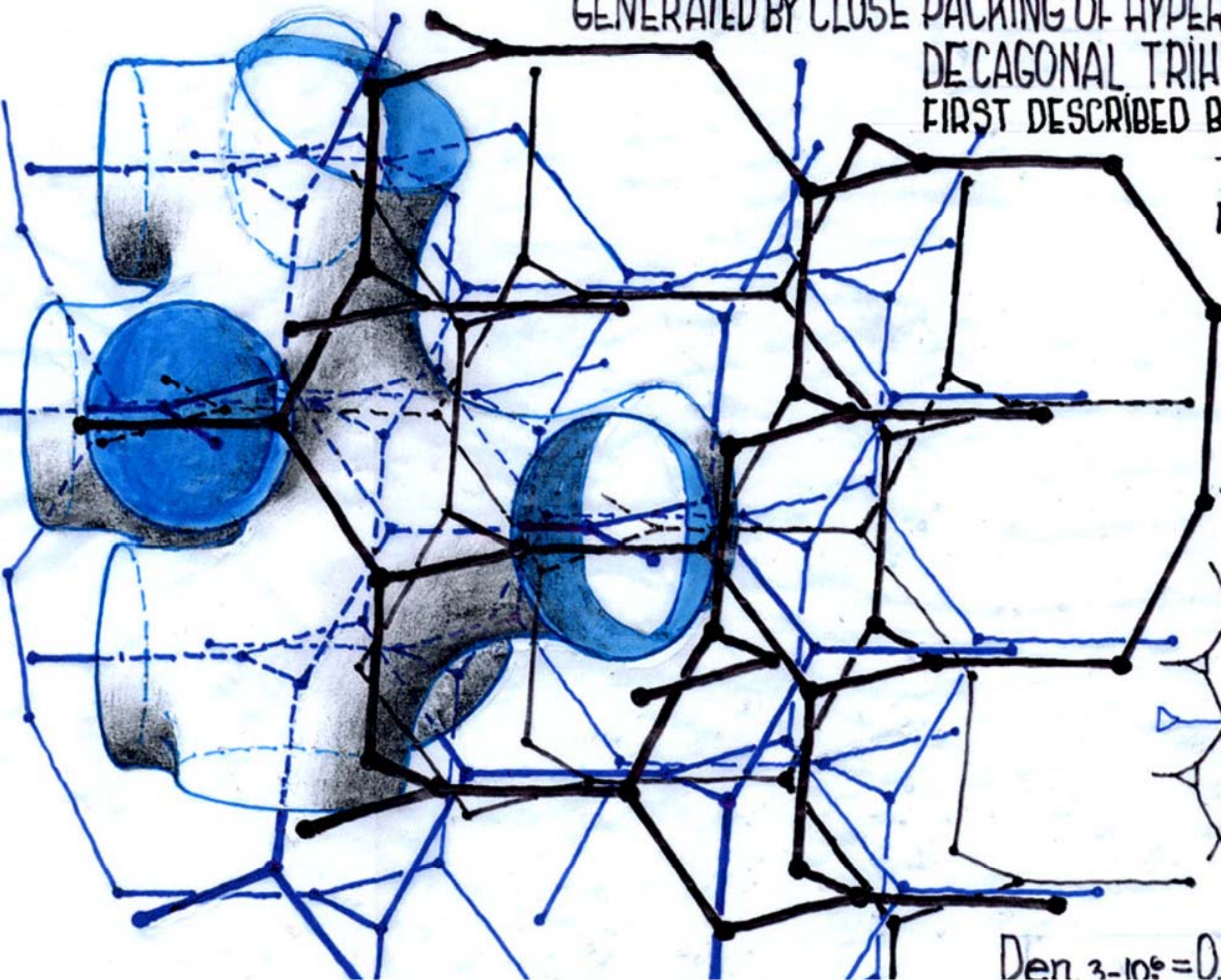


THE TRINITY OF THE DUAL PAIR OF NETWORKS AND THE ASSOCIATED PARTITION SURFACE.

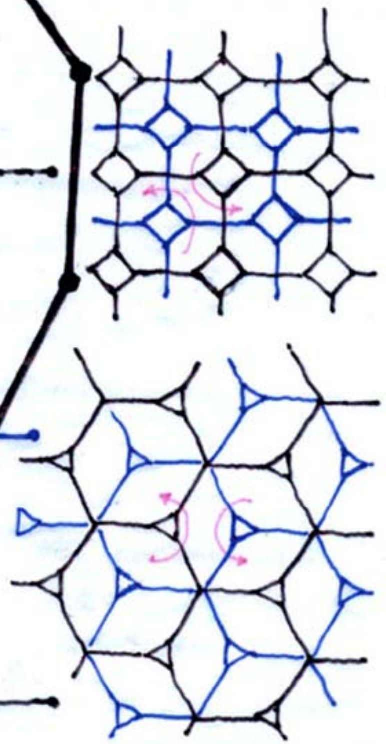


THE UNIFORM TRIVALENT (SELF-DUAL) P3-10_3^6 LATTICE, GENERATED BY CLOSE PACKING OF HYPERBOLIC SADDLE DECAGONAL TRIHEDRA

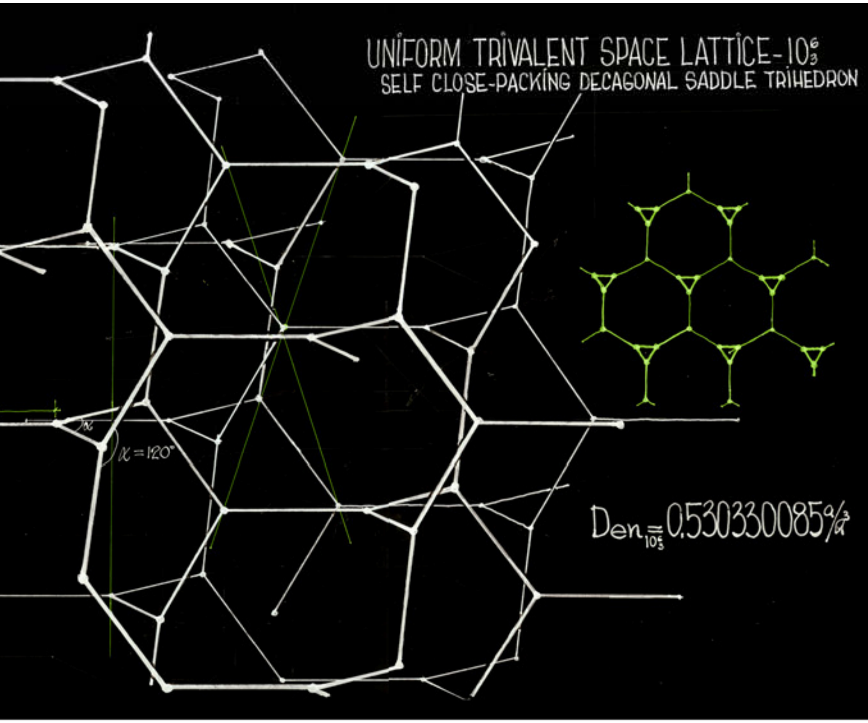
FIRST DESCRIBED BY A.F. WELLS-1962

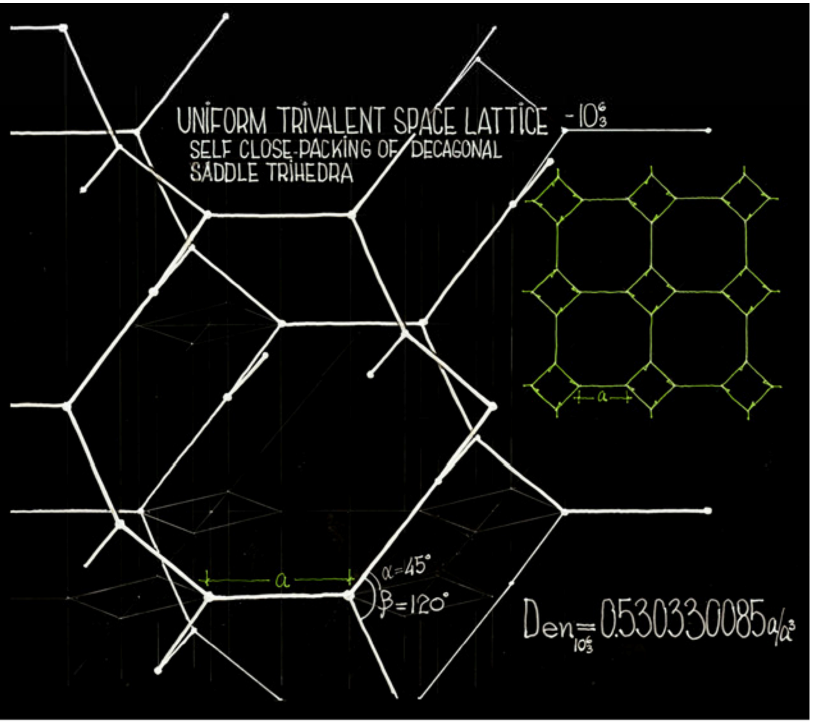


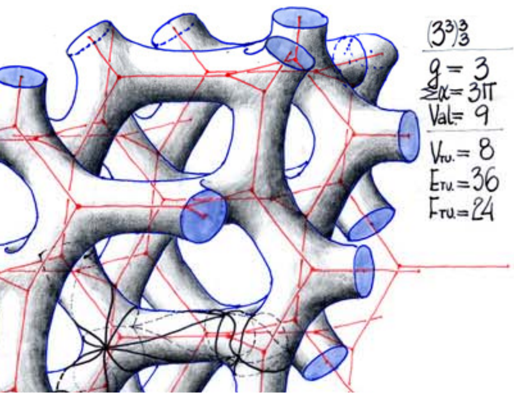
TWO TYPICAL PROJECTIONS

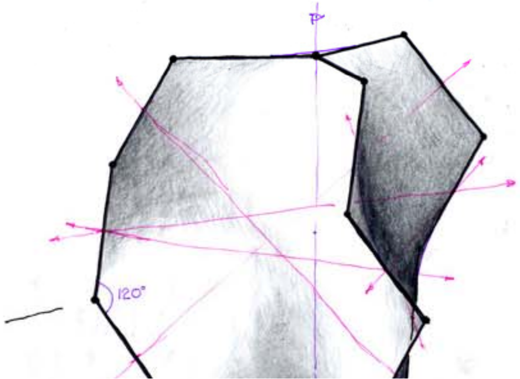


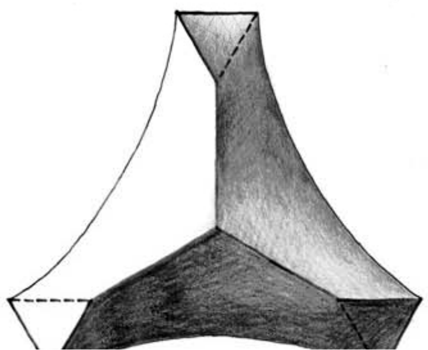
$Den_{3-10_3^6} = 0.530330085\ a/a^3$

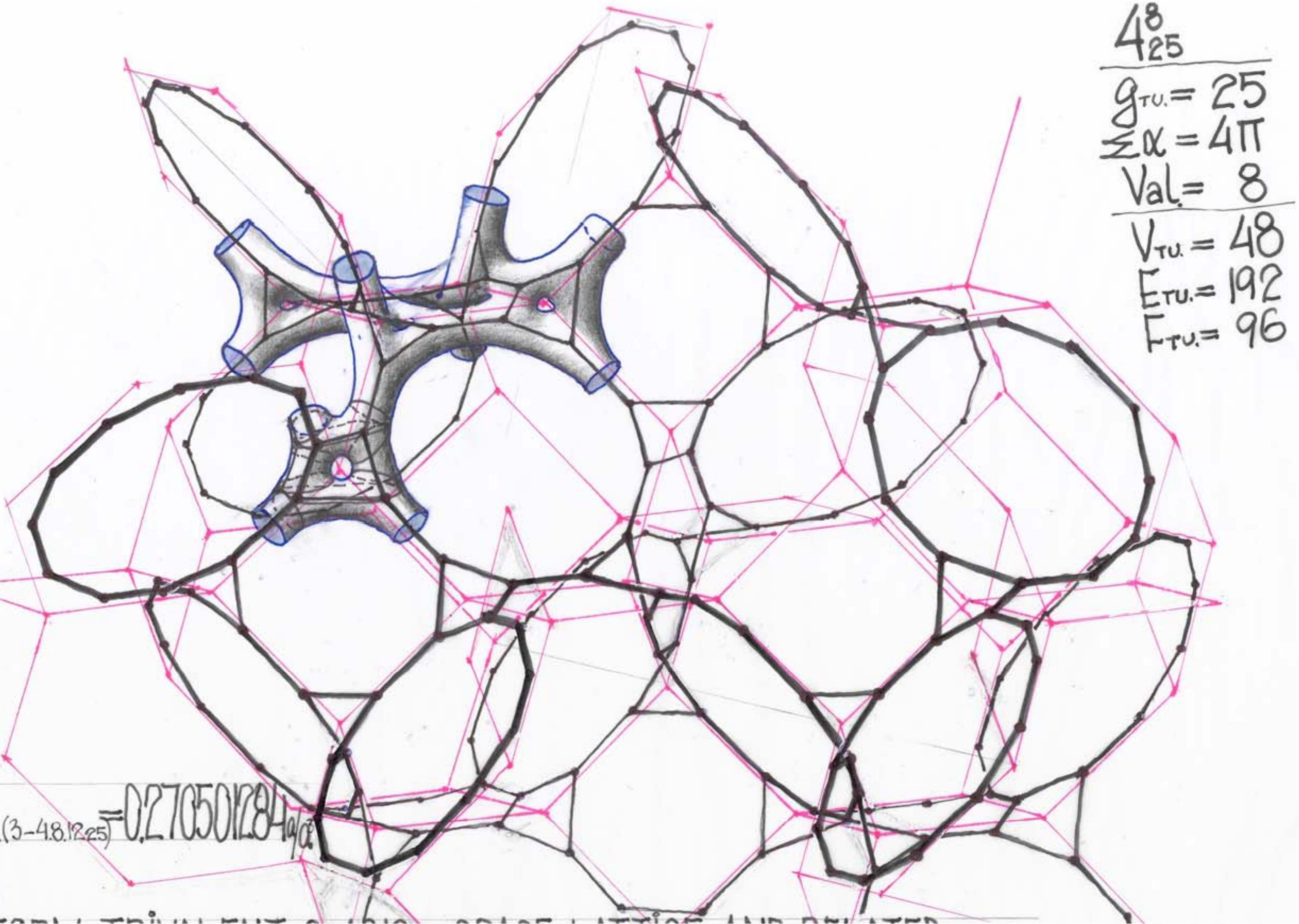

UNIFORM TRIVALENT SPACE LATTICE-10 6 3
SELF CLOSE-PACKING DECAGONAL SADDLE TRIHEDRON
α=120°
Den 10 6 3 = 0.530330085 a/α³


UNIFORM TRIVALENT SPACE LATTICE -10 6 3
SELF CLOSE-PACKING OF DECAGONAL SADDLE TRIHEDRA
a
α=45°
β=120°
Den 10 6 3 = 0.530330085 a/a³


(3³)3 3
g = 3
Σα = 3π
Val = 9
Vru = 8
Eru = 36
Fru = 24


120°





4^{8}_{25}
$g_{TU.} = 25$
$\Sigma\alpha = 4\pi$
Val. = 8
$V_{TU.} = 48$
$E_{TU.} = 192$
$F_{TU.} = 96$
(3-4.8.12.25) = 0.2705012784 a/α

etworks, as polyhedral tessellation configurations, rise to some familiar binding relations:

CT.U. = gT.U. = LT.U. – NT.U.+ 1,

J. & NT.U. stand for edge lines and vertex es, respectively).

Pav. F = 2E = V. Val.av. ;

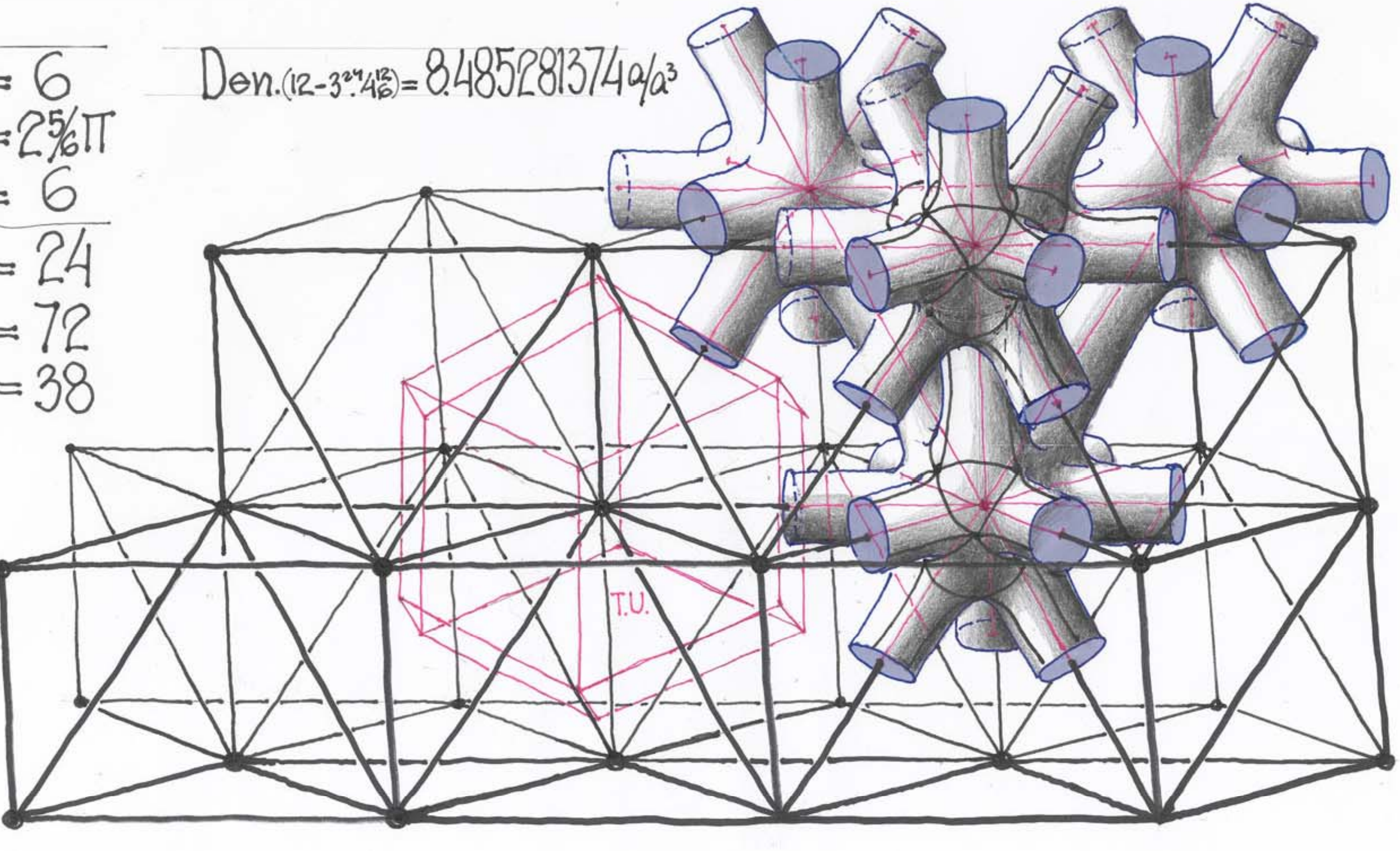
V (2Л –Σαav.) = 4Л(1 – g) - Descarte's theorem

V – E + F = 2(1 – g) **-** Euler's theorem;

= 6
= $2\frac{5}{6}\pi$
= 6

= 24
= 72
= 38

$\text{Den.}_{(12-3^{24}.4^{12}_{6})} = 8.485281374\, a/a^3$



$3-6.8^2_{49}$ Den.$_{(3-6.8^2_{49})}$ = 0.47140452 a/a^3

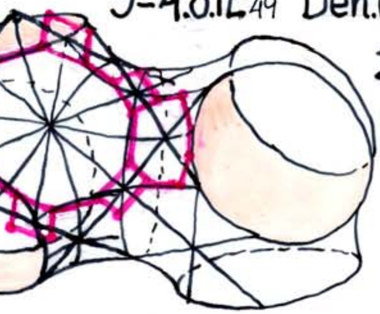
$\Sigma\alpha = 390°$

$Con_{T.U.} \equiv (g_{T.U.}) = 49$

$3-4.8.12_{49}$ Den.$_{(3-4.8.12_{49})}$ = 0.240236673 a/a^3

$\Sigma\alpha = 375°$

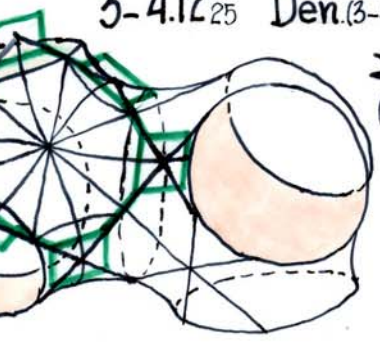
$Con_{T.U.} \equiv (g_{T.U.}) = 49$

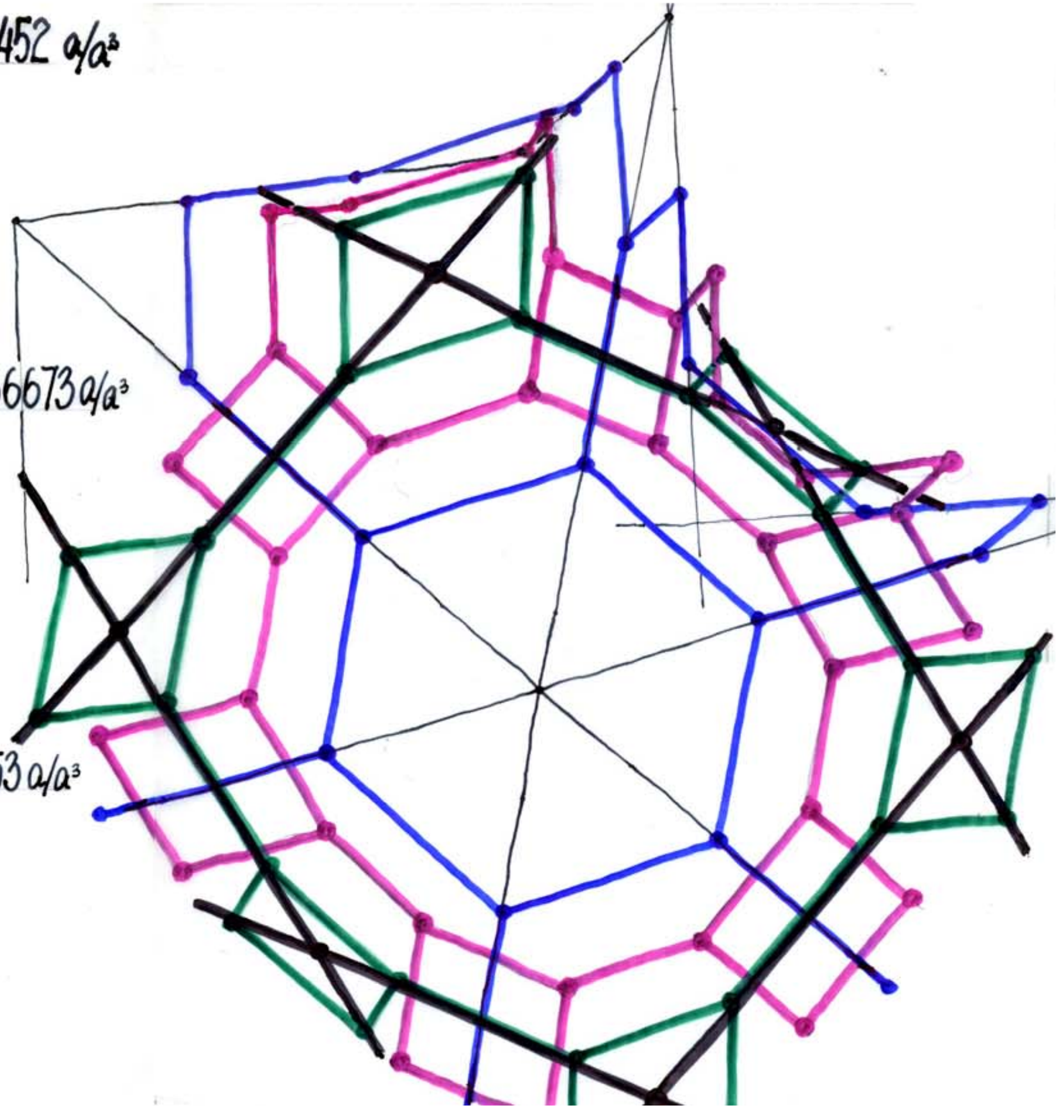


$3-4.12^2_{25}$ Den.$_{(3-4.12^2_{25})}$ = 0.319805153 a/a^3

$\Sigma\alpha = 390°$

$Con_{T.U.} \equiv (g_{T.U.}) = 25$

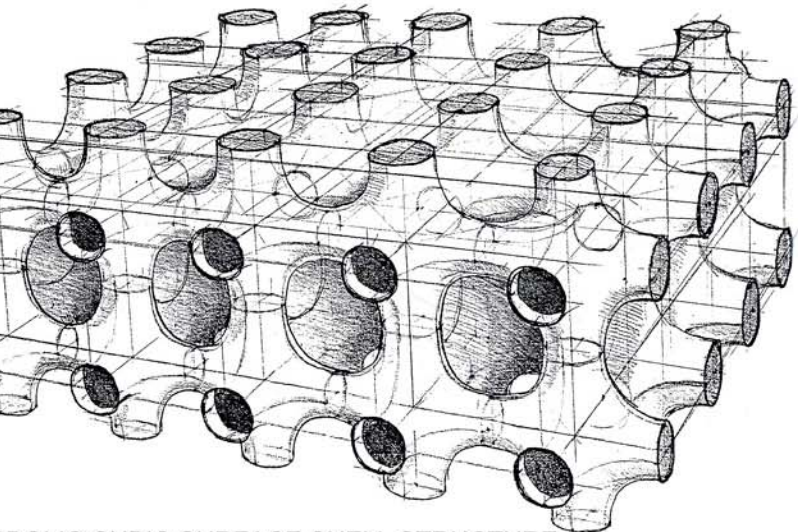




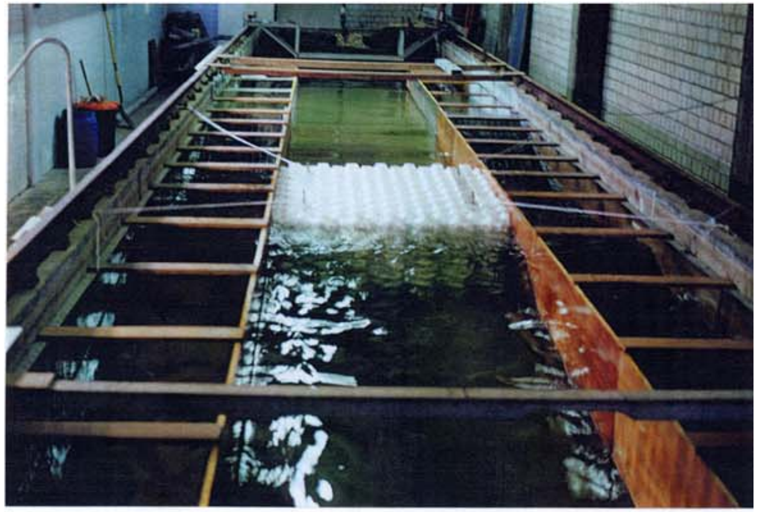
ATEGORIES OF UNIFORM) NETWORKS	Number of networks Found so far
Centroid related networks	5+13
Axis related networks	33
Plane related (Double Layer) networks	57
Centro-Axial-Plane related networks	5
Multi Layer Space networks Poly-vectorial Space networks	} ~160
Translation networks	$\rightarrow \infty$

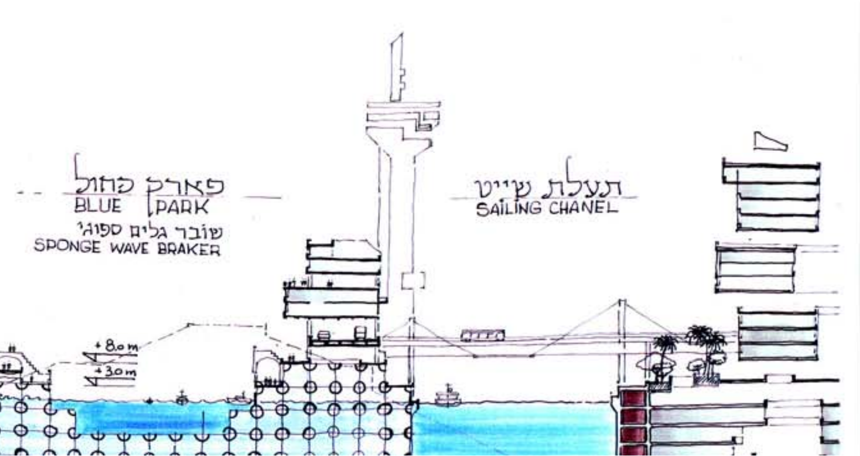
N

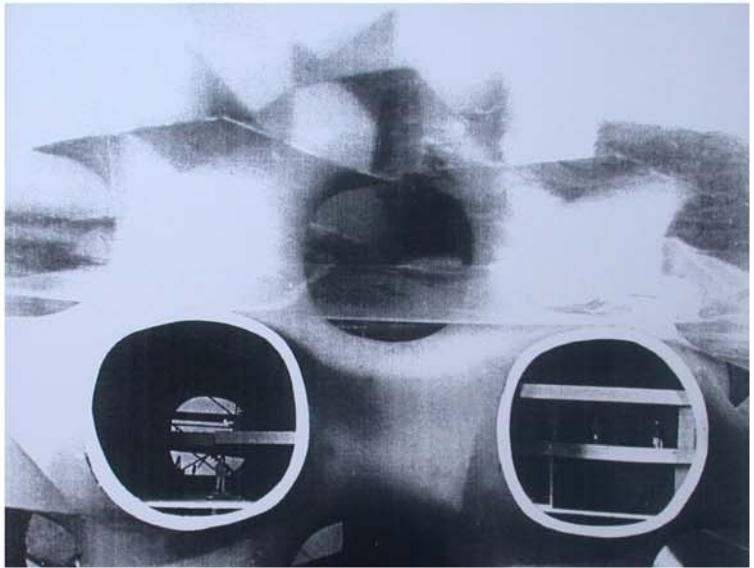
An assumption is formed that we are dealing with probably not more than few **hundreds of uniform space lattices in 3-D space and in view of the valency limiting values and symmetry constraints it seems that an exhaustive systematic search of these configurations is tenable.**

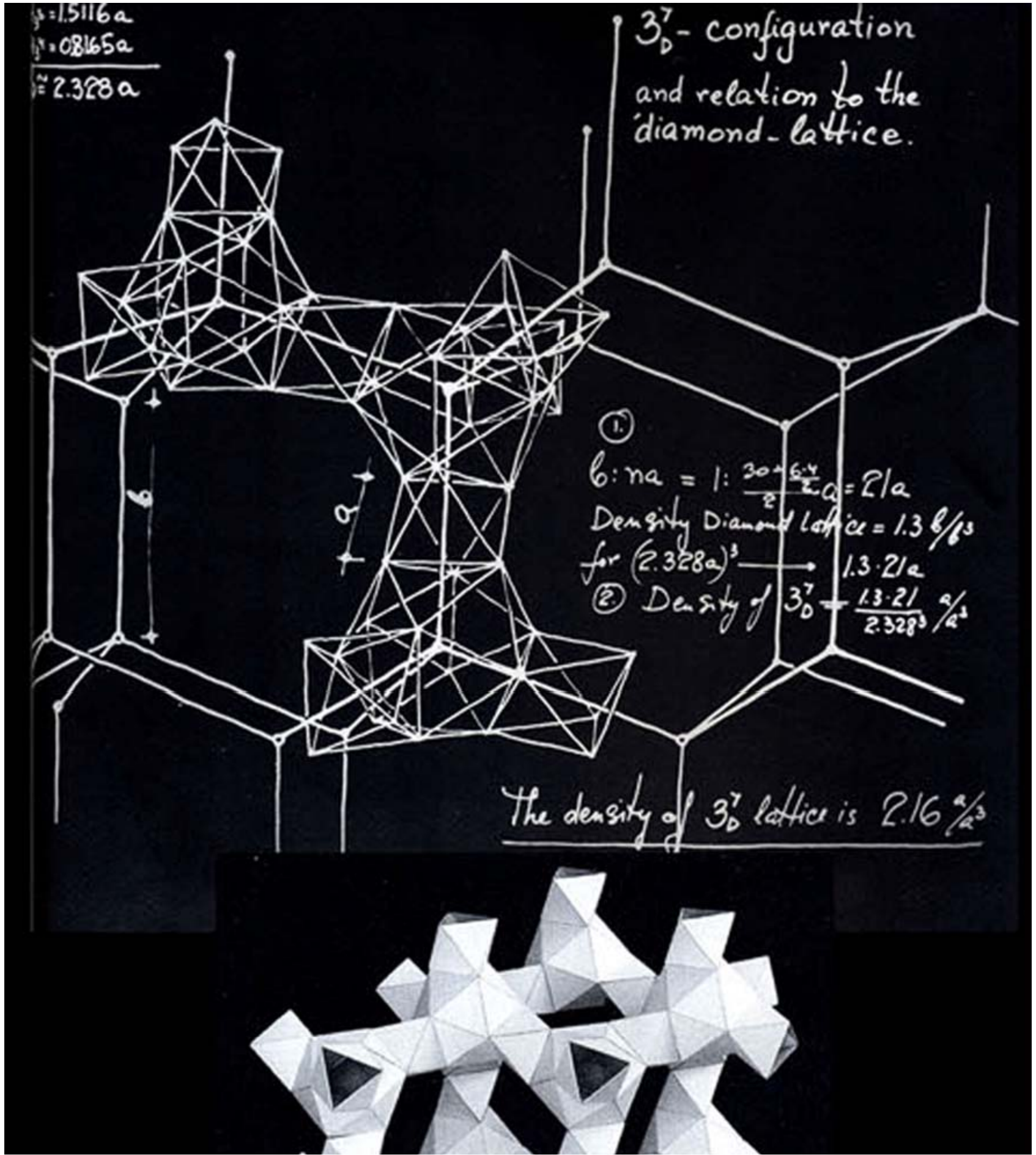


RBOLIC CUBIC SURFACE SHELL STRUCTURE

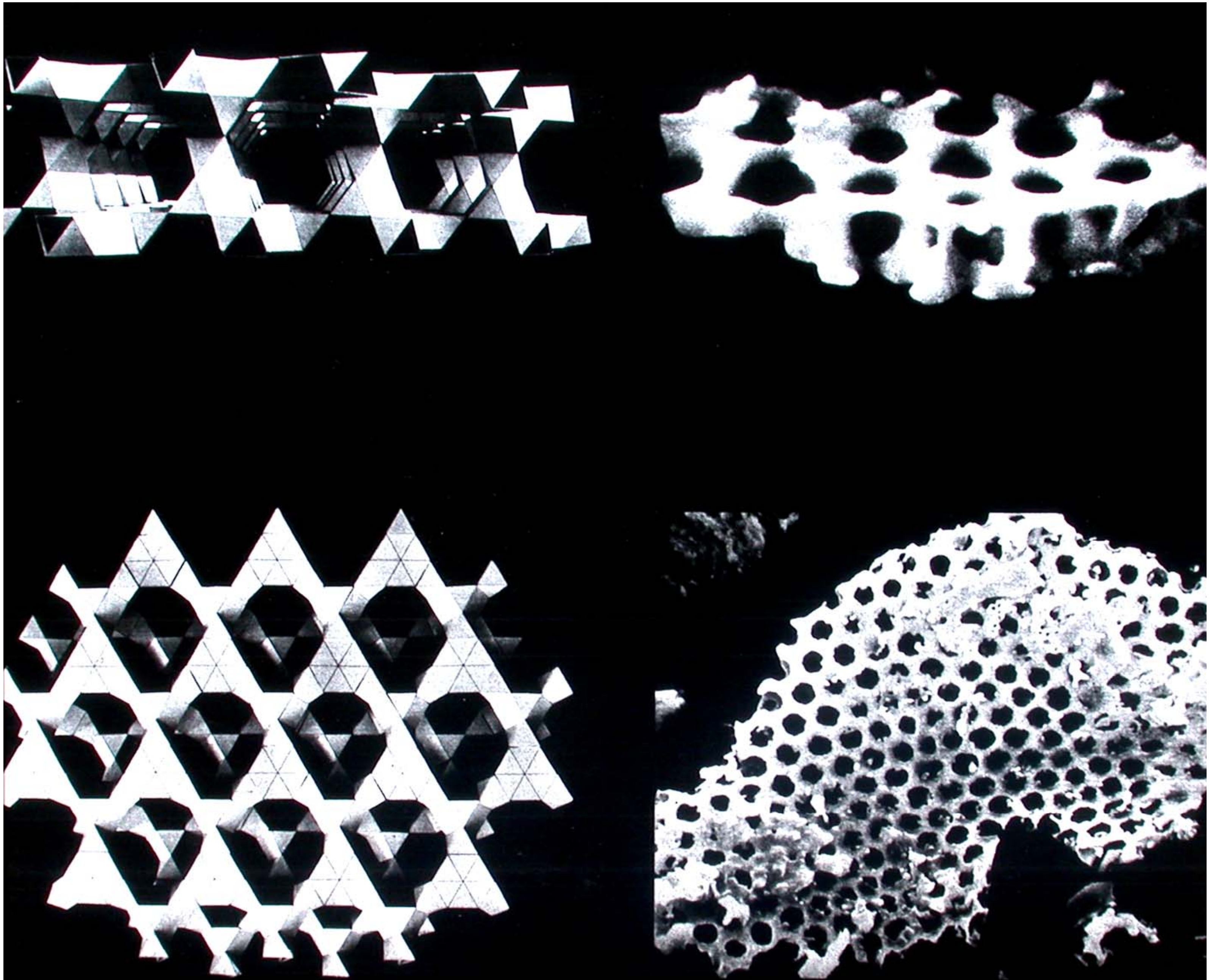




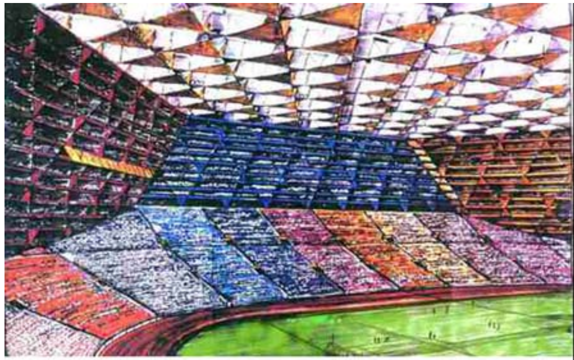


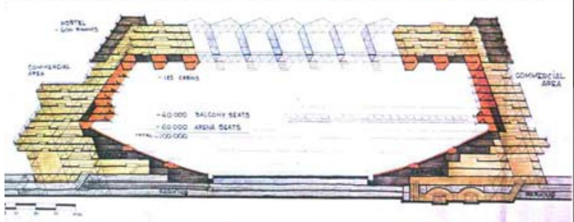


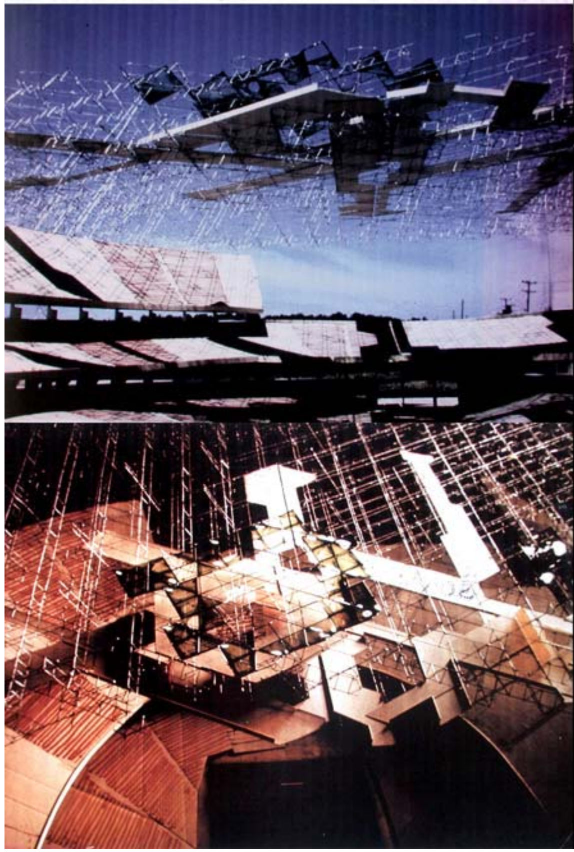
=1.5116 a
= 0.8165 a
= 2.328 a
3^7_D - configuration
and relation to the
'diamond-lattice.
1.
Density Diamond lattice = 1.3
for $(2.328a)^3$ → 1.3 · 21 a
2. Density of 3^7_D = $\frac{1.3 \cdot 21}{2.328^3}$
The density of 3^7_D lattice is 2.16

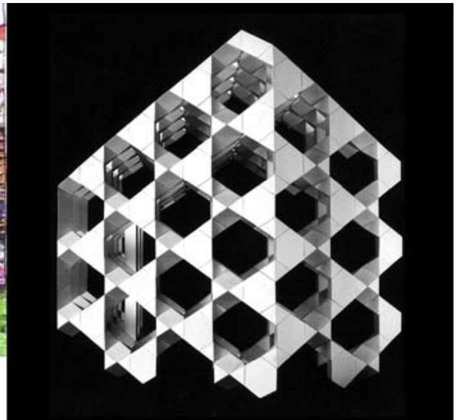


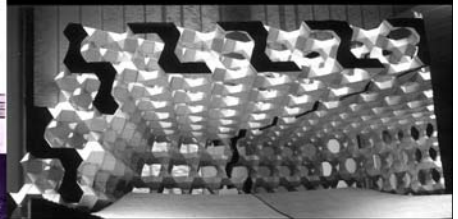
$3^3.4.3.4_3$ - I.P.L PLATE

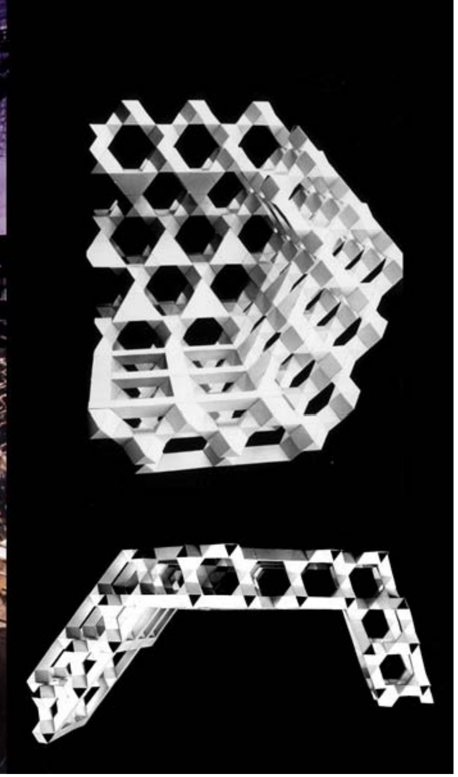



HOSTEL
COMMERCIAL AREA
COMMERCIAL AREA
+40.000 BALCONY SEATS
+60.000 ARENA SEATS
TOTAL 100.000









MERO D-8700 Würzburg
Steinachstraße 5
Telefon (0931) 41030

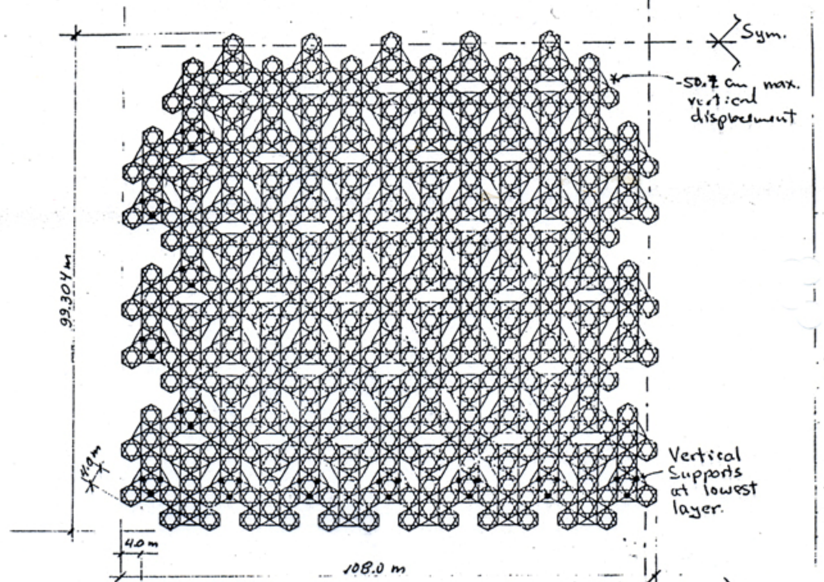
Blatt: I.P.L 200 m x 200 m

ST.B.-Nr.:

IPL- Plate. 200 x 200 m
:effective structural height = 5 modules = 16.3259 m.

Model for structural analysis ~ 1/4 of plate

Sym.



Significant quantities for structural analysis model (~1/4 system):

Number of nodes: 2145
Number of members: 7023, 2 lengths: 4.0 m for modules sides and
Material properties: of steel $4 \times \sqrt{2} = 5.6508$ m to diagonalise cuboctahedron-sq
Imposed load: uniform: 1.0 kN/m² (= 100 kg/m²), dead weight of structure include
Supports: vertical, at the sides of plate (+ symmetry constraints at sym. axes).
Maximum vertical displacement = - 50.7 cm at the middle of the plate,
e.i. ~ $\frac{1}{400}$ of span.
(Additional horizontal displacements up to 10.0 cm)
Maximum forces: - 167 Tons (compression), 130 Tons (Tension)

Dimensioning results (for the whole plate with sides 200 m x 200 m = 40 000 m²)

Tubes: 28 092, total length: 119 009.3 m weight = 1343.41 T
Nodes: 8580, weight = 124.04 T
Σ 1467.45 T → 1500 Tons → 37.5 kg/m²

Tube size distribution

ϕ 88.9 x 3.4	57%
108. x 3.6	17.2%
127. x 3.6	8.8%
139.7 x 4.0	6. %
159 x 6.3	8. %
168.3 x 8.0	2. %
219.1 x 6.3/ .. 8.8/10/12.5/16.	~ 1 %

Node size distribution

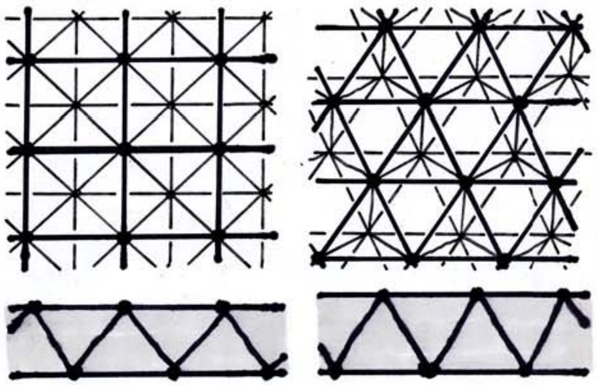
out / flats	
ϕ 110/ 58	22.5 %
132/ 118	20.0 %
155/ 138	32. %
180/ 158	10.3 %
198/ 178	8.2 %
228/ 208	4. %
250/ 228; 300/ 268; 350/ 288	3. %

Notes: With w = 2.0 kN/m² tubes weight = 1443.3 T + nodes weight = 158.24 T = 1600 T → 40 kg/m²
With w = 1.0 kN/m² tubes weight = 1175.4 T + nodes weight = 124.13 T = 1300 T → 37.5 kg
but starting dimensioning with ϕ 60.3 x 2.9 and ϕ 76 ...

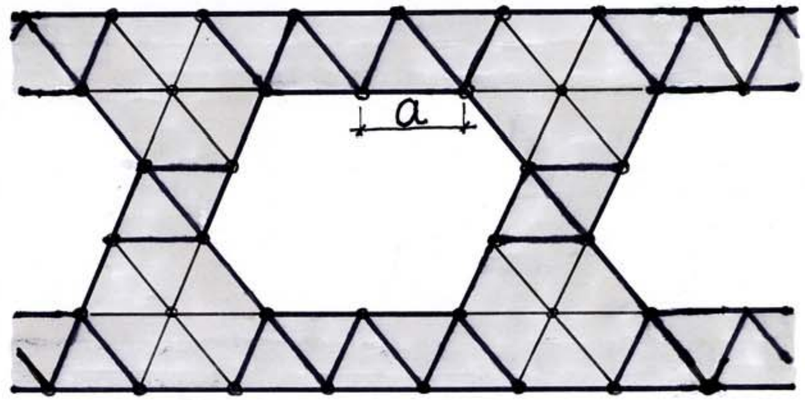
COMPARATIVE MORPHOLOGICAL CHARACTERISTICS OF THE OCTET DOUBLE-LAYER AND THE INFINITE SPONGE POLYHEDRAL LATTICE SPACE TRUSS STRUCTURES.

"STATE OF THE ART SPACE TRUSS SOLUTIONS WITH A SPAN-RECORD OF 108m (OSAKA EXPO, 1969-1970), AND ~200kg/sqm OF DEAD WEIGHT.
THE OCTATRUSS SPACE FRAME

OCTET-LATICE DERIVATIVES DOUBLE LAYERED

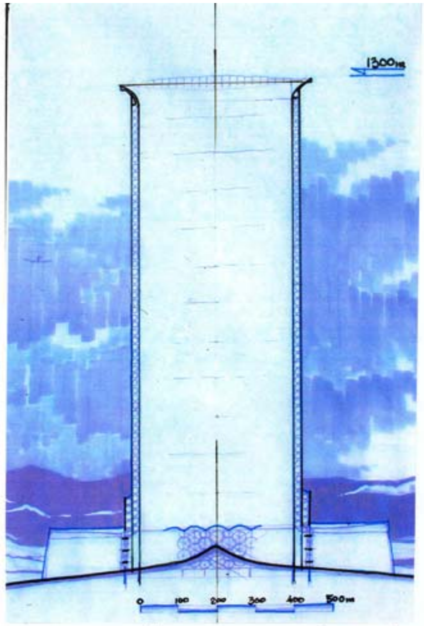


INFINITE POLYHEDRAL SPONGE LATTICE SIX LAYERED-$3^3 4.3.4_3$ I.P.L TRUSS

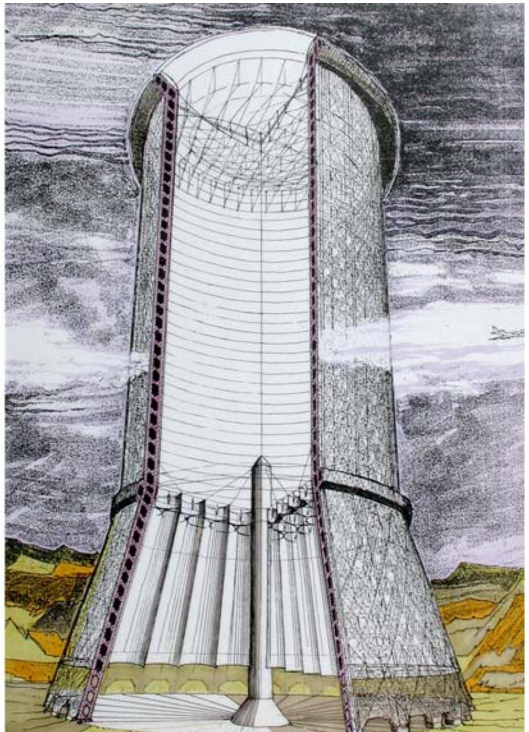


	Octet-lattice derivatives (double layered)		Infinite polyhedral sponge lattice
•STRUCTURAL DEPTH	h = 0.7071 a	h = 0.8165a	H = 4.0825 a
•EDGE VALENCY	Val. = 8	Val. = 9	Val. = 6
•SPATIAL DENSITY	D_S = 8.4852 a/a³		D_S = 1.5922 a/a³
•PROJECTED DENSITY	D_P = 8.000 a/a²	D_P = 10.392 a/a²	D_P = 8.6603 a/a²
•STRUT MATERIAL IN THE NEUTRAL ZONE	50%	33%	20%
•STRUT-SPAN RATIO	l = 13÷16a	l = 15÷18a	L = 50÷60a

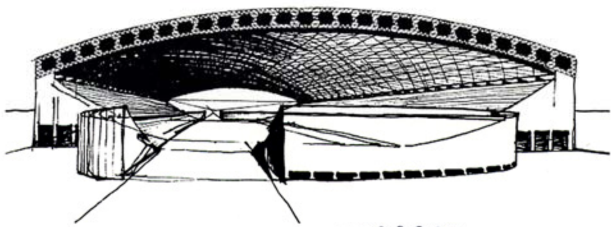
THE I.P.L-$3^3 4.3.4_3$ SPACE FRAME, CAPABLE OF SOLVING FREE SPANS OF UP TO 200÷300m WITHIN THE PREVAILING STRUCTURAL-ECONOMICAL CONSTRAINTS.



AERO-ELECTRIC POWER STATION, UTILIZING HOT-DRY DESERT AIR (BY DAN ZASLAVSKI). WITH I.P.L STRUCTURE OF 1300m HIGH.



AERO-ELECTRIC POWER STATION I.P.L TOWER



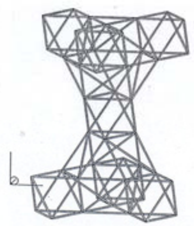
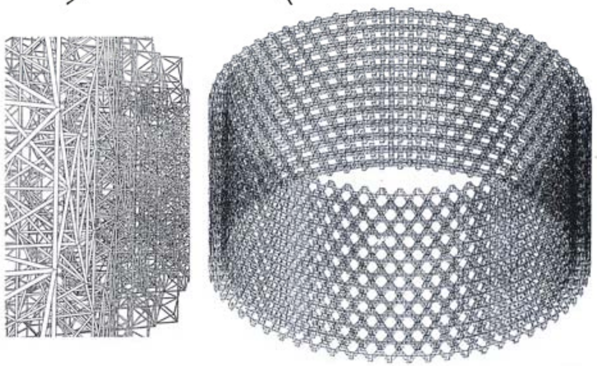


Figure 2. Basic repetitive "column" substructure



CYLINDRIC $3^3.4.3.4_3$- I.P.L SPACE TRUSS

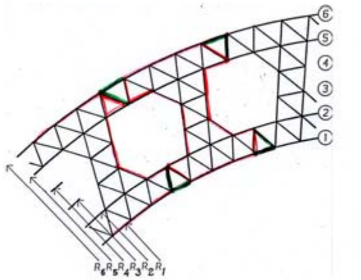
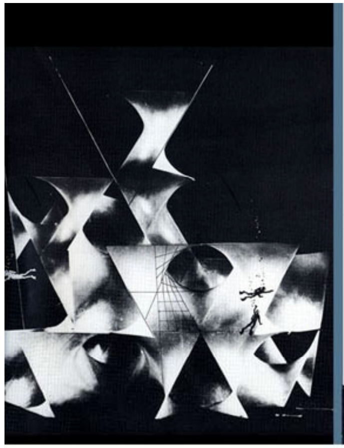
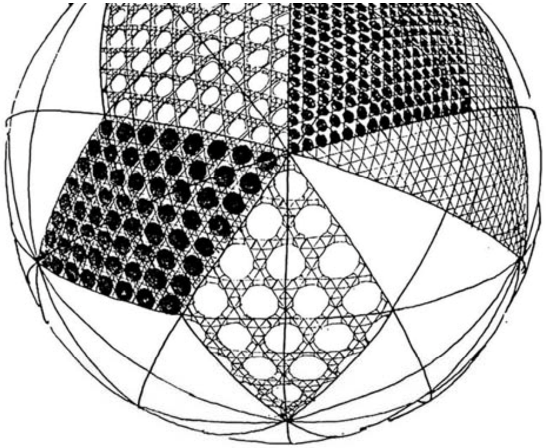
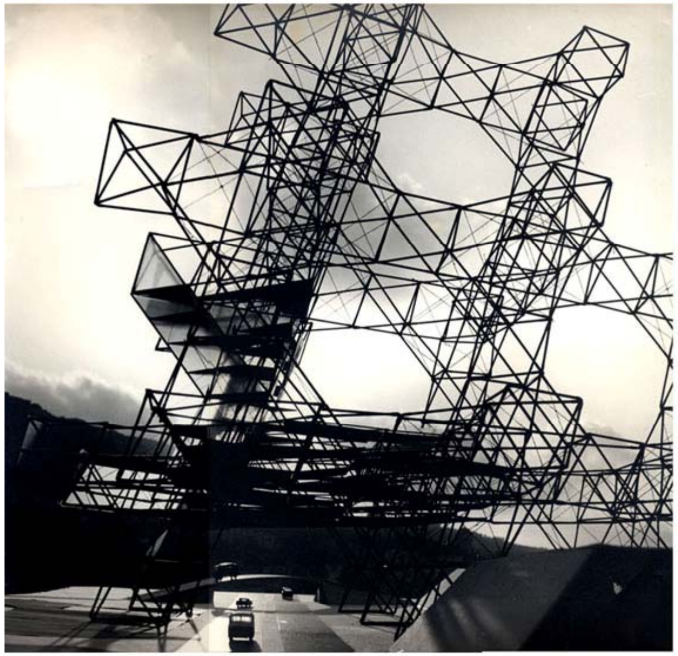


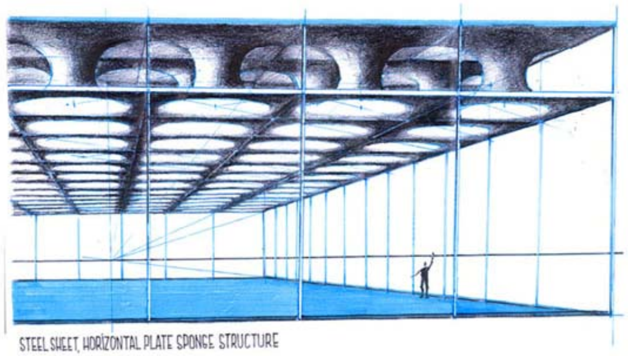
Figure 3. Six layered cylindrical IPL shell showing the different radii and the resulting variations in the typical rod lengths; $R_6 > R_1$



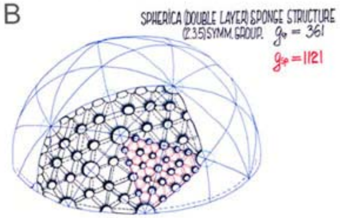




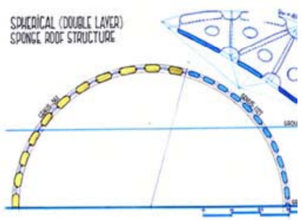
A

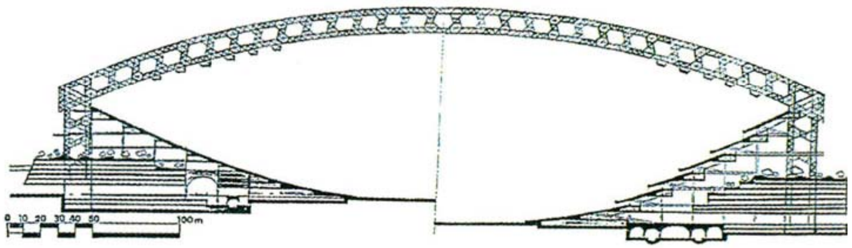

STEEL SHEET, HORIZONTAL PLATE SPONGE STRUCTURE

B


SPHERICA (DOUBLE LAYER) SPONGE STRUCTURE
(2,3,5) SYMM. GROUP.

C

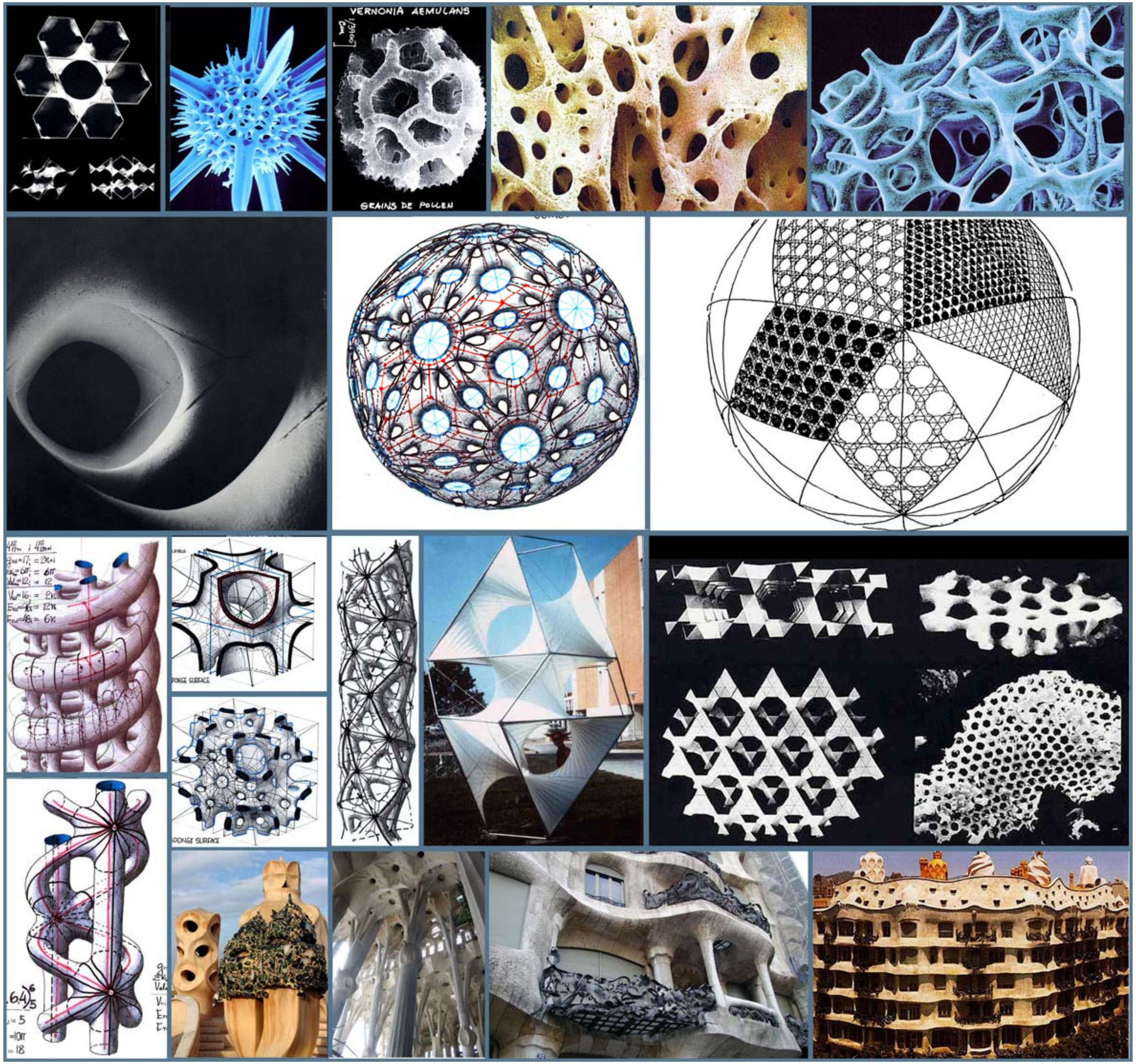

SPHERICAL (DOUBLE LAYER)
SPONGE ROOF STRUCTURE


0 10 20 30 40 50 100m

3D networks and the associated hyperbolical sponge surfaces seem to pose a critical aspect in all 'material sciences' and as an extension of graph-theory, dealing geometrically with any plurality that may exist, of focal entities and their inter-relations.

After investing in the systematic research of the topic, the author claims enumerating, categorizing and graphically describing, ***so far,*** *about 270 uniform 3D space networks and related hyperbolical sponge surface configurations.*

The effort is meant to support an evolution of new imagery which might influence scientific exploration and inspire art, architecture and innovative space structures.


VERNONIA AEMULANS
GRAINS DE POLLEN