UNIFORM NETWORKS, SPONGE SURFACES AND UNIFORM SPONGE POLYHEDRA IN 3-D SPACE

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The diversity of shapes and forms which meets the eye is overwhelming. **They shape our environment: physical, mental, intellectual**. Theirs is a dynamic milieu; time induced transformation, flowing with the change of light, with the relative movement of the eye, with physical and biological transformation and the evolutionary development of the perceiving mind.

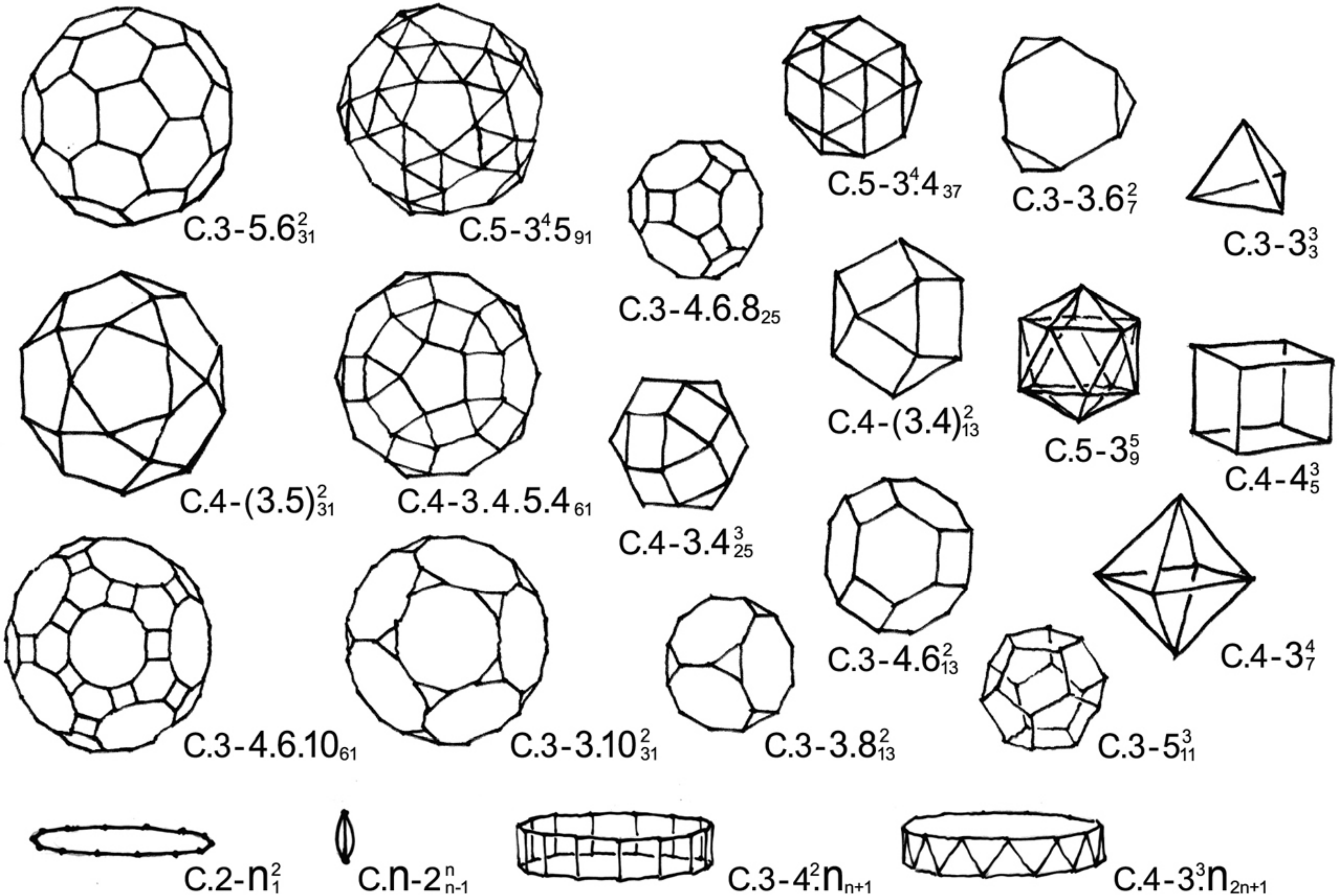
"**Our study of natural form**", the essence of morphology, "**is part of that wider science of form which deals with the forms assumed by nature** under all aspects and conditions, and in a still wider sense, with **forms which are theoretically imaginable"**…..(On Growth and Form – D'Arcy Thompson), "Theoretically" to imply that we are dealing with causal- rational forms.

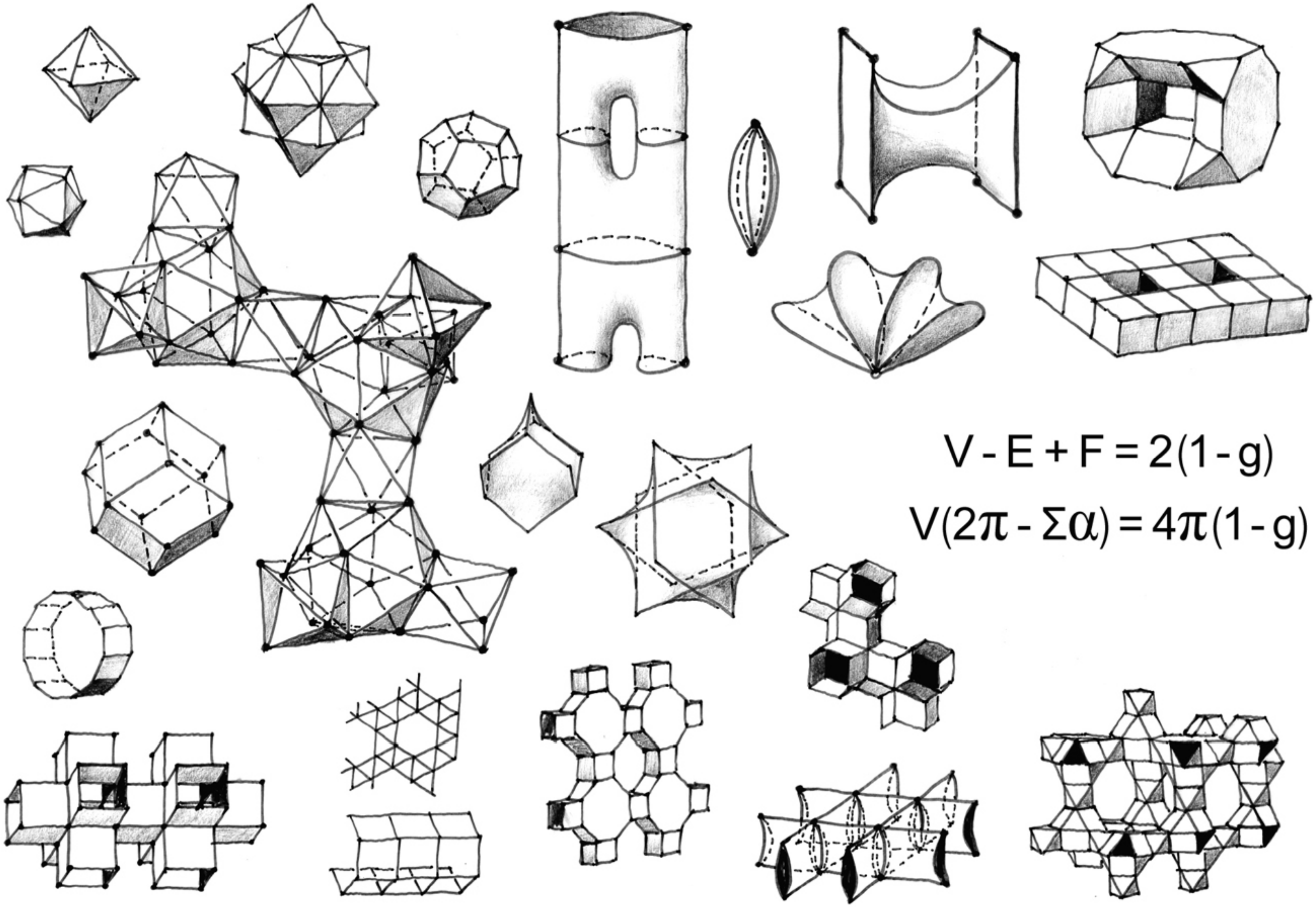
"It is the business of logic to invent purely artificial structures of elements and relations. Sometimes one of these structures is close enough to a real situation to be allowed to represent it. And then, because the logic is so tightly drawn, we gain insight into the reality which was previously withheld from us".

A particular interest should be focused on two principal categories of structures: Structures which are shaped like polyhedral solids or continuous enveloping tessellated surfaces, subdividing the entire space into two complementary sub-spaces. Conspicuous are those relating to sponge-like labyrintian, polyhedral space dividing surfaces

The second category of structures, populating 3D space, describes **polytopal interrelating and interconnected arrays of** (sometimes) energized **point-wise entities which could be represented as diagrams with a network or space lattice characteristics**.
Diagrams of this kind may represent the structure of almost any abstract or physical plurality that may exist, in the world of phenomena of the biological-physical-material domain, on every possible scale, from the nano-molecular to the cosmological.

It is these polyhedral and network lattice structures and their extended derivatives which shape our physical-natural or artificial man-conceived environment and provide for our mental pictures of its architecture


C.3-5.6$^2_{31}$
C.5-3^4.5$_{91}$
C.3-4.6.8$_{25}$
C.5-3^4.4$_{37}$
C.3-3.6$^2_7$
C.3-3$^3_3$
C.4-(3.5)$^2_{31}$
C.4-3.4.5.4$_{61}$
C.4-(3.4)$^2_{13}$
C.5-3$^5_9$
C.4-4$^3_5$
C.4-3.4$^3_{25}$
C.3-4.6$^2_{13}$
C.4-3$^4_7$
C.3-4.6.10$_{61}$
C.3-3.10$^2_{31}$
C.3-3.8$^2_{13}$
C.3-5$^3_{11}$
C.2-n$^2_1$
C.n-2$^n_{n-1}$
C.3-4^2.n$_{n+1}$
C.4-3^3.n$_{2n+1}$



V - E + F = 2(1 - g)
V(2π - Σα) = 4π(1 - g)

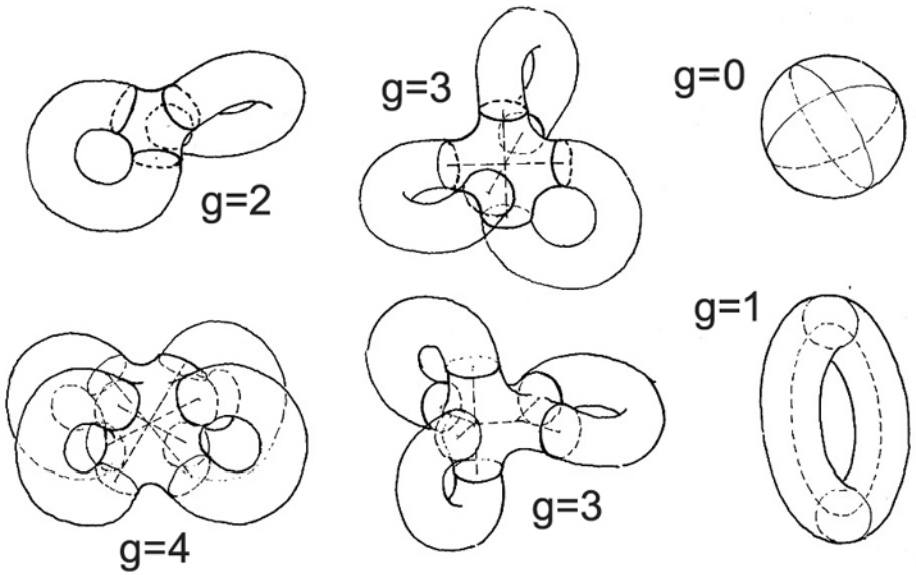
Definitions

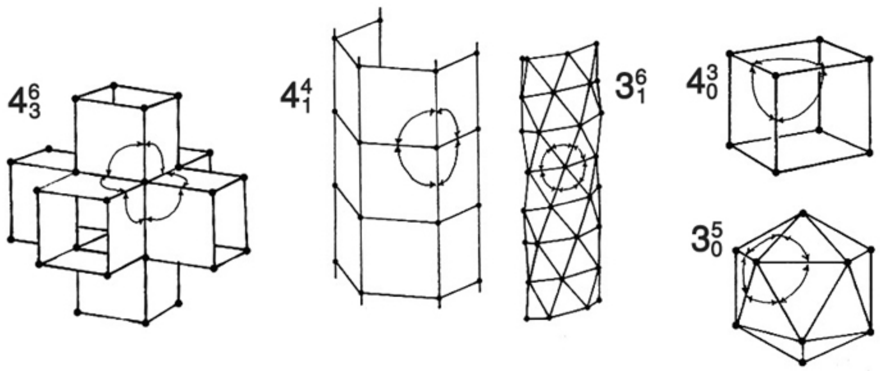
A POLYGONAL REGION of order n, for n≥1, is a point set, topologically equivalent to a circular disc with a boundary divided into n edges by set of n vertices. It may have curved edges, maybe non-planar and two edges of the same region may be matched (Stewart) .

Polyhedral map drawing on a sponge surface must lead to polygonal faces which may constitute, under a suitable topological transformation, a plane polygonal region.

A POLYHEDRON - P is a connected, unbounded 2-dimensional manifold, formed by a set of simply connected polygonal regions of order n, for n≥o, arranged so that each edge of each region is matched with exactly one other edge of the same, or another region and vertices are matched only as required by the matching of edges. It implies that **one and the same, or two, and no more than two distinct polygonal regions (faces) meet at each edge**. The restriction of vertex matching in the definition means there is only one circuit of polygonal regions at each vertex of P.

UNIFORM POLYHEDRON is a polyhedron with the same repeating vertex figure and the same cyclic order of polygonal faces about each of its vertices





	4_0^3	3_0^5	4_1^4	3_1^6	4_3^6
$\Sigma\alpha$	270	300	360	360	540
Val.	3	5	4	6	6
g	0	0	1	1	3

Rene Descartes, in the first half of the 17th century, while referring to **convex regular polyhedra**, stated that:
"The total angular deficit, of the sum of the angular deficits, taken over all the vertices of a convex polyhedron, equals 4Л for (all) regular polyhedra": $\partial . V = (2Л - \Sigma\alpha)\ V = 4Л$

The total average curvature value $\sum \alpha_{av.}$ of a vertex region of a sponge polyhedron may be expressed as :

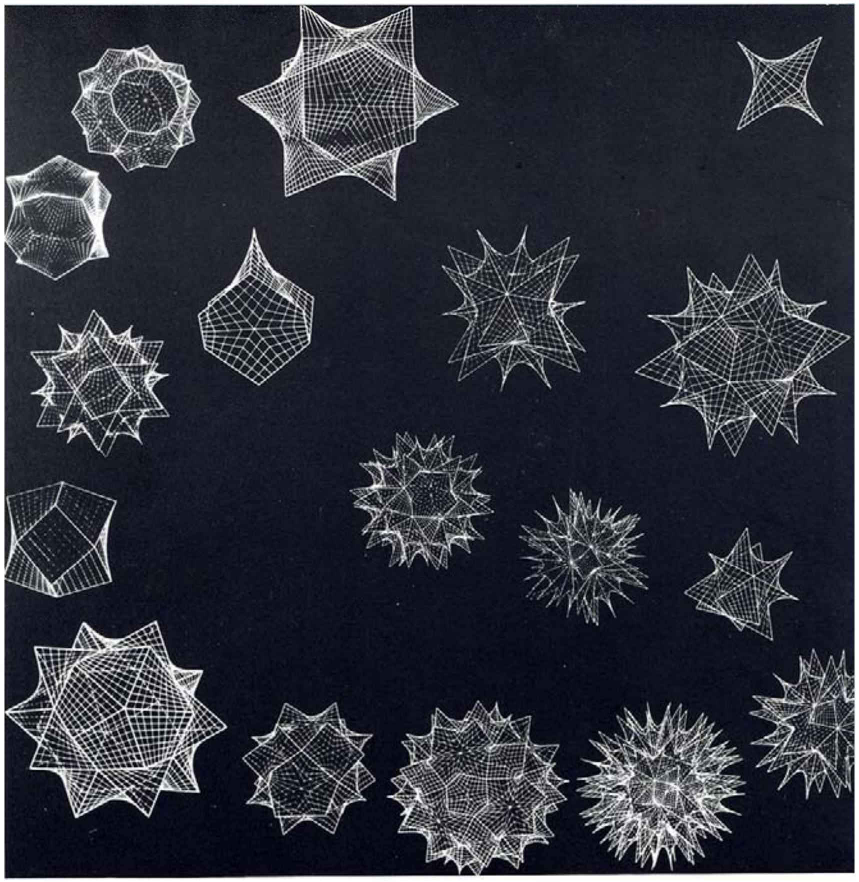
$$\sum \alpha_{av.} = 2\pi \left[1-\frac{2(1-g_{T.U})}{V_{T.U.}}\right],$$

as derived from Descartes' (expanded) theorem, (with $V_{T.U.}$ representing the number of vertices in a translation unit, when the polyhedron is of a periodic nature).

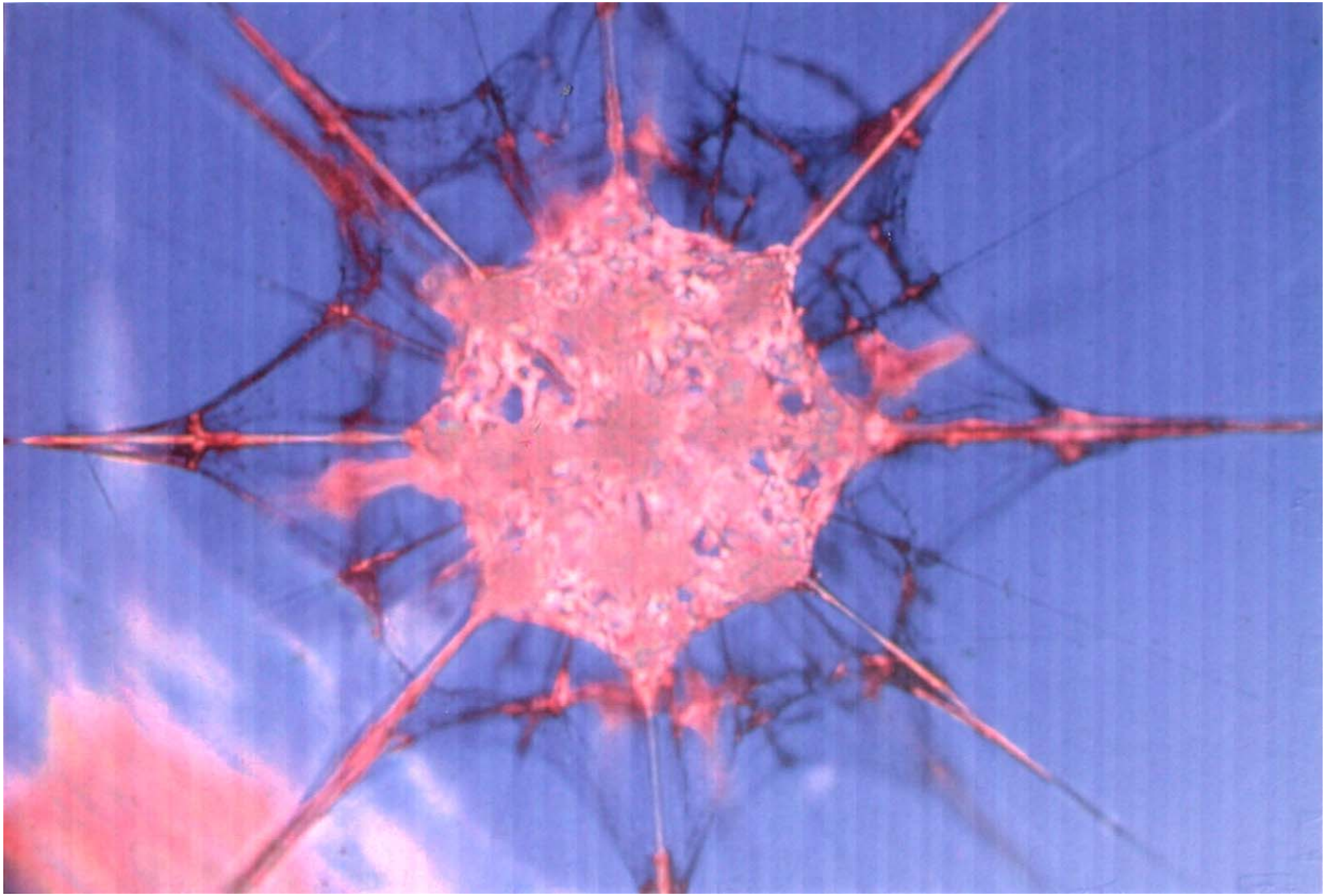
Leonhard Euler, the founder of topology, stated in the so-called Euler's Theorem: **"The number *V*-E+F=K**, (V, E, F, stand for vertices, Edges and Faces, respectively, with K, called the characteristic of the manifold)**, is the same for all permissible maps on the manifold."**

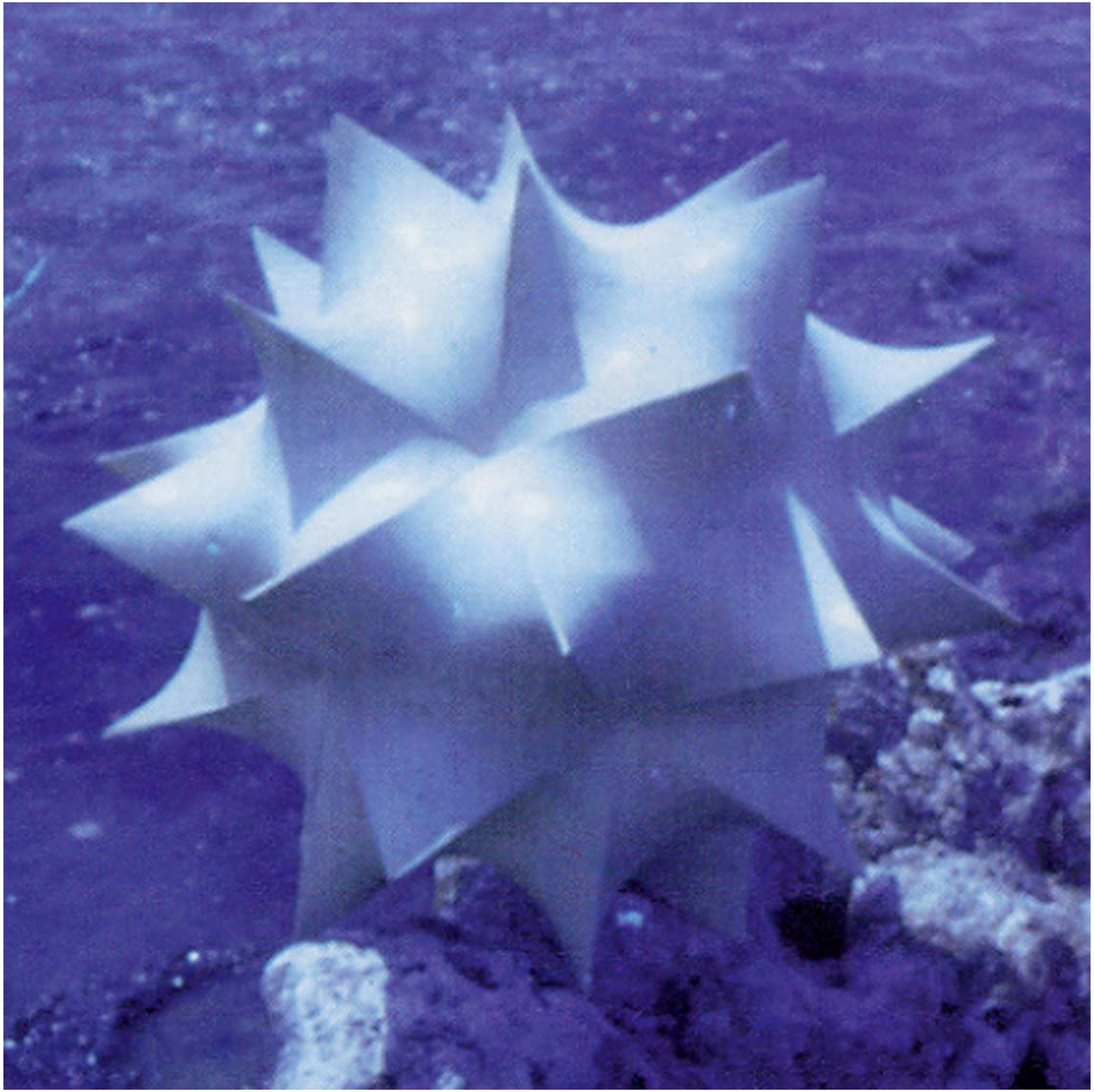
Each of the sponge surfaces may be mapped with a grid, representing eventually a sponge polyhedron

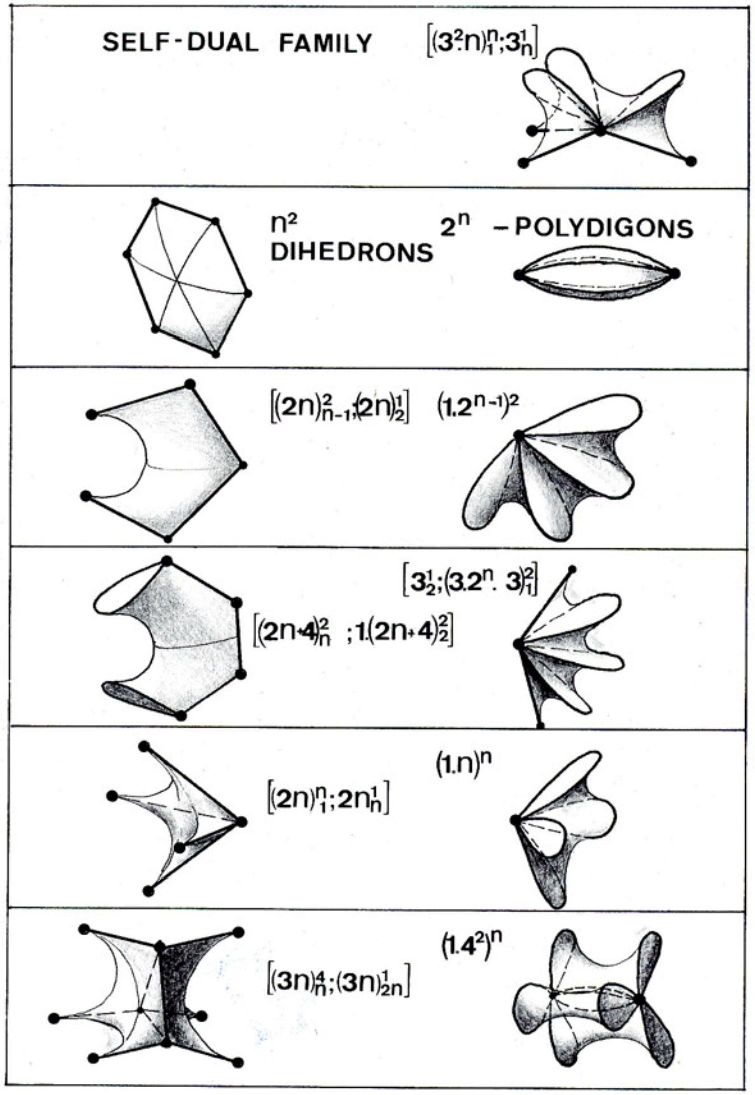
Generally speaking, **any far-reaching definition of polyhedra is admissible as long as it does not violate Euler's formula**.:



Finite Saddle Polyhedra.

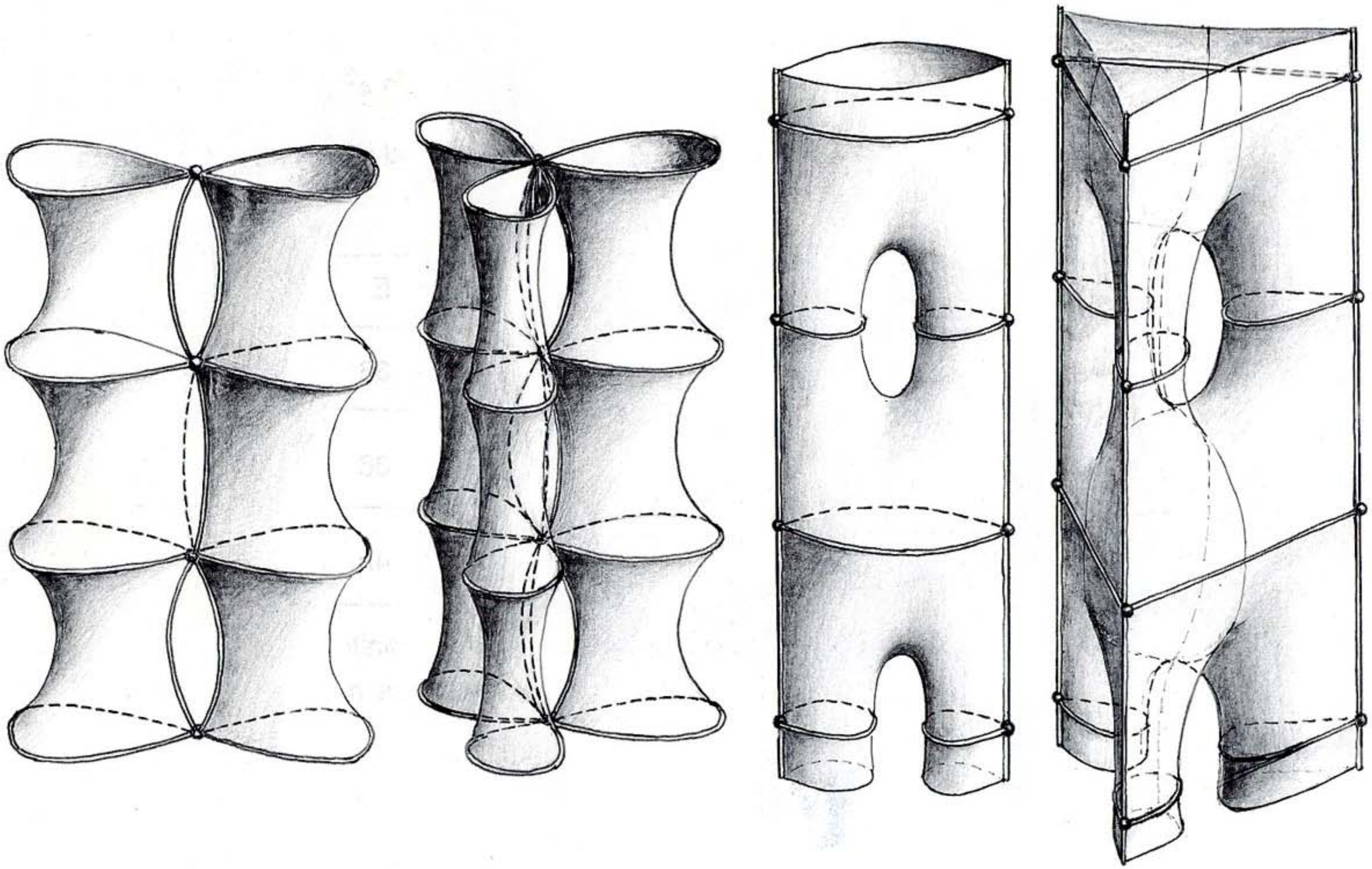


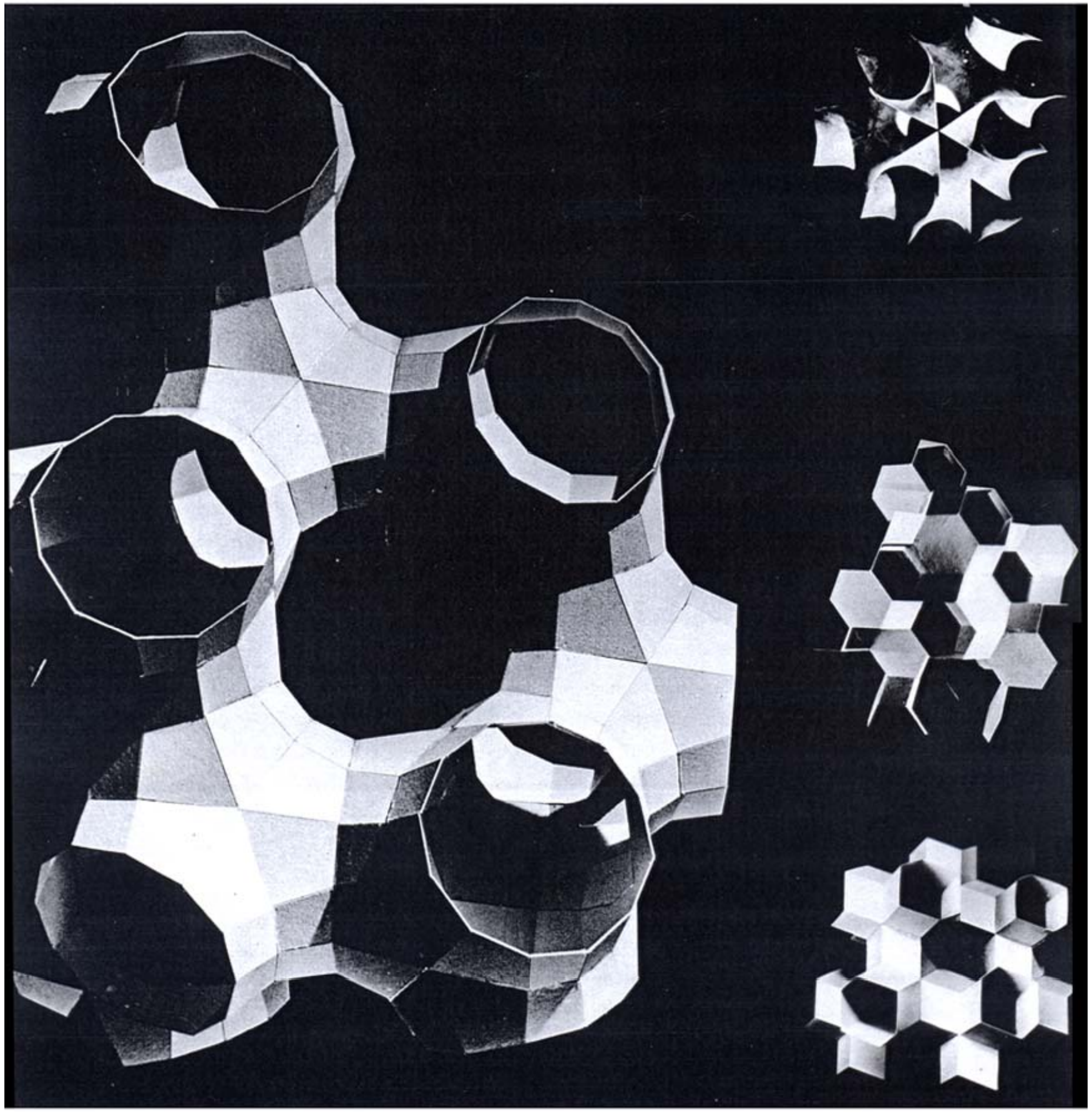




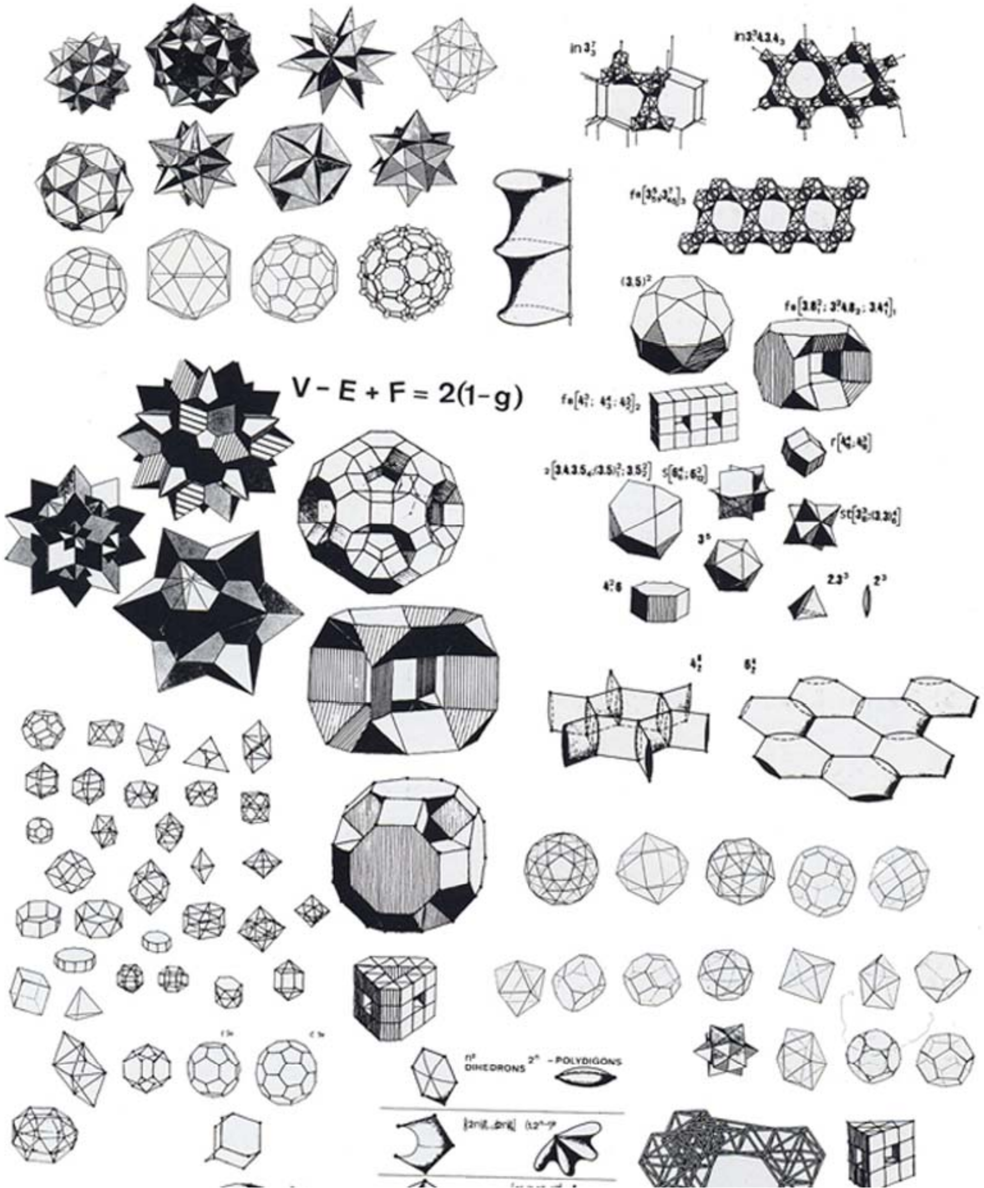
Floral polyhedra, families within the g=0 domain Aaranged according to their dual pairs.

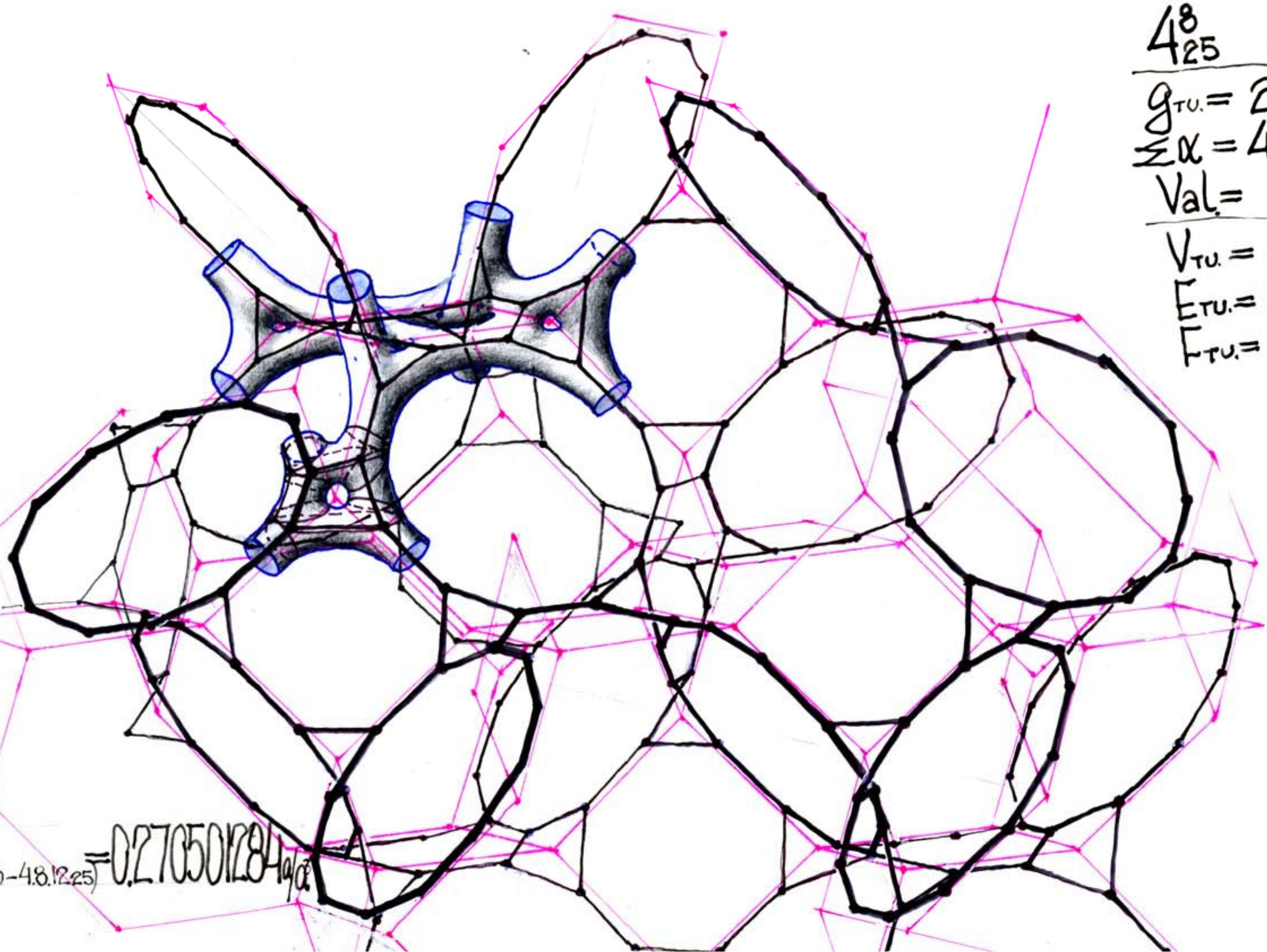
Periodic Floral Infinite
Polyhedra.





Periodic, uniform and non-uniform infinite sponge polyhedra related to a minimal (hyperbolic) surface of g =3 which subdivides space between two diamond lattices.


V − E + F = 2(1−g)
DIHEDRONS
– POLYDIGONS



4^{8}_{25}

$g_{TU} = 25$

$\Sigma\alpha = 4\pi$

$Val = 8$

$V_{TU} = 48$

$E_{TU} = 192$

$F_{TU} = 96$

The following mathematical relations dominate the field of **all spherical, toroidal and hyperbolical uniform polyhedra** in 3D space:

Val. $V_{T.U.} = 2E_{T.U.} = F_{T.U.}P_{AV.}$

$\mathbf{Con._{T.U.} = E_{T.U.} - V_{T.U.} + 1}$

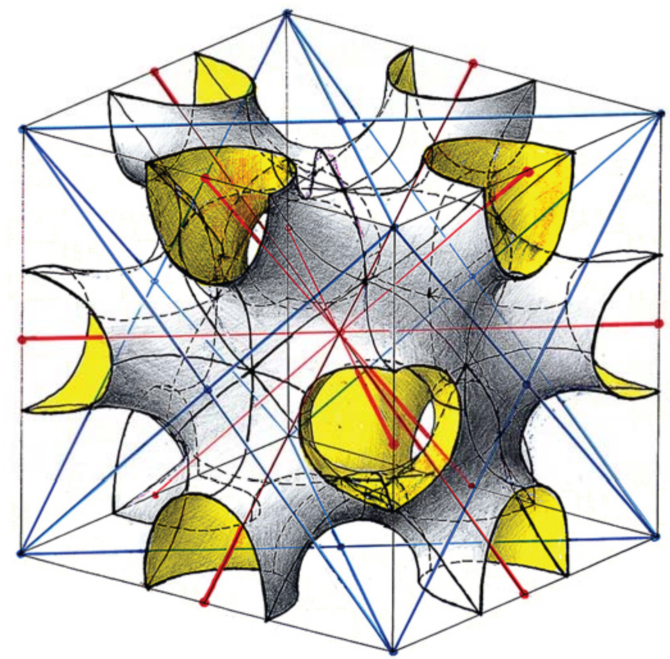
$\mathbf{V_{T.U.} (2Л - \Sigma\alpha_{AV.}) = 4Л (1\text{-}g_{T.U.})}$ **–** Descartes's Formula,

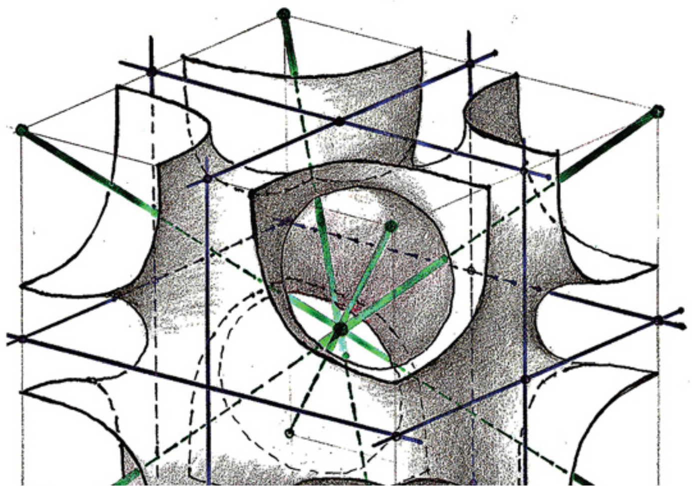
$\mathbf{V_{T.U.} - E_{T.U.} + F_{T.U.} = 2(1\text{-}g_{T.U.})}$ **–** Euler's Formula,

with all parameters derived from a Translation (or a repeat) - Unit (T.U.), with Val**.,** Pav., Con., *Σ*α and g, representing the valency in a vertex, average polygon, the network's connectivity, the total vertex

Generally speaking, **uniform 3D networks** (lattices) **come in dual** (reciprocal) **pairs and are associated with continuous hyperbolical sponge surfaces which subdivide the space between the two.**

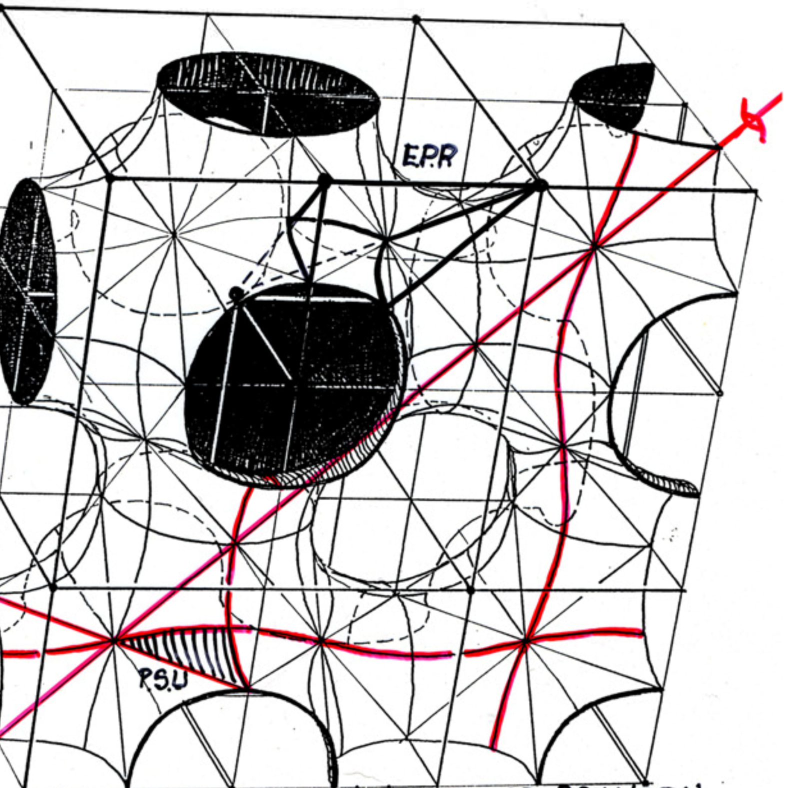
This TRINITY of the dual network pair and the associated reciprocal sponge surface, is the most conspicuous, all pervading geometric-topological phenomenon of our 3D space, associated with its order and organization, and more than anything else **determines the way we perceive and**

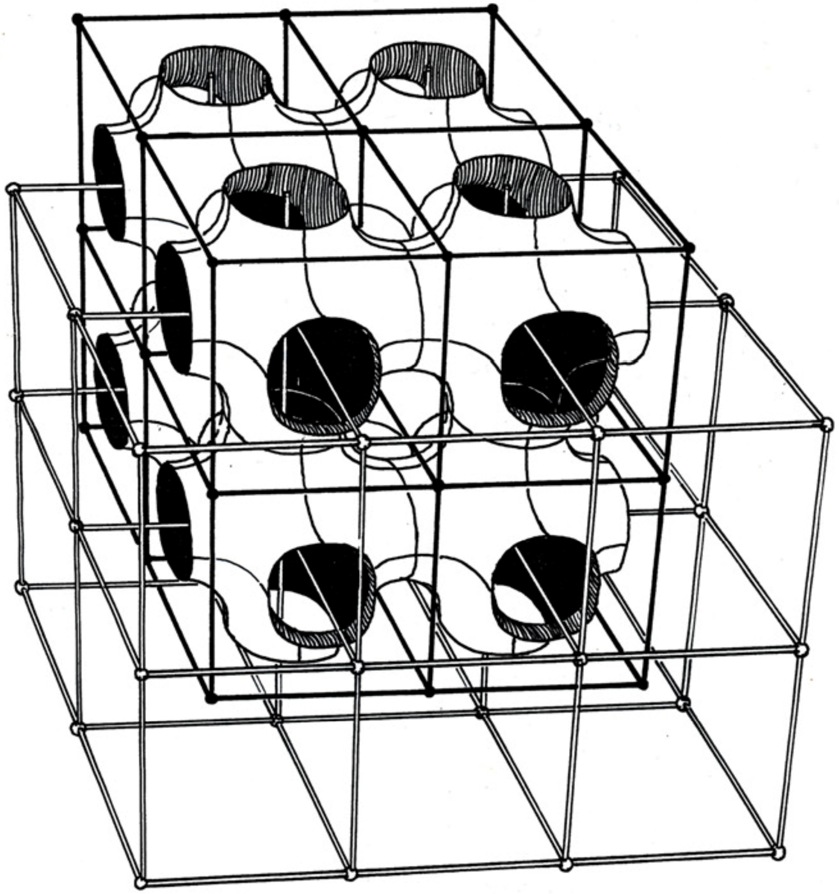




Connectivity (Con.) of a multiple vertex-edge configuration represents the maximal number of edge disconnections which still leave the configuration un-separated into two parts.

. In a 'Trinity', connectivity value is shared by both dual networks, and conforms with genus-g value of the reciprocal partition surface.

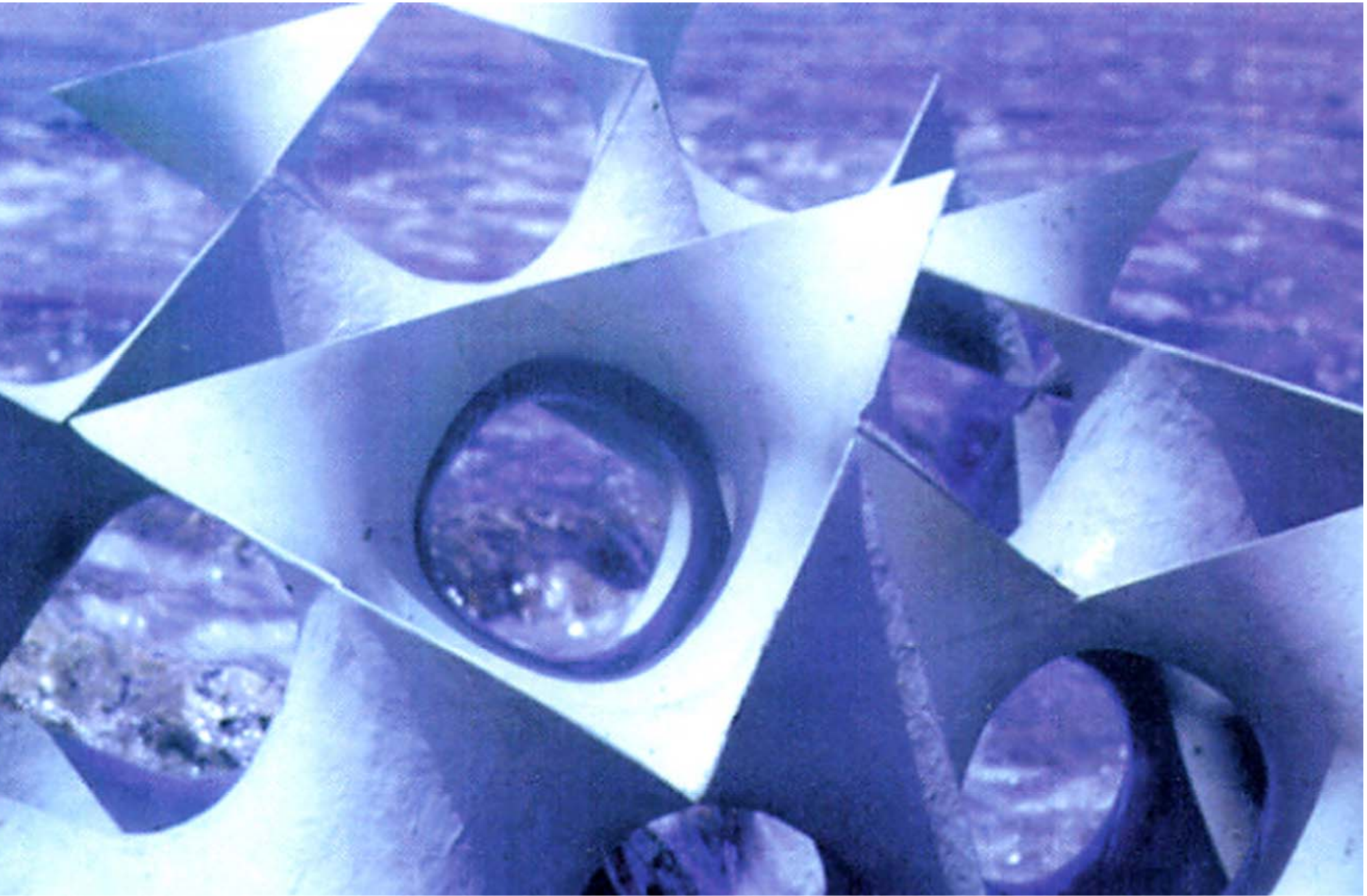


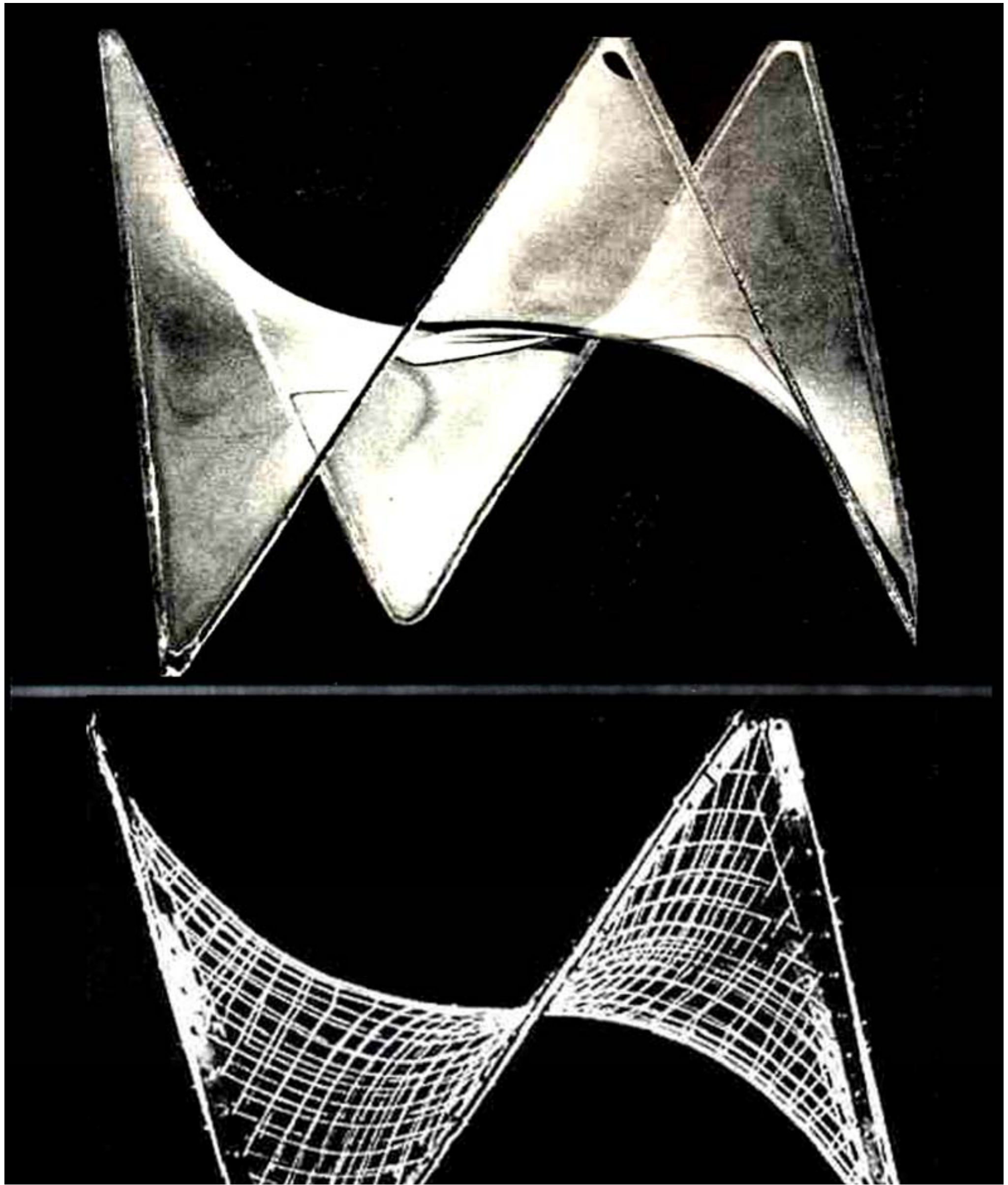


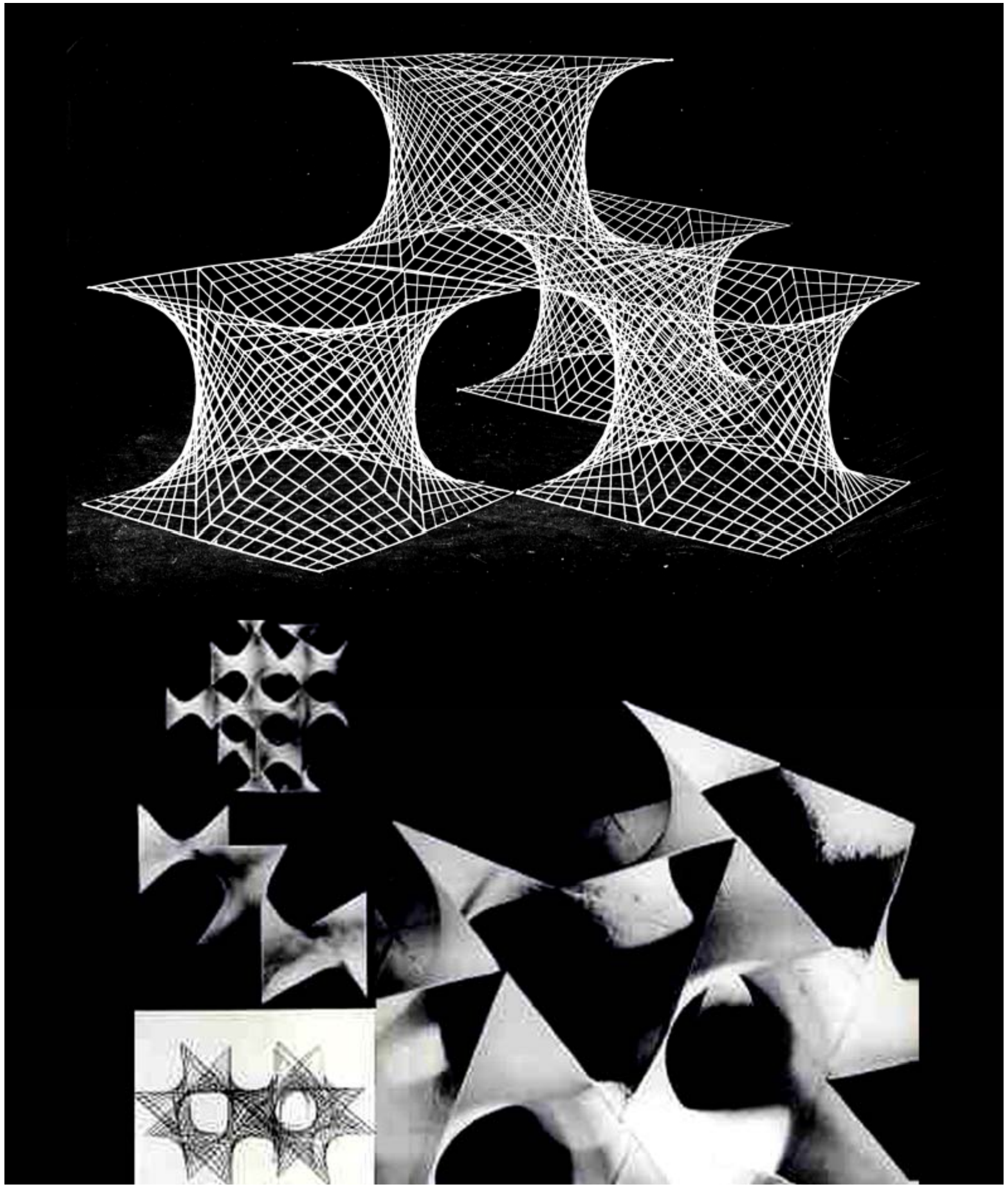
BOLIC SURFACE SUBDIVIDING SPACE BETWEEN
COMPLEMENTARY (DUAL) SPACE NETWORKS.

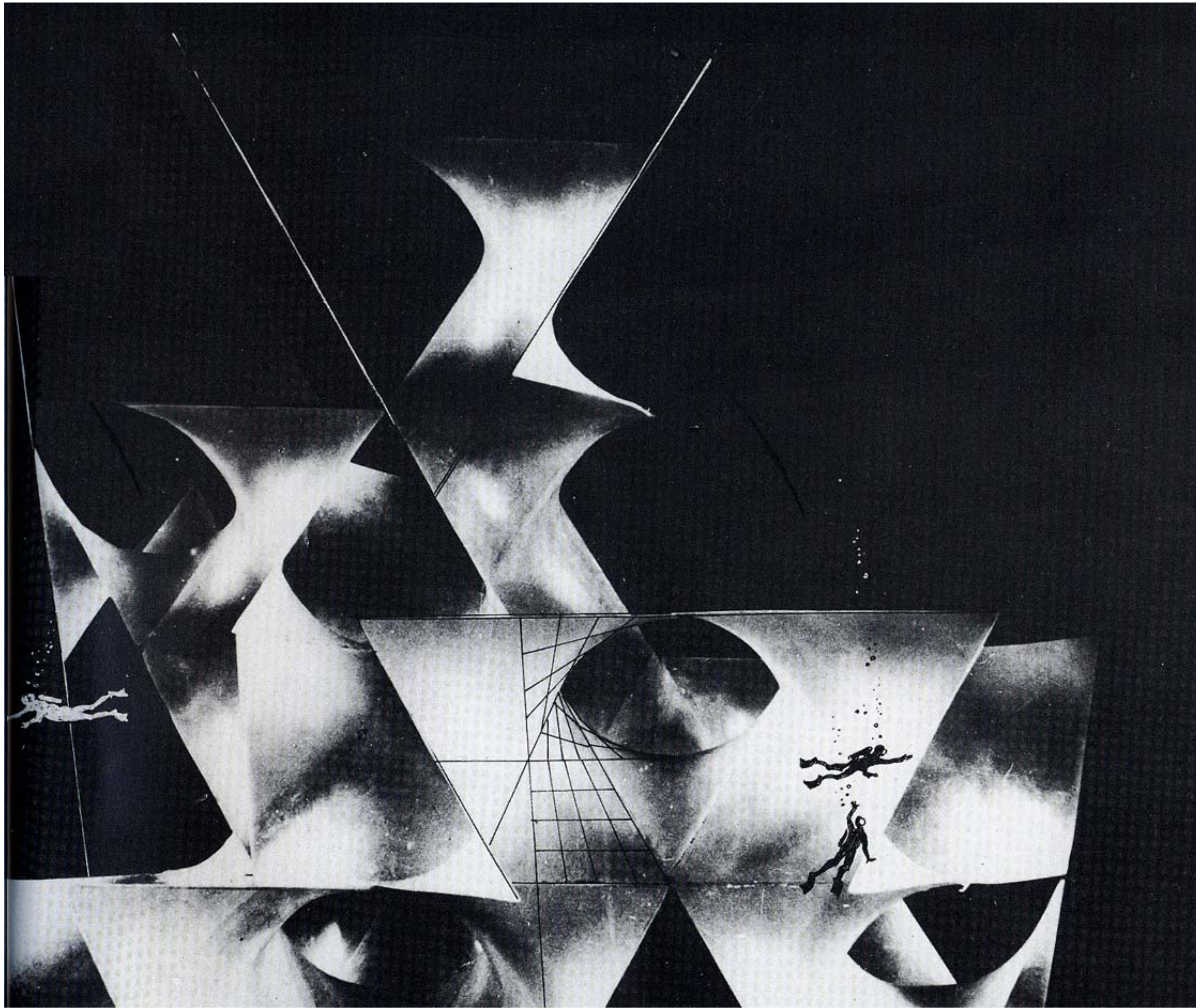
E OF THE PERIODIC HYPERBOLIC SURFACES
WITH VERY HIGH DEGREE OF REGULARITY.

– A REPETITIVE PERIODIC SURFACE UNIT.

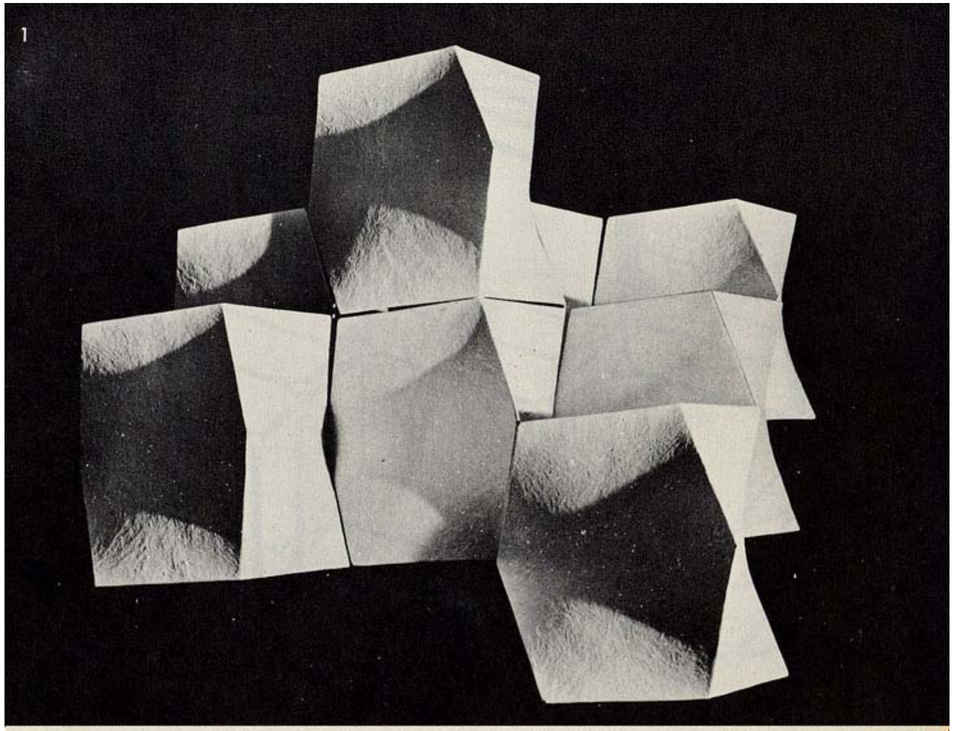








1



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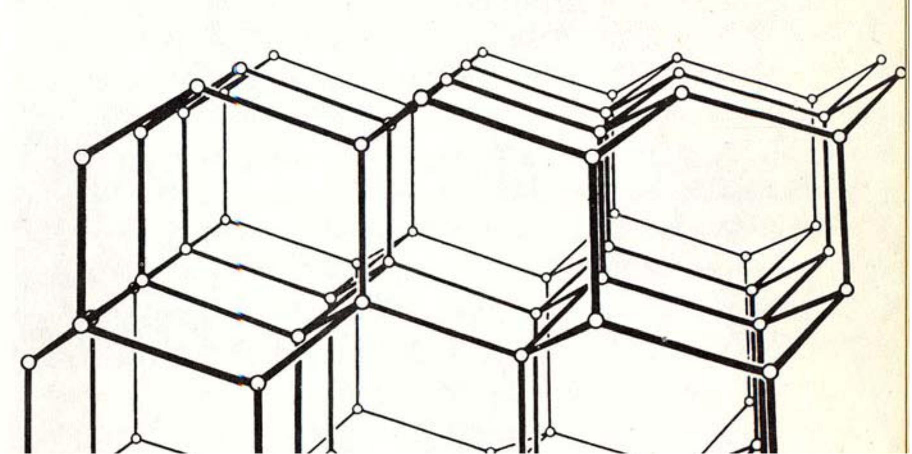
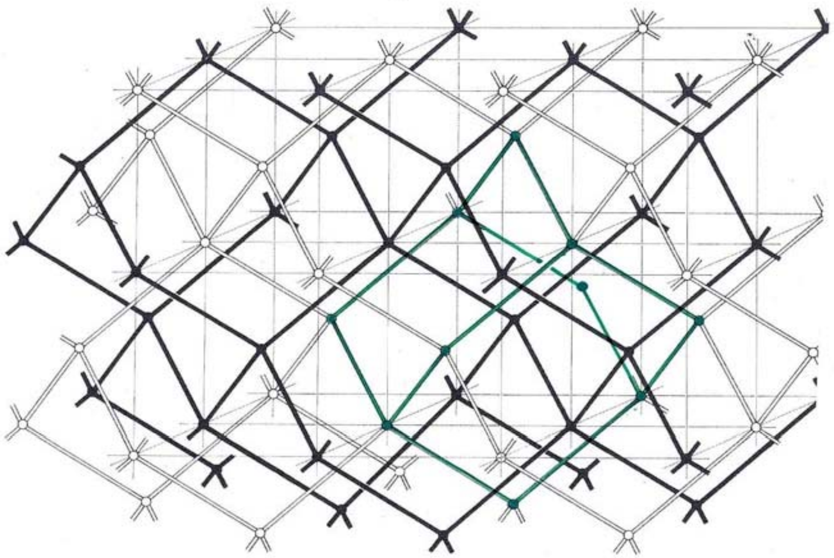
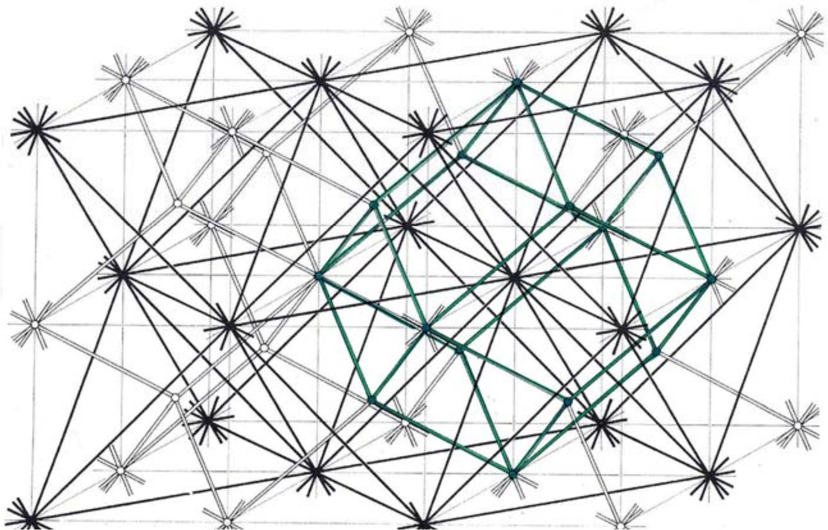
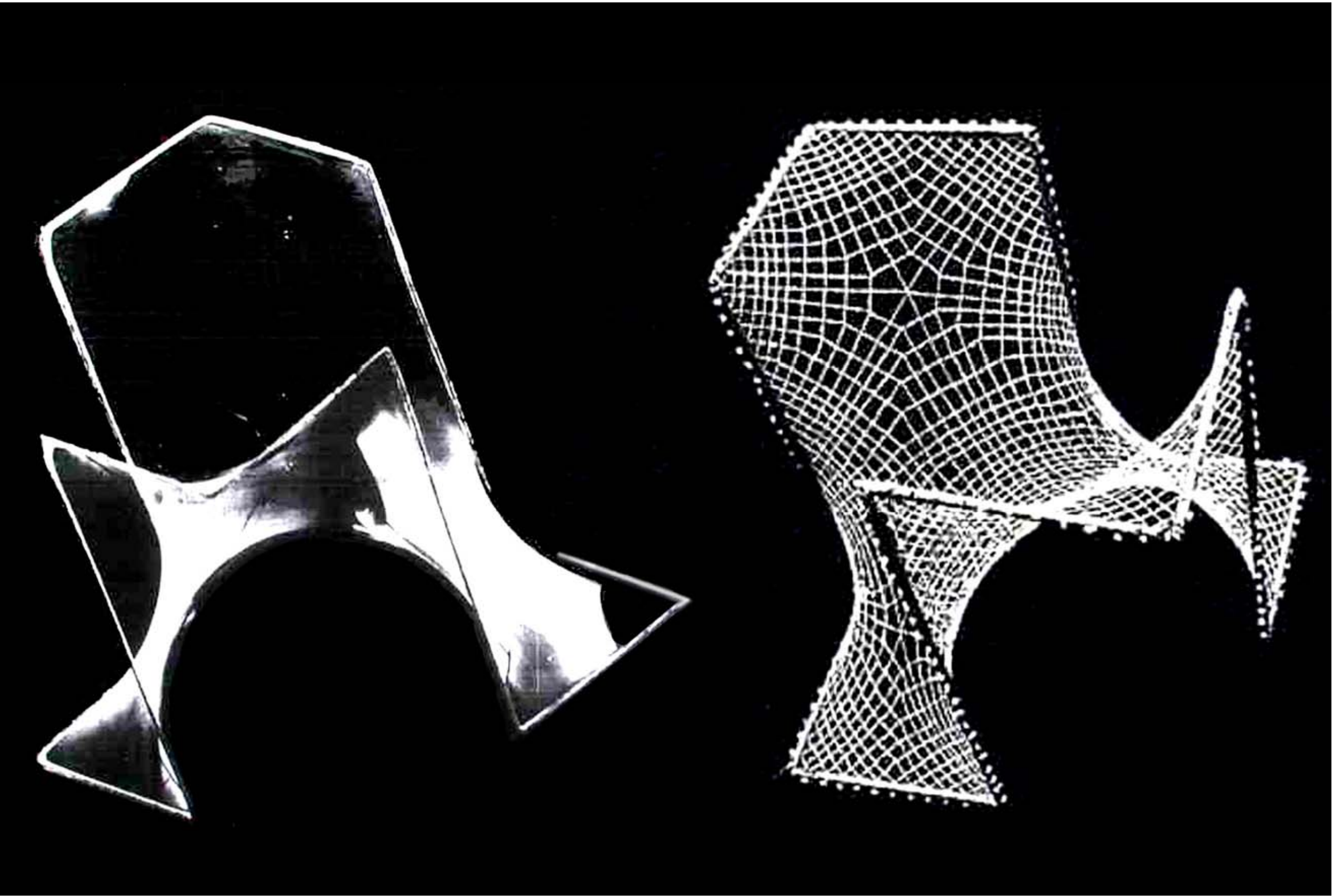
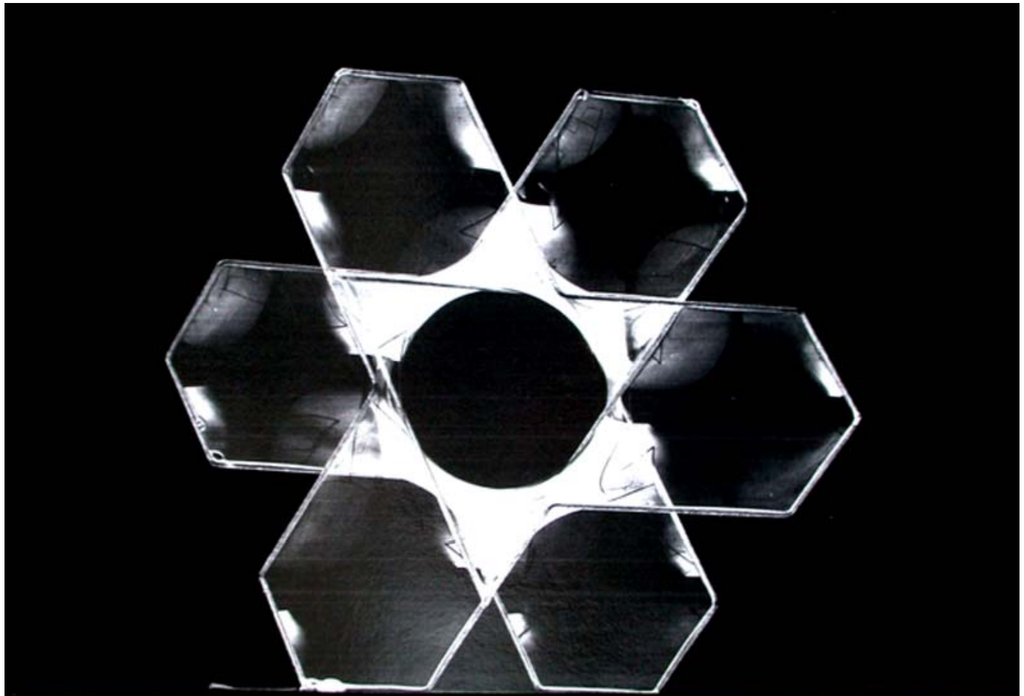
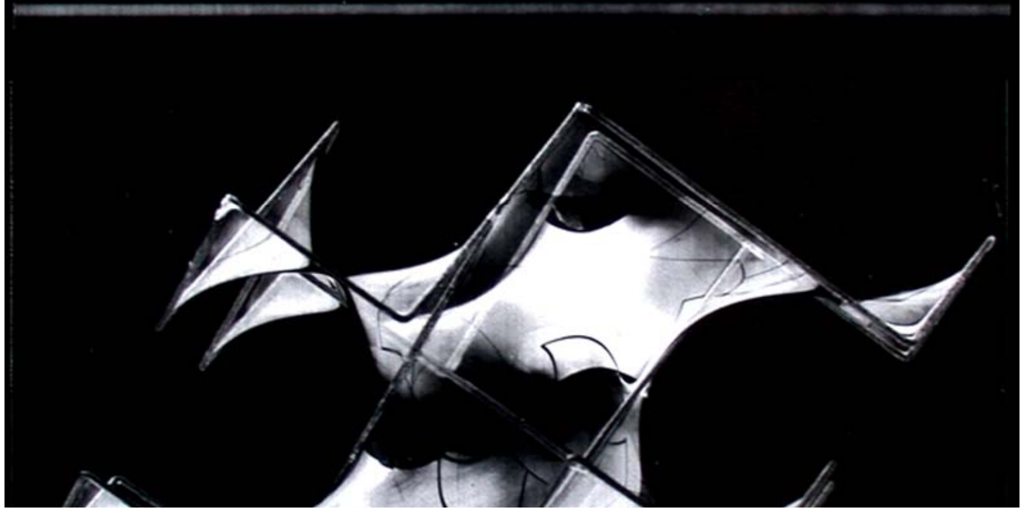


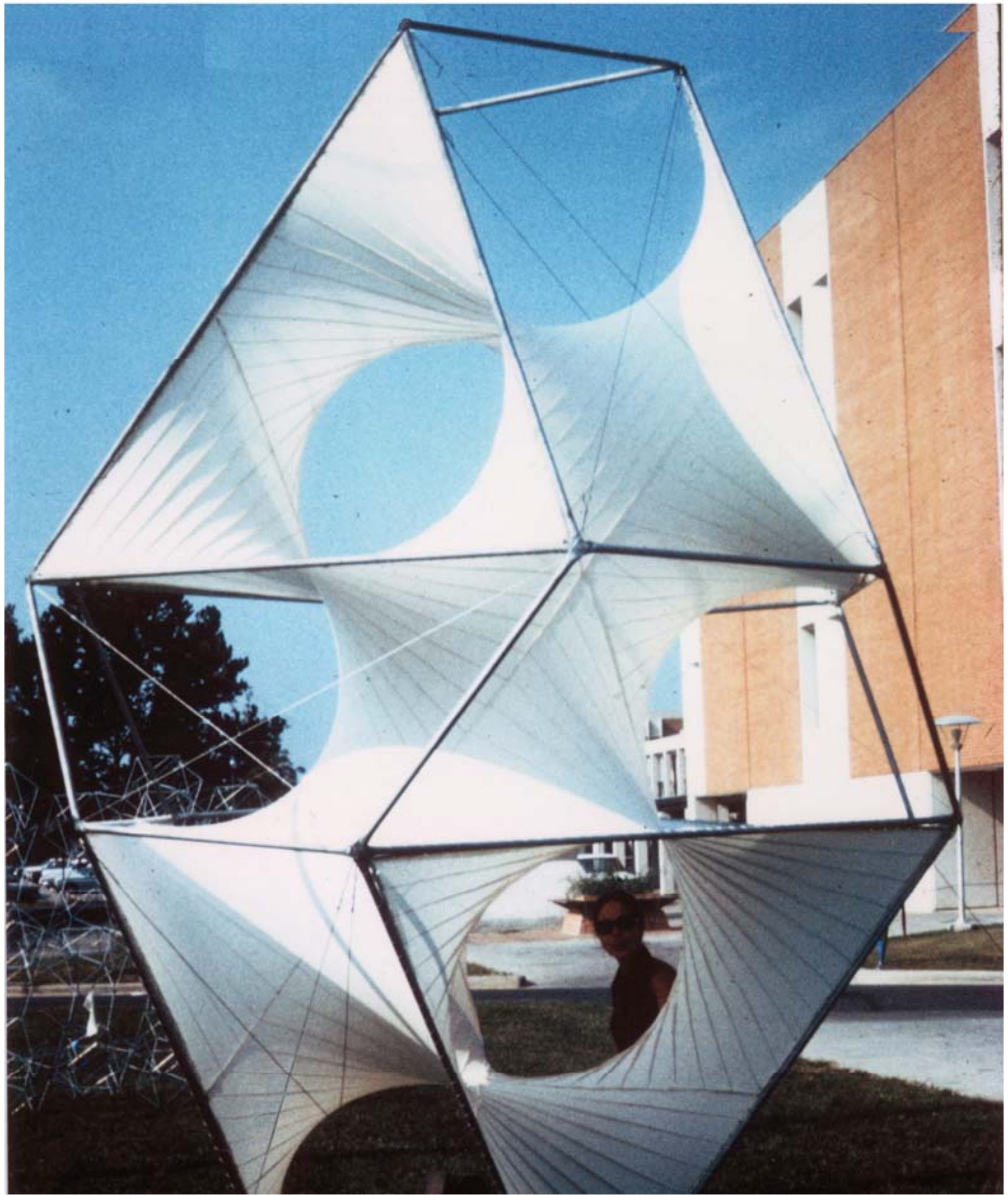


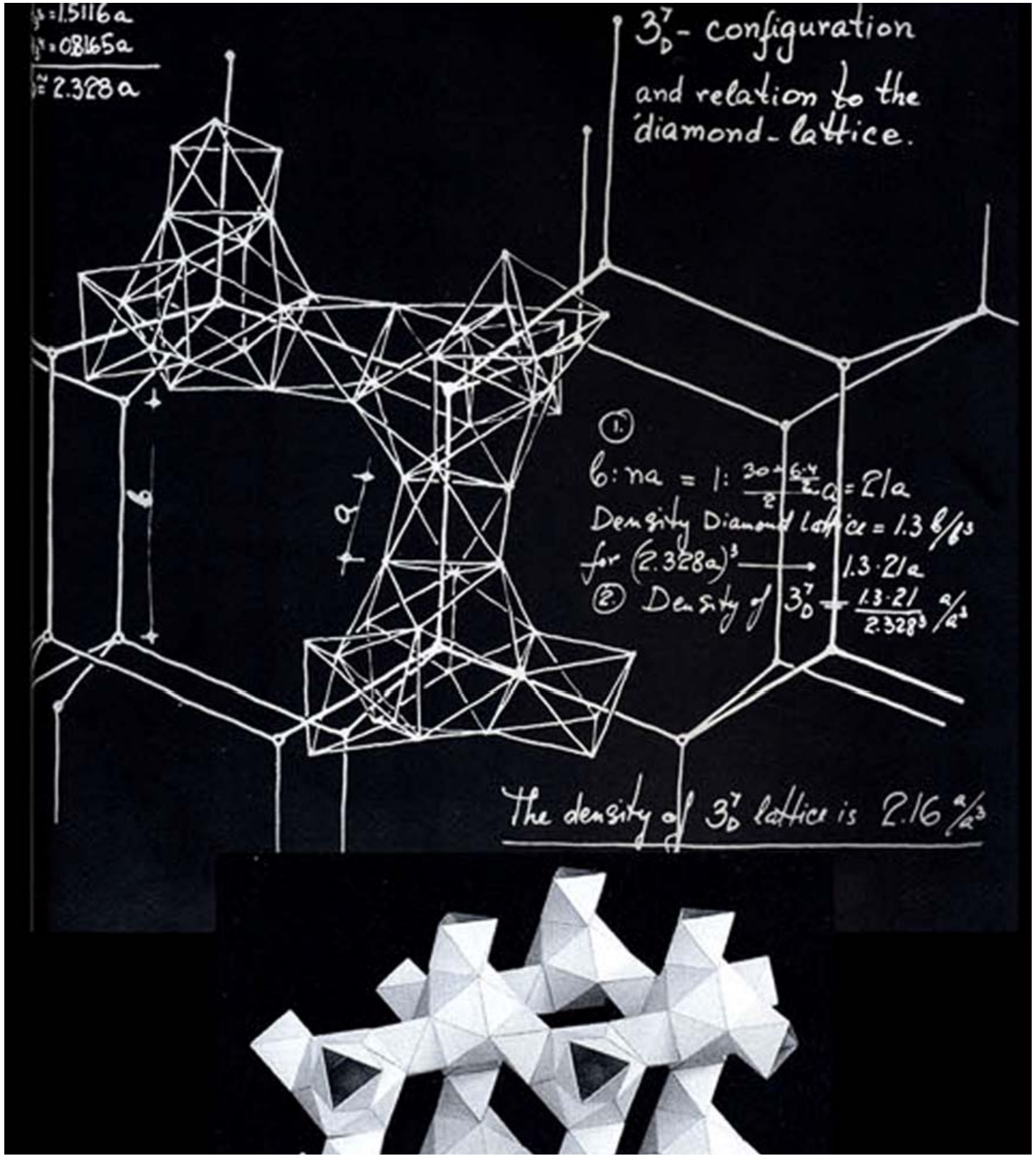




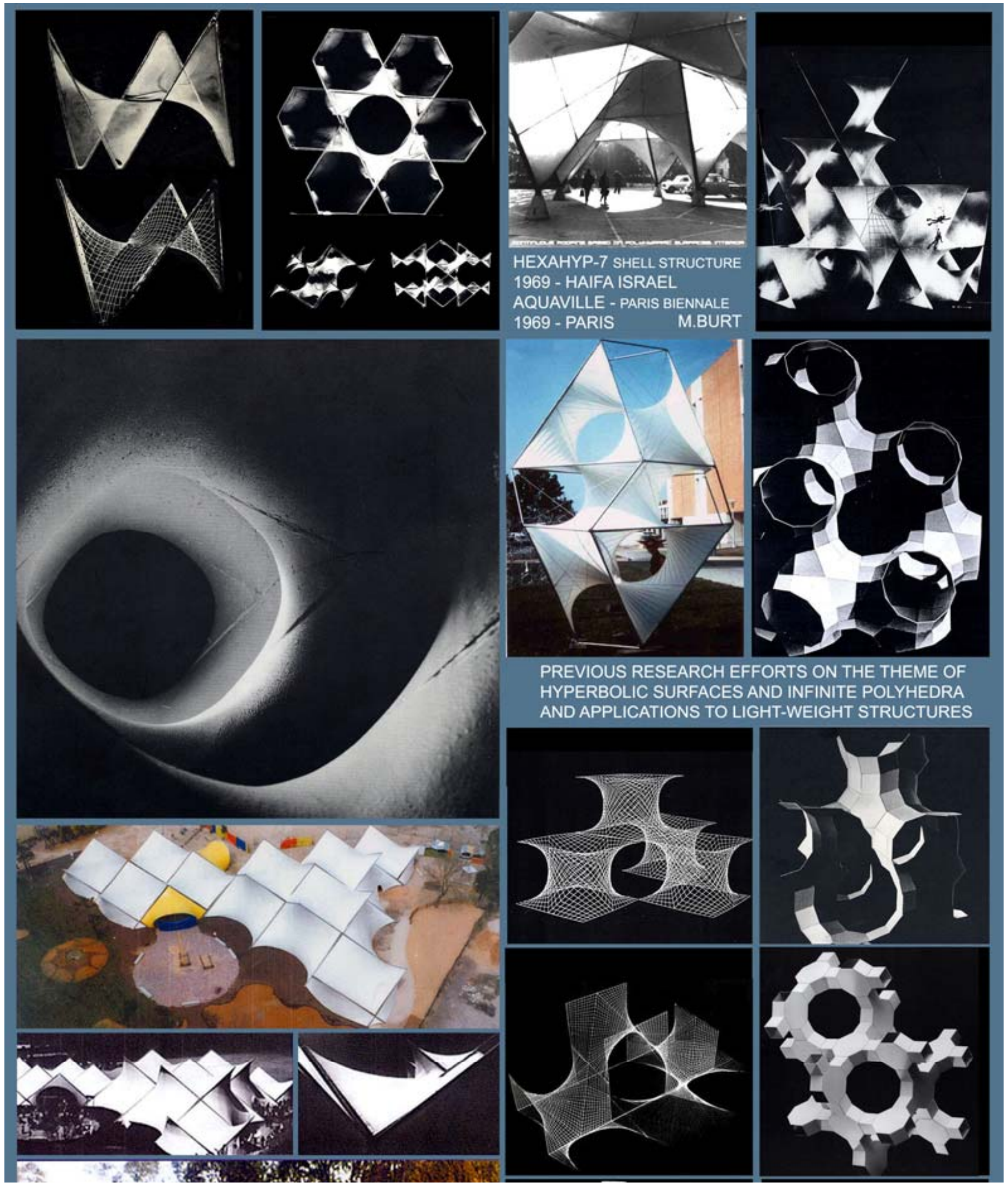








=1.5116a
=0.8165a
= 2.328a
3^7_D - configuration
and relation to the
'diamond-lattice.
(1.)
Density Diamond lattice = 1.3
for $(2.328a)^3$ → 1.3·21a
(2.) Density of 3^7_D
The density of 3^7_D lattice is 2.16

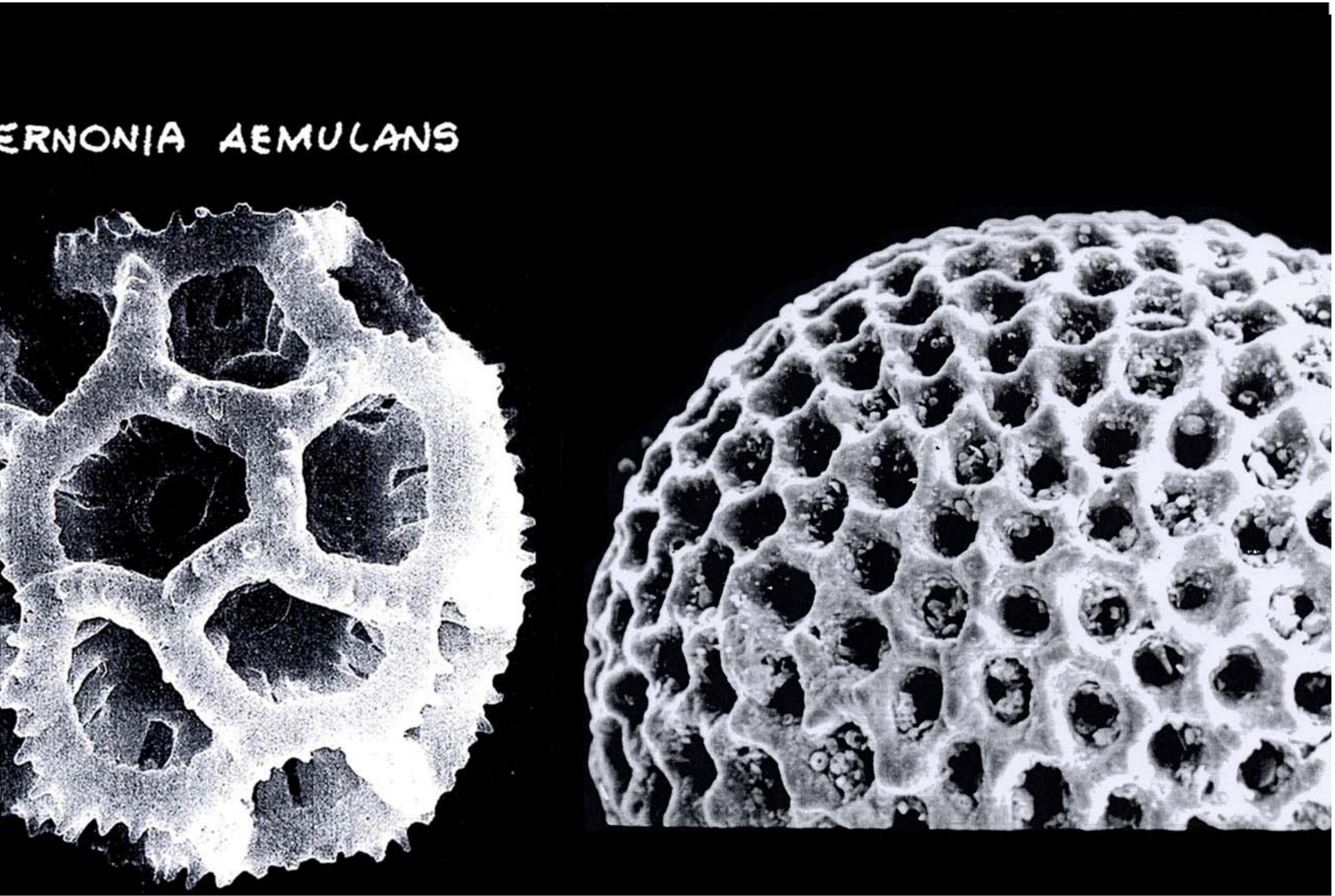

HEXAHYP-7 SHELL STRUCTURE
1969 - HAIFA ISRAEL
AQUAVILLE - PARIS BIENNALE
1969 - PARIS M.BURT
PREVIOUS RESEARCH EFFORTS ON THE THEME OF
HYPERBOLIC SURFACES AND INFINITE POLYHEDRA
AND APPLICATIONS TO LIGHT-WEIGHT STRUCTURES

By defining as 'morphic' those processes which display a movement toward greater 3-dimensional spatial order, symmetry of form (Whyte-1969) and **morphology as the logical preoccupation with and manipulation of those processes,** than the **research nto the nature of networks and the associated sponge surfaces may be classified as the essence of morphology.**

Nature is saturated with sponge structures on every possible scale of physical-biological reality. The term was first adopted in biology: **"Sponge: any member of the phylum Porifera, sessile aquatic animals, with single cavity in the body, with numerous pores. The fibrous skeleton of such an animal, remarkable for its power of sucking up water".** (Wordsworth dictionary).

Of course the term applied to **'spherical sponges'. It turns out that the key characteristic of porosity is attributable to a much wider morphological phenomenon.**

continuous 3D networks may be considered as a polyhedral tessellation of some unbounded (finite or infinite) surface which should be considered as the network's Genetic Surface (Ge.S.). All possible polyhedral tessellations of a common genetic surface share same $\Sigma\alpha_{AV.}$ and g Values..


ERNONIA AEMULANS

The dominant features of the periodic sponge polyhedral tessellations which determine the **parameter values** are the **genetic surface, its repeat (E.P.R) or translation unit, the mode of its perforation and its (n) layer arrangement.**

$.4^2.11^2.3^2.11^2.4^2.11.4^2.11^2.3^2.11^2.4^2.11.3.6.4^2.6^2.3^3.6^2.4^2.6_{601}$

$.5) = 601$

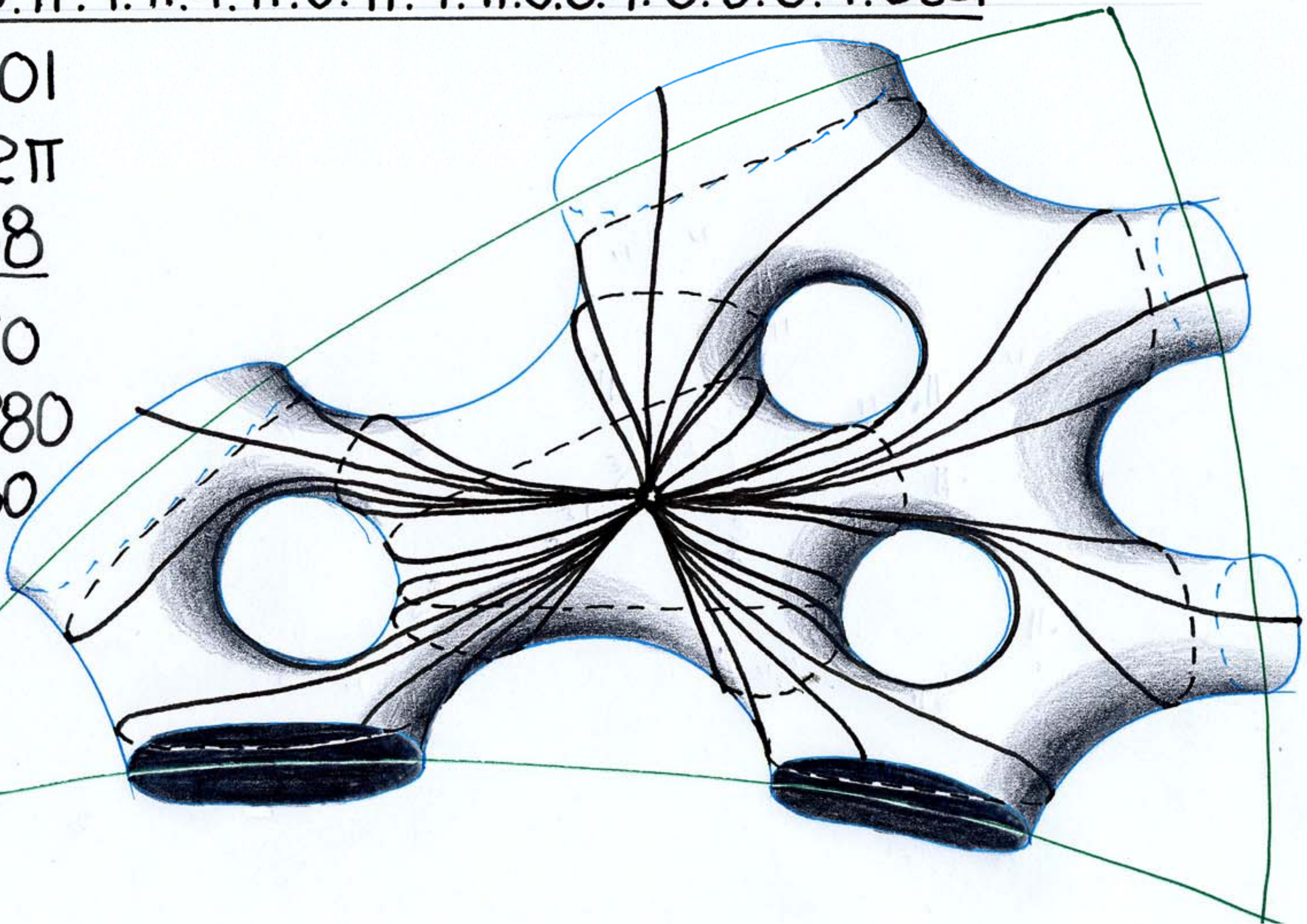
$= 22\pi$

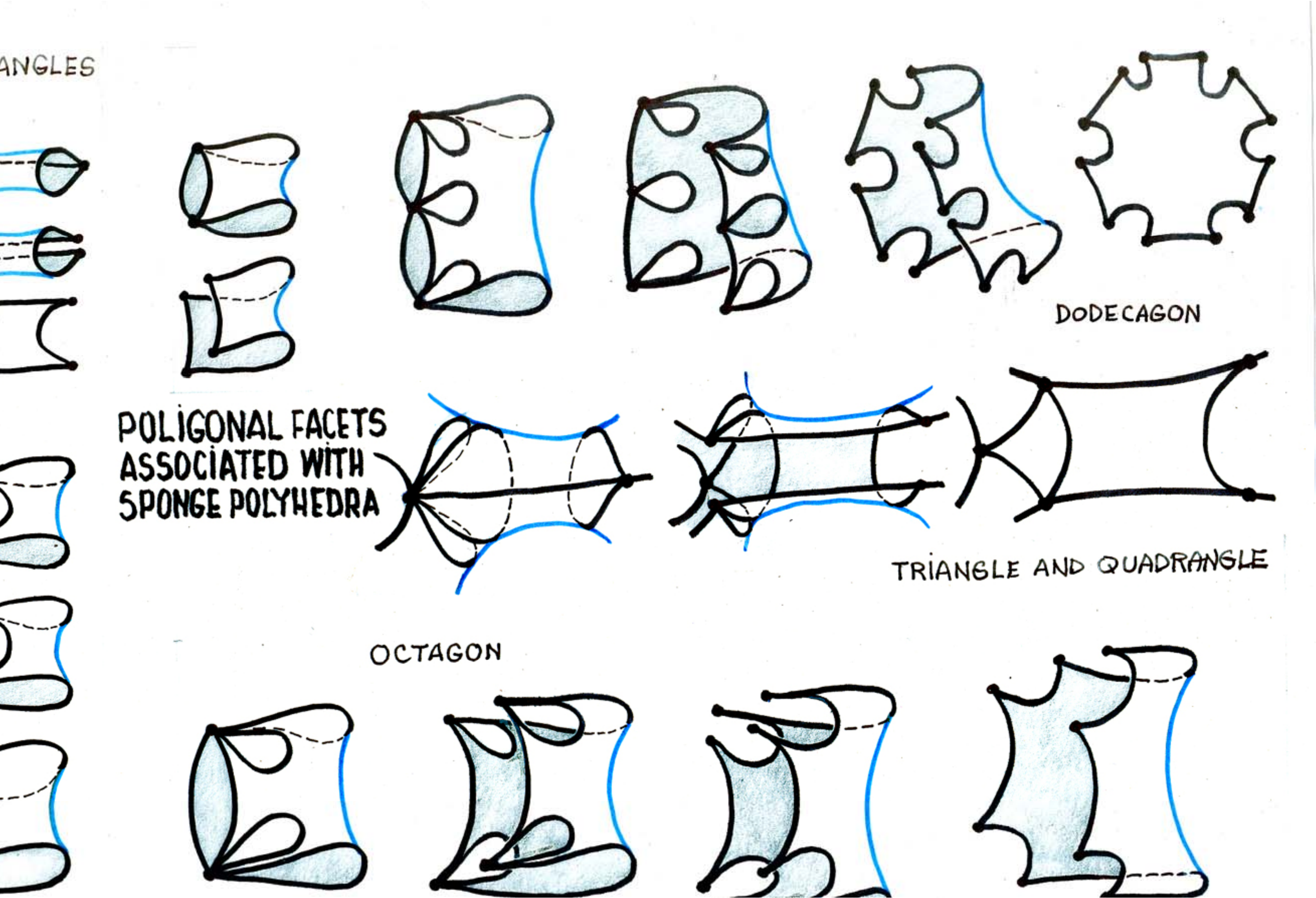
$. = \underline{38}$

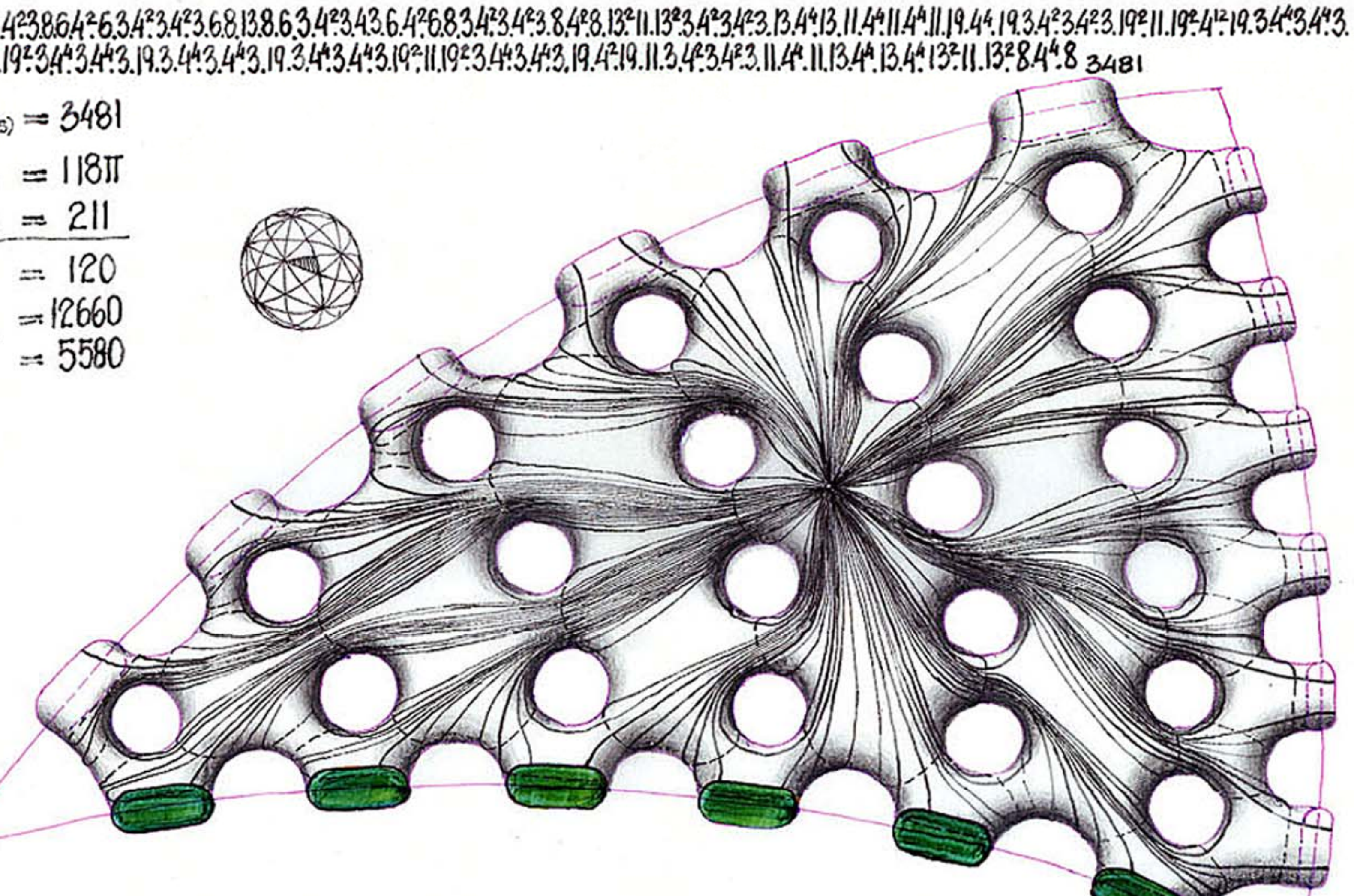
$= 120$

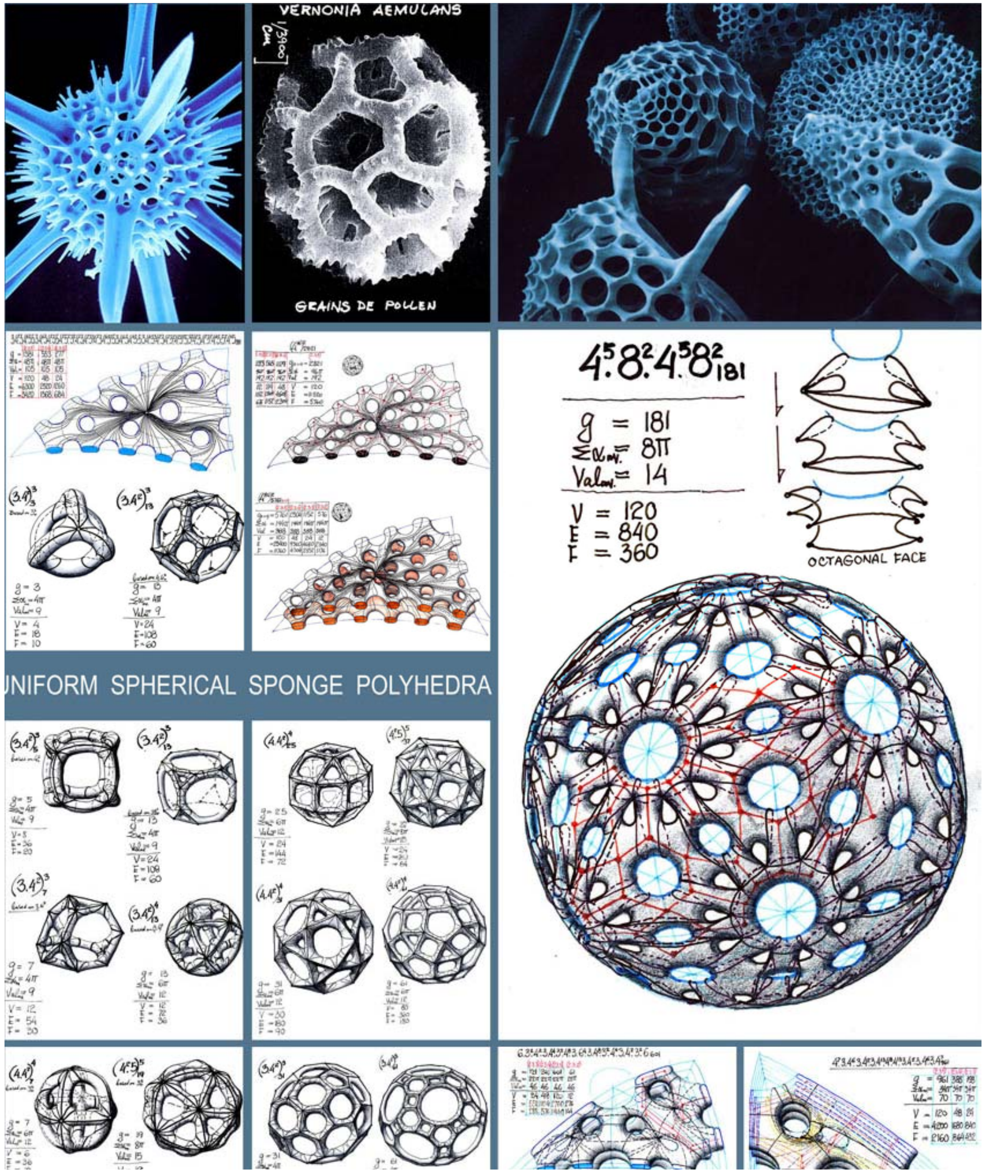
$= 2280$

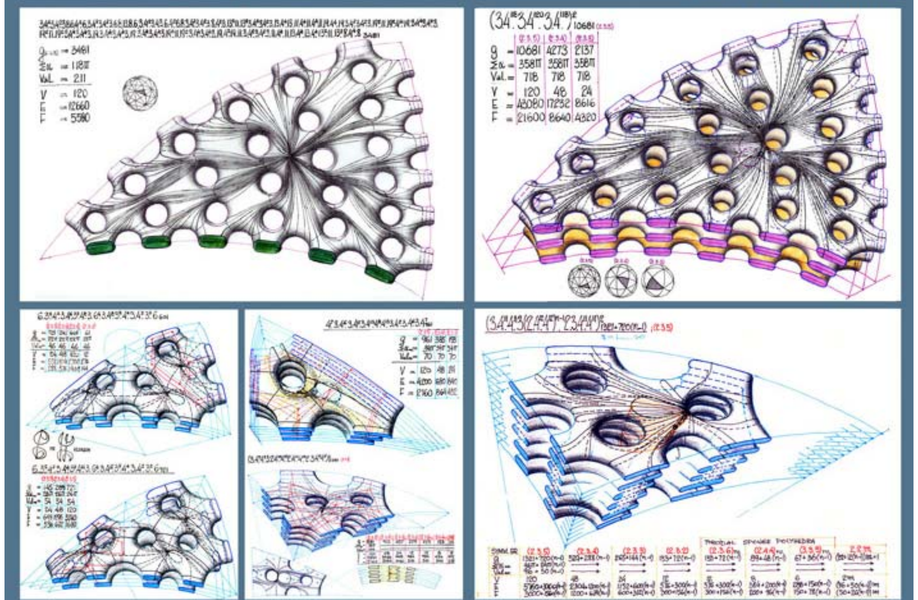
$= 960$




ANGLES
DODECAGON
POLIGONAL FACETS
ASSOCIATED WITH
SPONGE POLYHEDRA
TRIANGLE AND QUADRANGLE
OCTAGON

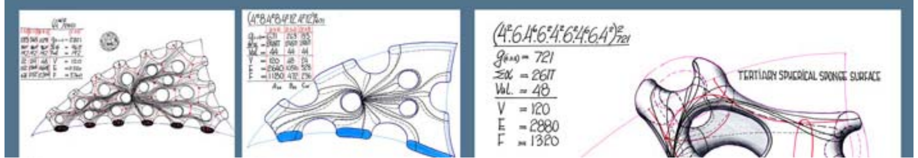

$.4^2.3.8.6.4^2.6.3.4^2.3.4^2.3.6.8.13.8.6.3.4^2.3.4.3.6.4^2.6.8.3.4^2.3.4^2.3.8.4^2.8.13^2.11.13^2.3.4^2.3.4^2.3.13.4^4.13.11.4^4.11.4^4.11.19.4^4.19.3.4^2.3.4^2.3.19^2.11.19^2.4^{12}.19.3.4^4.3.4^4.3.$
$.19^2.3.4^4.3.4^4.3.19.3.4^4.3.4^4.3.19.3.4^4.3.4^4.3.19^2.11.19^2.3.4^4.3.4^4.3.19.4^2.19.11.3.4^2.3.4^2.3.11.4^4.11.13.4^4.13.4^4.13^2.11.13^2.8.4^4.8$ 3481
s) = 3481
= 118π
= 211
= 120
= 12660
= 5580

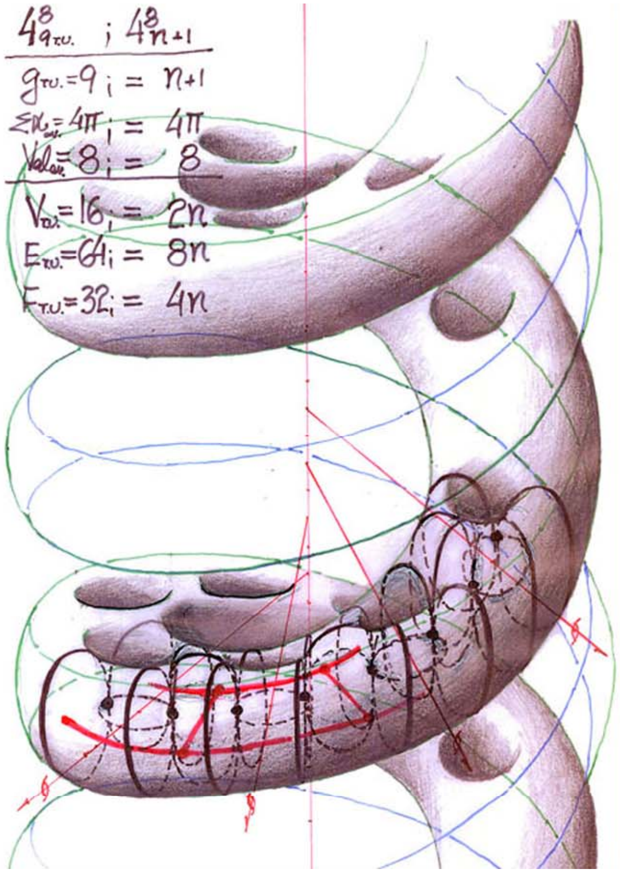

VERNONIA AEMULANS
GRAINS DE POLLEN
UNIFORM SPHERICAL SPONGE POLYHEDRA
4.5.8.2.4.5.8.2 181
g = 181
Valen = 14
V = 120
E = 840
F = 360
OCTAGONAL FACE



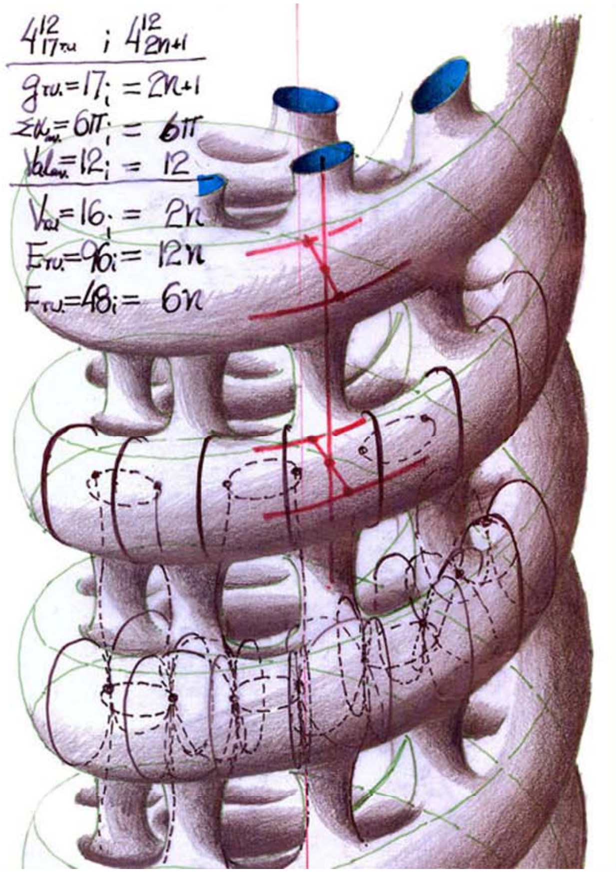
SINGLE AND MULTI LAYERED, UNIFORM, SPHERICAL SPONGE POLYHEDRA

MICHAEL BURT, 2008





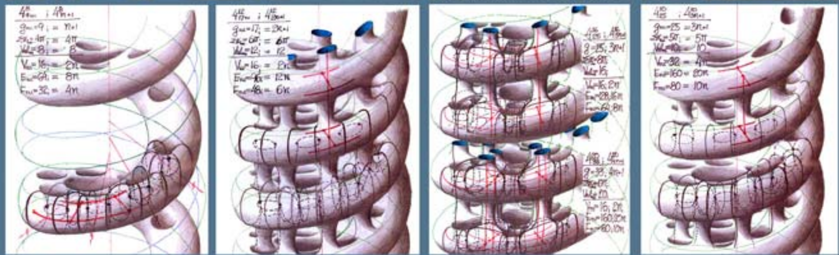
g TU.=9 ; = n+1
= 4π ; = 4π
=8 ; = 8
V =16 ; = 2n
E TU.=64 ; = 8n
F TU.=32 ; = 4n



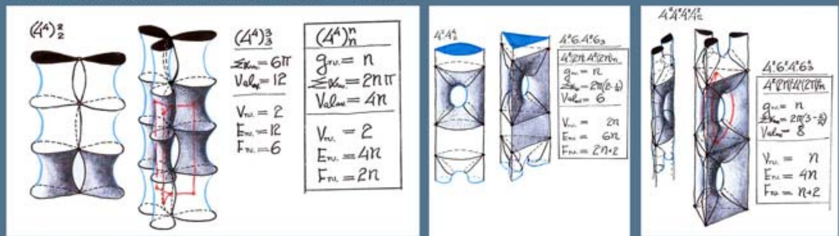
g TU.=17 ; =2n+1
= 6π ; = 6π
=12 ; = 12
V =16 ; = 2n
E TU.=96 ; = 12n
F TU.=48 ; = 6n

PONT DU GARD

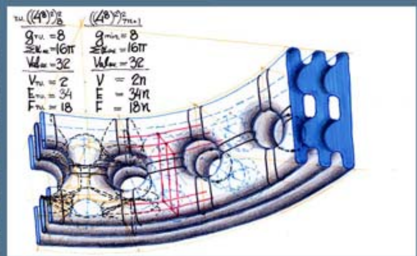
PARK GUEL - GAUDI

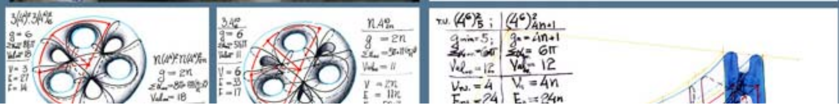


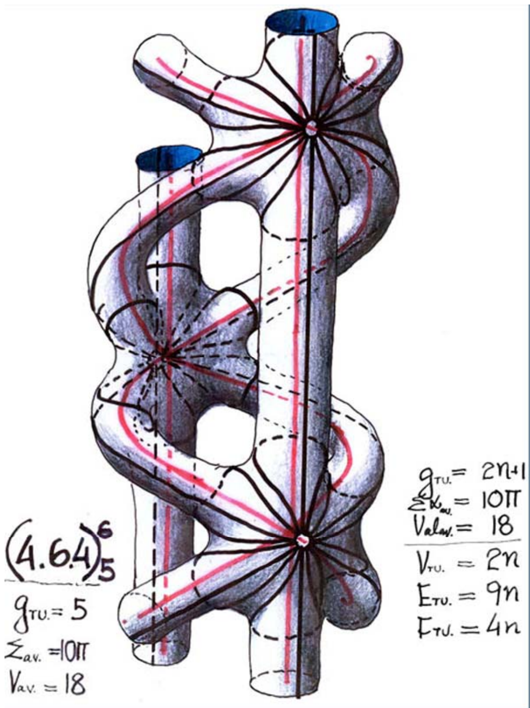
PRIMITIVE UNIFORM SPONGE POLYHEDRA M.BURT

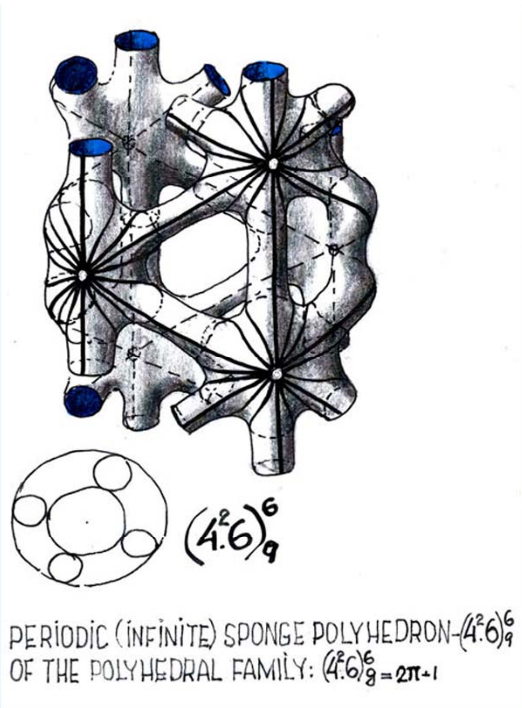


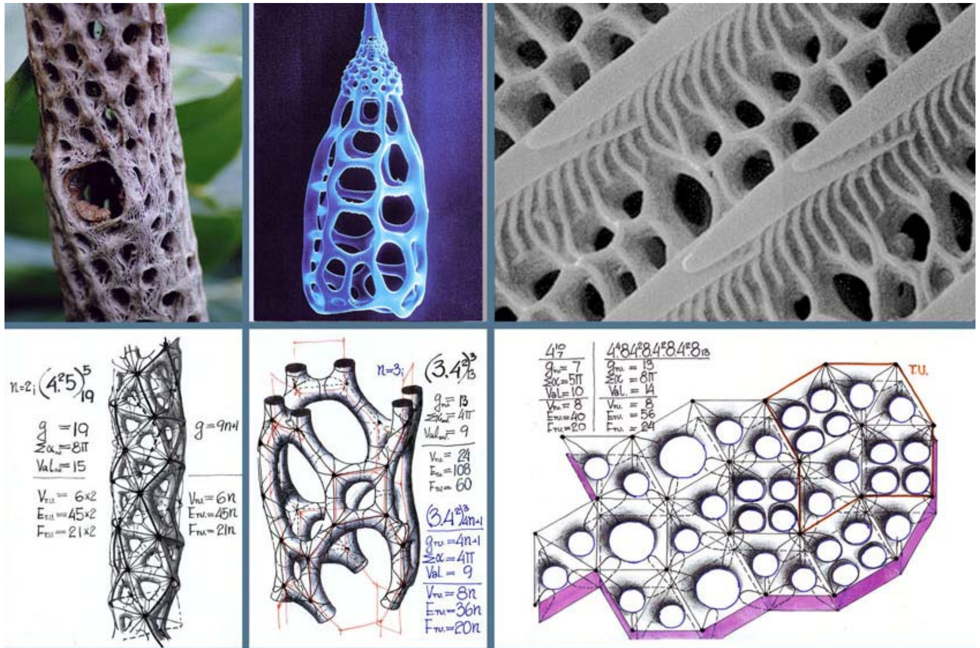
CASA MILLA - GAUDI

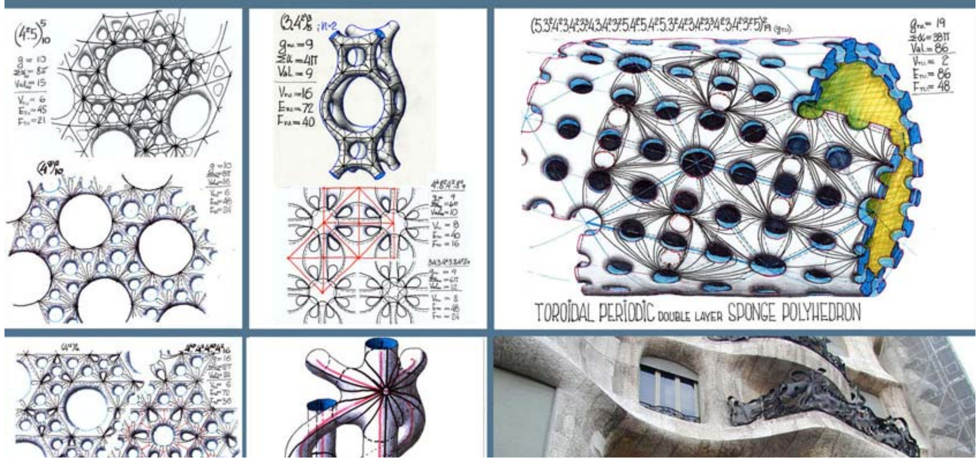


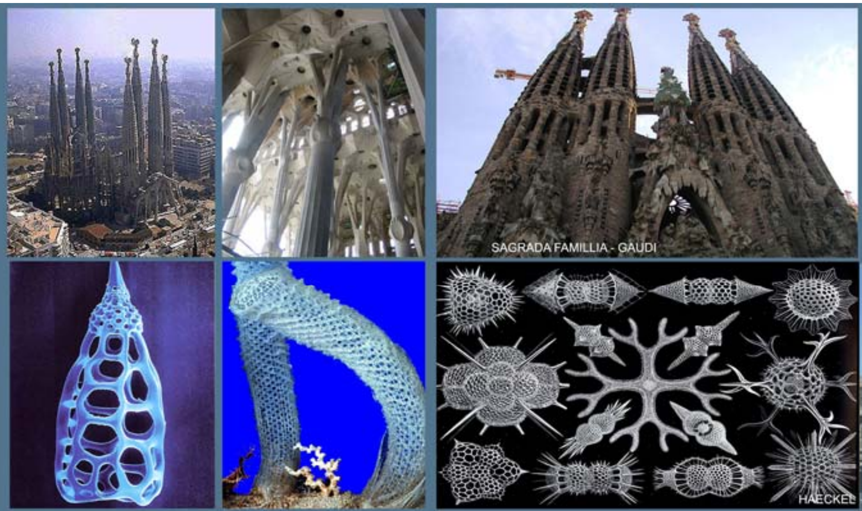



(4.6.4)6 5
gTU = 5
Σa.v. = 10π
Va.v. = 18
gTU = 2n+1
Σa.v. = 10π
Va.v. = 18
VTU = 2n
ETU = 9n
FTU = 4n

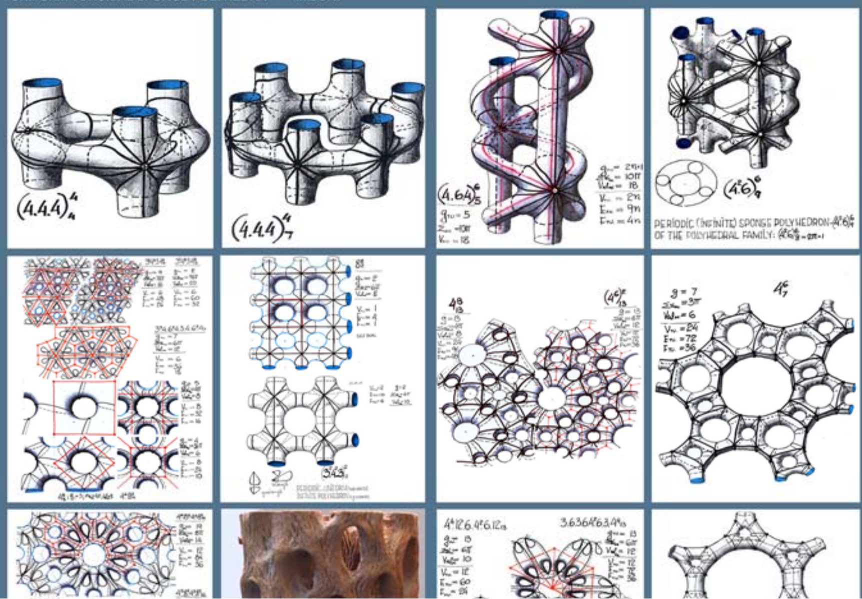

(4².6)6 9
PERIODIC (INFINITE) SPONGE POLYHEDRON–(4².6)6 9
OF THE POLYHEDRAL FAMILY: (4².6)6 g = 2n+1

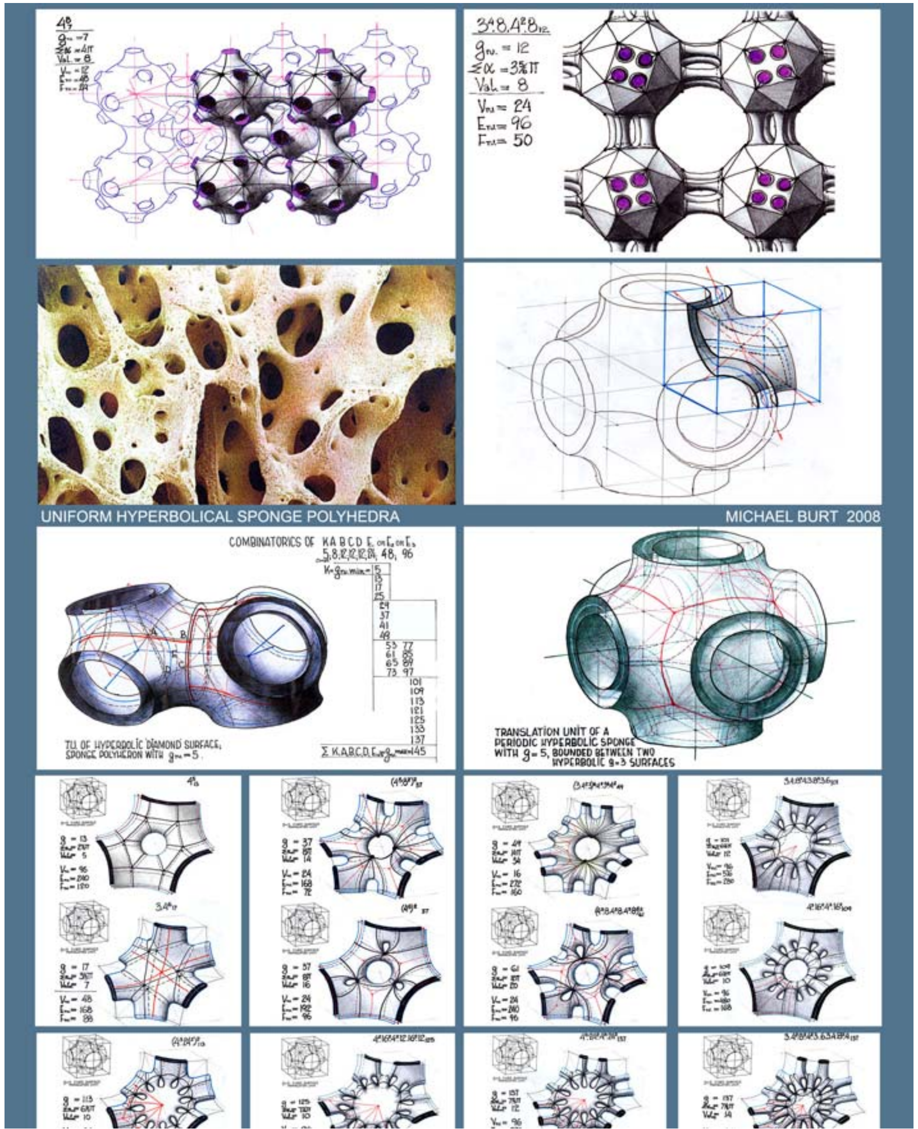

T.U.

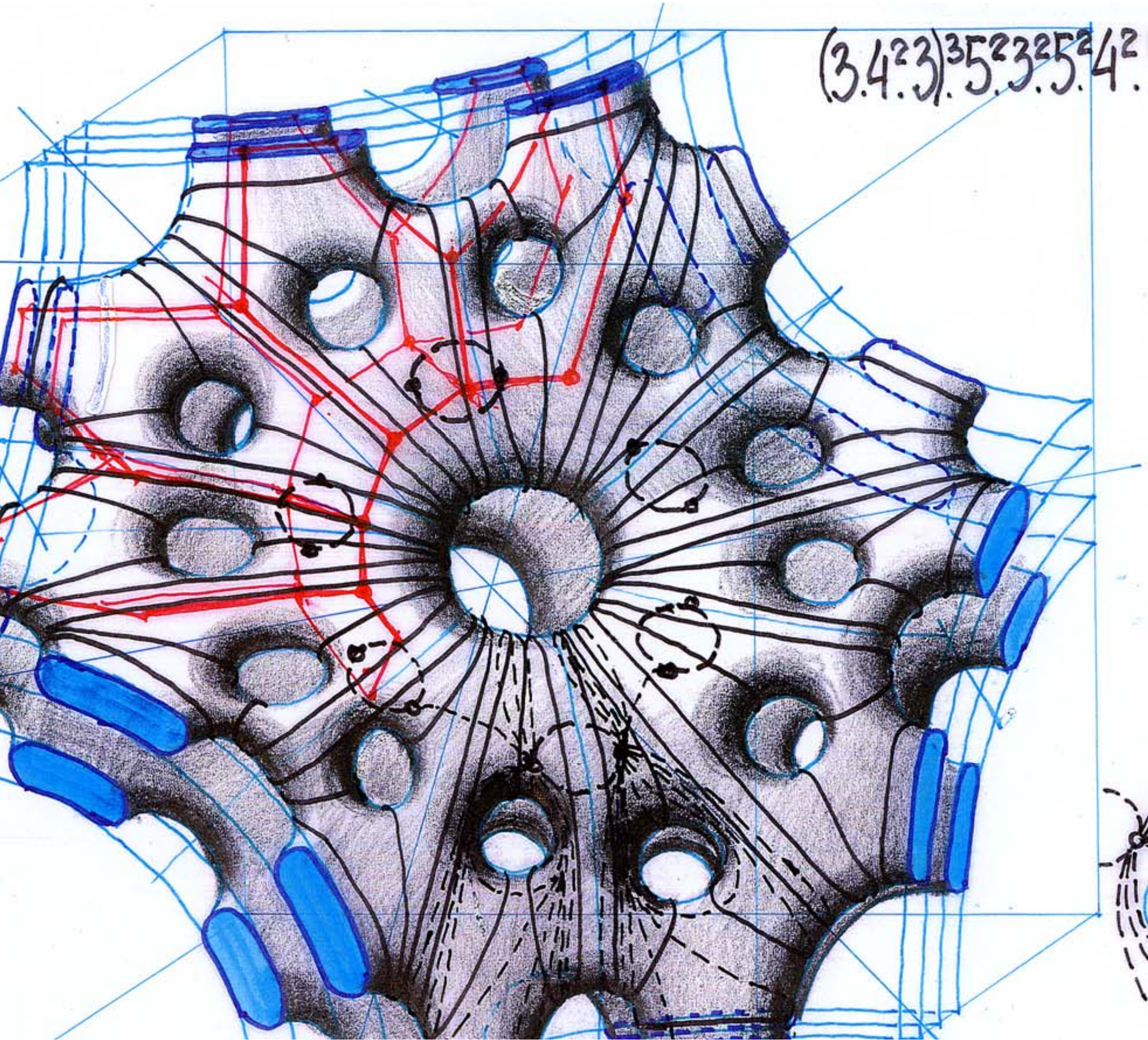

TOROIDAL PERIODIC DOUBLE LAYER SPONGE POLYHEDRON



UNIFORM TOROIDAL SPONGE POLYHEDRA M.BURT




UNIFORM HYPERBOLICAL SPONGE POLYHEDRA
MICHAEL BURT 2008
Val. = 8
V = 24
E = 96
F = 50
TU OF HYPERBOLIC DIAMOND SURFACE, SPONGE POLYHEDRON WITH g = 5.
TRANSLATION UNIT OF A PERIODIC HYPERBOLIC SPONGE WITH g = 5, BOUNDED BETWEEN TWO HYPERBOLIC g = 3 SURFACES



$(3.4^2.3)^3.5^2.3^2.5^2.4^2.5.3.4^2.3.5.4^2.5^2.3^2.5^2_{337}$

$g_{TU.} = 337$

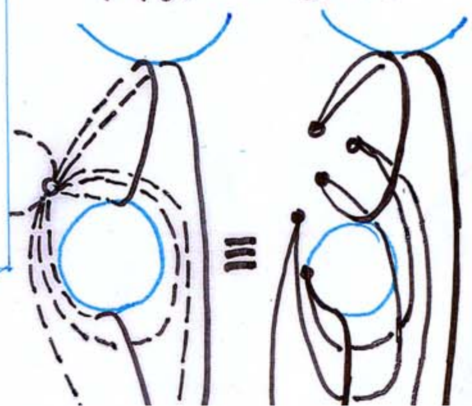
$\sum \alpha_{av.} = 16\pi$

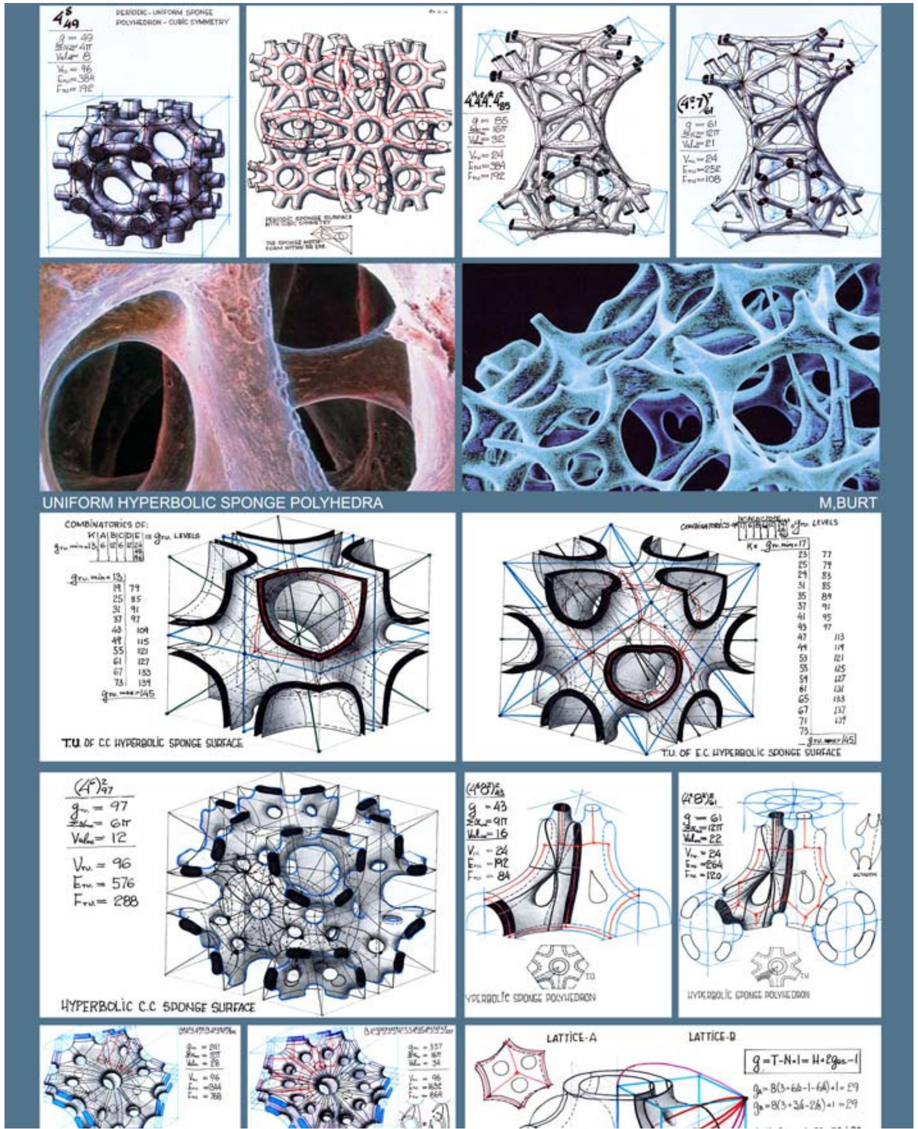
$Val_{av.} = 34$

$V_{TU.} = 96$

$E_{TU.} = 1632$

$F_{TU.} = 864$



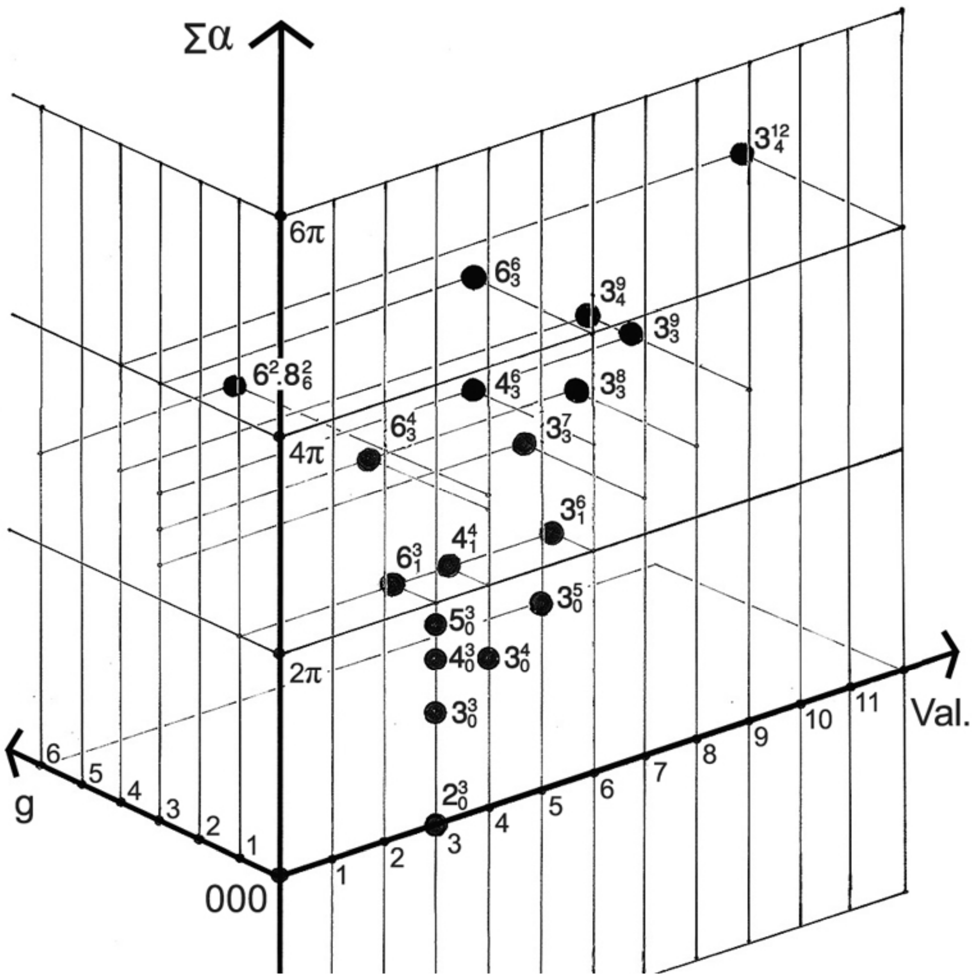

PERIODIC - UNIFORM SPONGE
POLYHEDRON - CUBIC SYMMETRY
UNIFORM HYPERBOLIC SPONGE POLYHEDRA
M,BURT
T.U. OF C.C HYPERBOLIC SPONGE SURFACE
T.U. OF E.C. HYPERBOLIC SPONGE SURFACE
HYPERBOLIC C.C SPONGE SURFACE
HYPERBOLIC SPONGE POLYHEDRON
LATTICE-A
LATTICE-B

‘THE PERIODIC TABLE OF THE POLYHEDRAL UNIVERSE’

The periodic Table of the Polyhedral Universe is a **tabular arrangement of all known and hypothetical polyhedra, which comply with the celebrated Euler’s formula** . The ‘Table’ is constructed on the basis of **thoroughly selected primary parameters of the polyhedral phenomenon,** namely the $\mathbf{Val._{AV.}}$ – (Average Valency – number of edges of a polyhedron which meet in a vertex), $\boldsymbol{\Sigma\alpha._{AV.}}$ – (Average Sum of angles of the face polygons in a vertex, and in a wider sense, the total average curvature of a vertex region), and **g** – (genus of the 2-d manifold of the polyhedron)

e primary parameters of *V*al.$_{AV.}$, $\Sigma\alpha_{AV.}$
d *g*, seem to capture the essence of the
lyhedral topological nature, and when
ed as coordinates of a Cartesian 3D
ace, provide for an environment, in
hich every conceivable individual 3D
lyhedron has a unique point
presentation .

ll shared properties are posing as
scernible, mathematically embraced
cation patterns



Σα
6π
4π
2π
000
g
1
2
3
4
5
6
Val.
1
2
3
4
5
6
7
8
9
10
11
3^{12}_{4}
6^{6}_{3}
3^{9}_{4}
3^{9}_{3}
$6^{2}.8^{2}_{6}$
4^{6}_{3}
3^{8}_{3}
6^{4}_{3}
3^{7}_{3}
3^{6}_{1}
4^{4}_{1}
6^{3}_{1}
3^{5}_{0}
5^{3}_{0}
4^{3}_{0}
3^{4}_{0}
3^{3}_{0}
2^{3}_{0}

Polyhedra with E=12 within the g=0 domain.

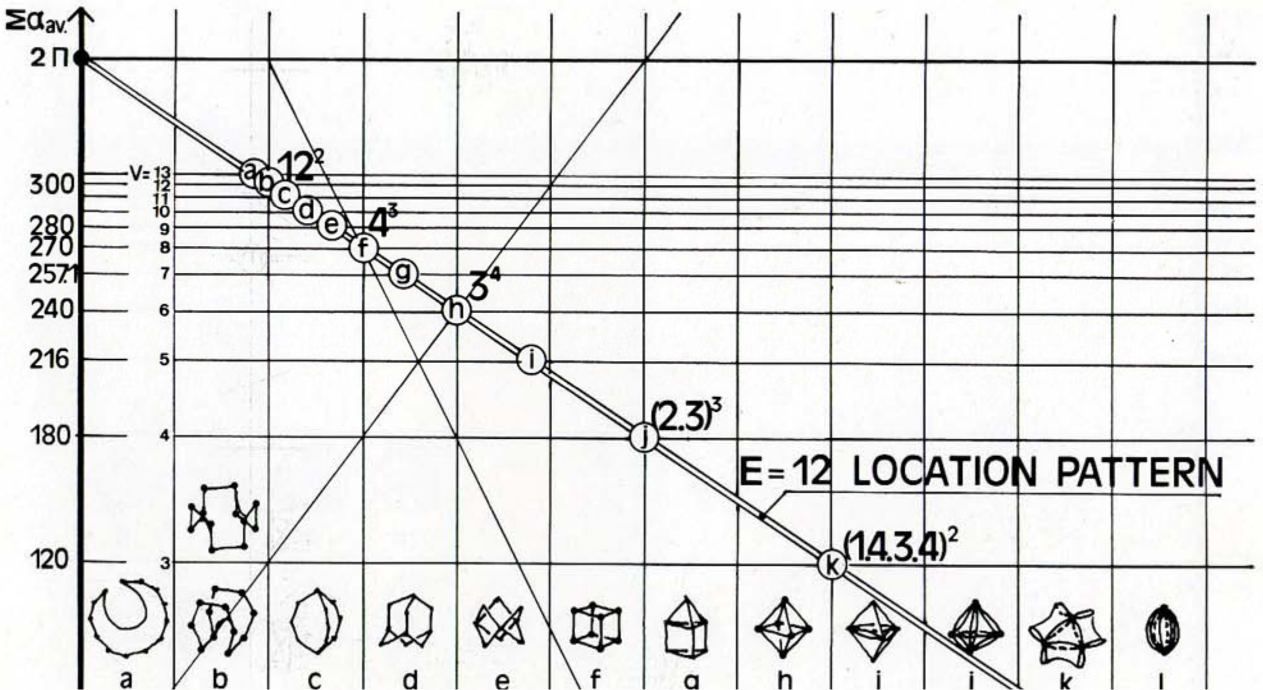
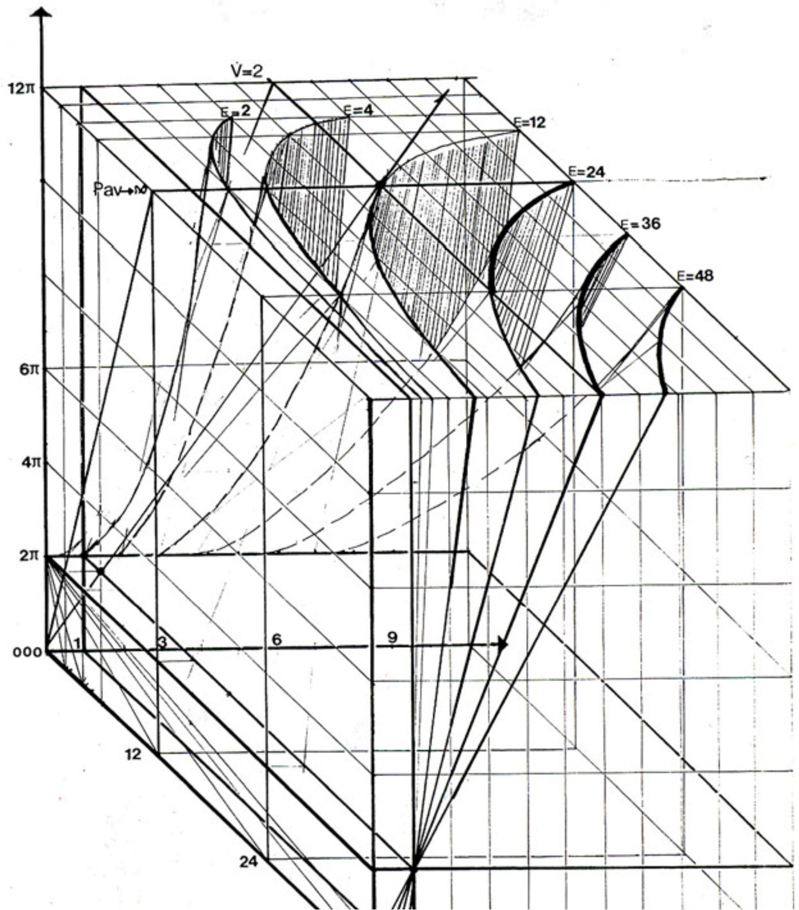
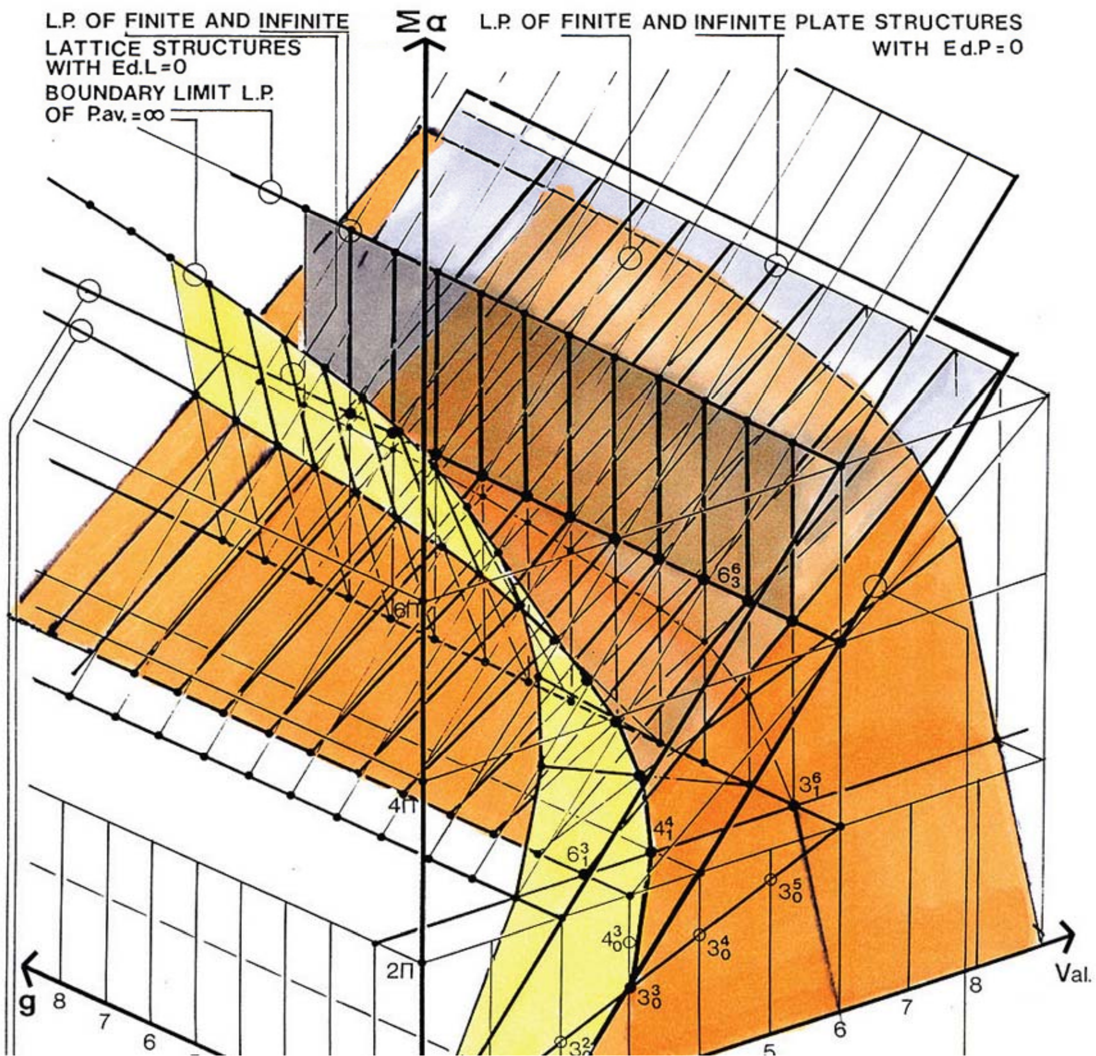
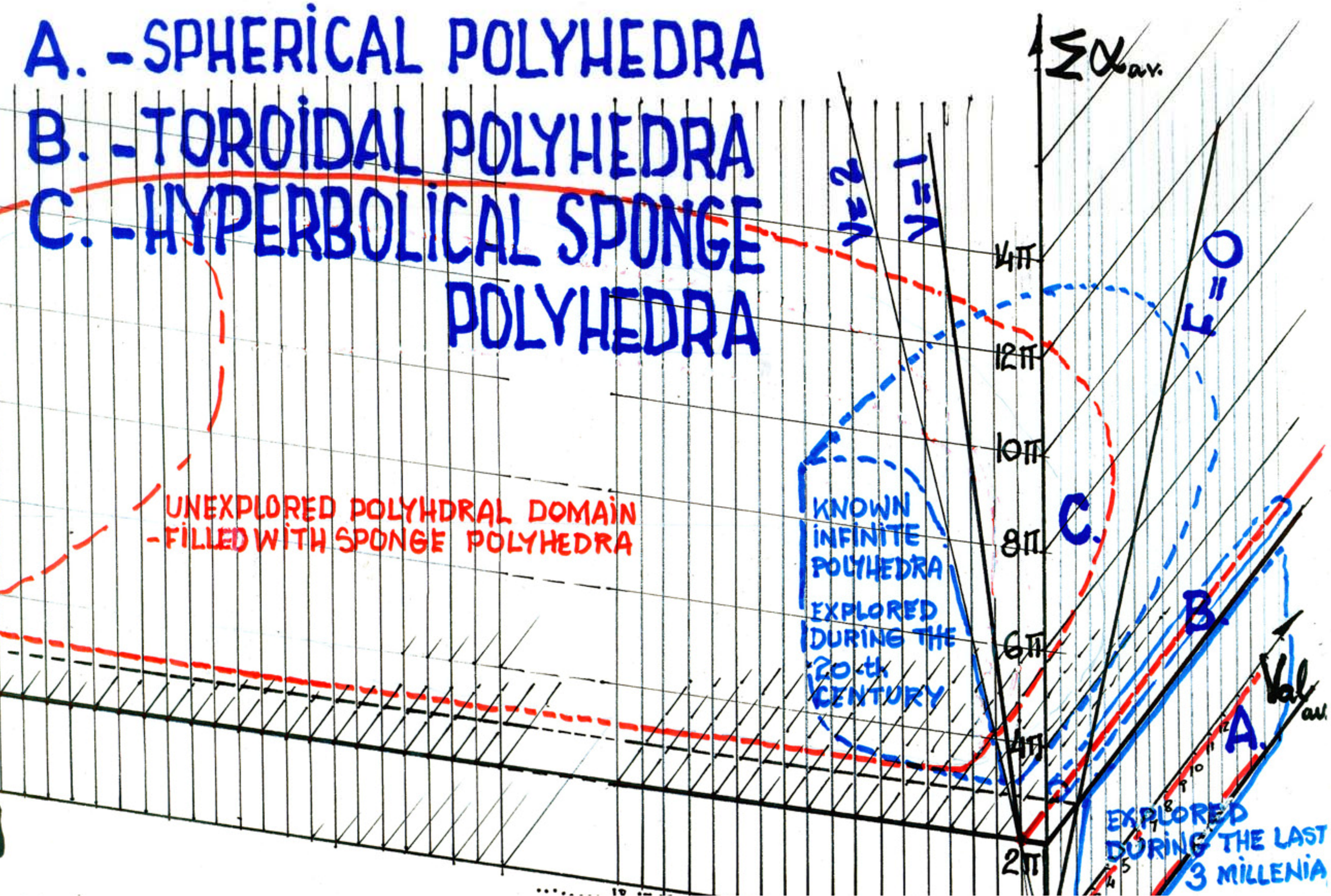


Fig. 34
Quadratic doubly curved surfaces as location patterns of polyhedra sharing the same number of edges.





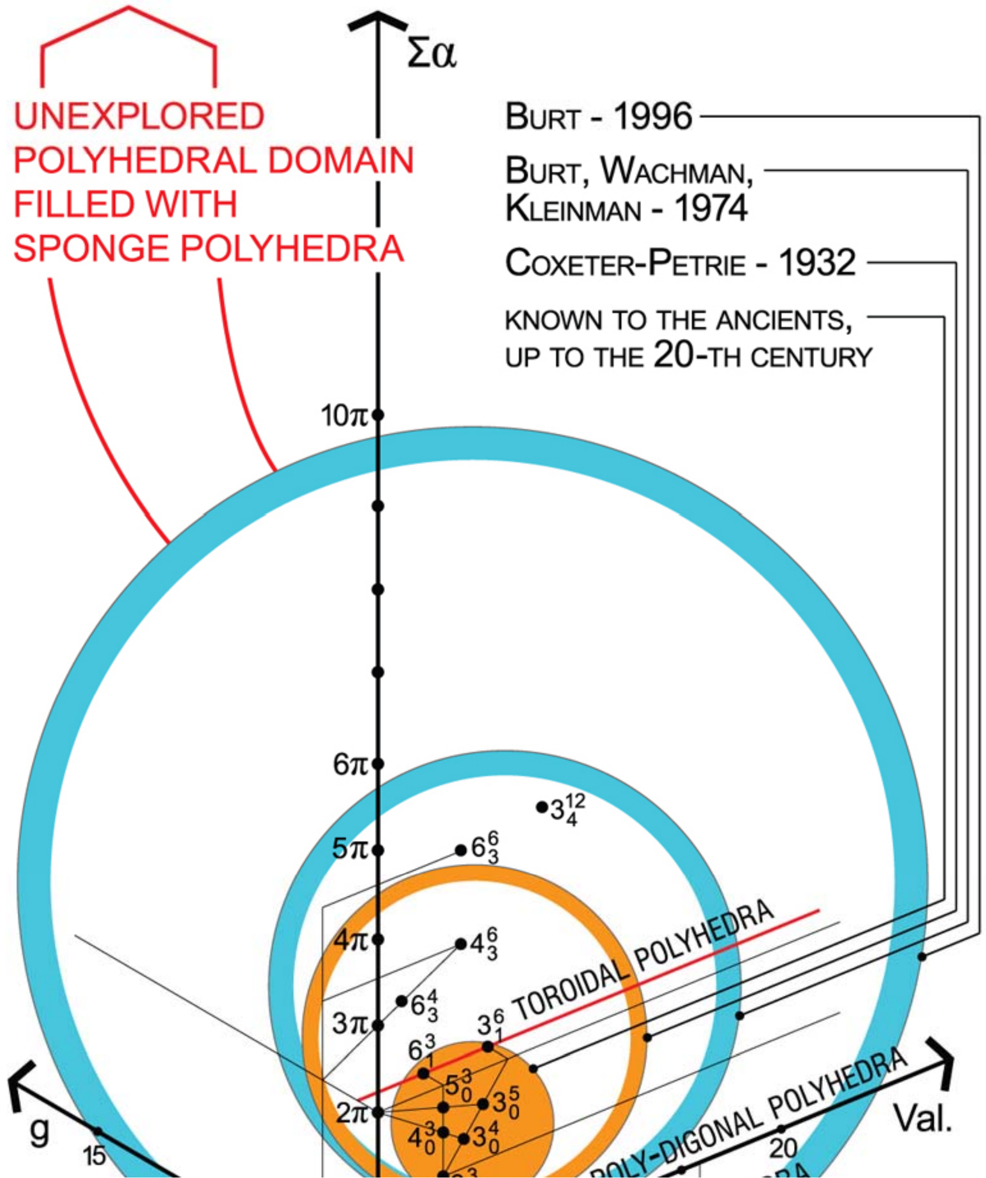
L.P. OF FINITE AND INFINITE
LATTICE STRUCTURES
WITH Ed.L=0
BOUNDARY LIMIT L.P.
OF P.av.=∞
Σα
L.P. OF FINITE AND INFINITE PLATE STRUCTURES
WITH Ed.P=0
6П
4П
2П
g
Val.


A. -SPHERICAL POLYHEDRA
B. -TOROIDAL POLYHEDRA
C. -HYPERBOLICAL SPONGE POLYHEDRA
UNEXPLORED POLYHDRAL DOMAIN
-FILLED WITH SPONGE POLYHEDRA
KNOWN INFINITE POLYHEDRA
EXPLORED DURING THE 20-th CENTURY
EXPLORED DURING THE LAST 3 MILLENIA
Σα av.
Val av.
V=2
V=1
V=0
14π
12π
10π
8π
6π
4π
2π
A.
B.
C.

THE EXPANDING BOUNDARIES OF THE ‘POLYHEDRAL UNIVERSE’

“How far, in terms of the primary parameter values of val.; Σα and g, the theoretically imaginable uniform polyhedra phenomenon may expand (?)” is a mind provoking question and a worthy intellectual challenge for every 3D space explorer.

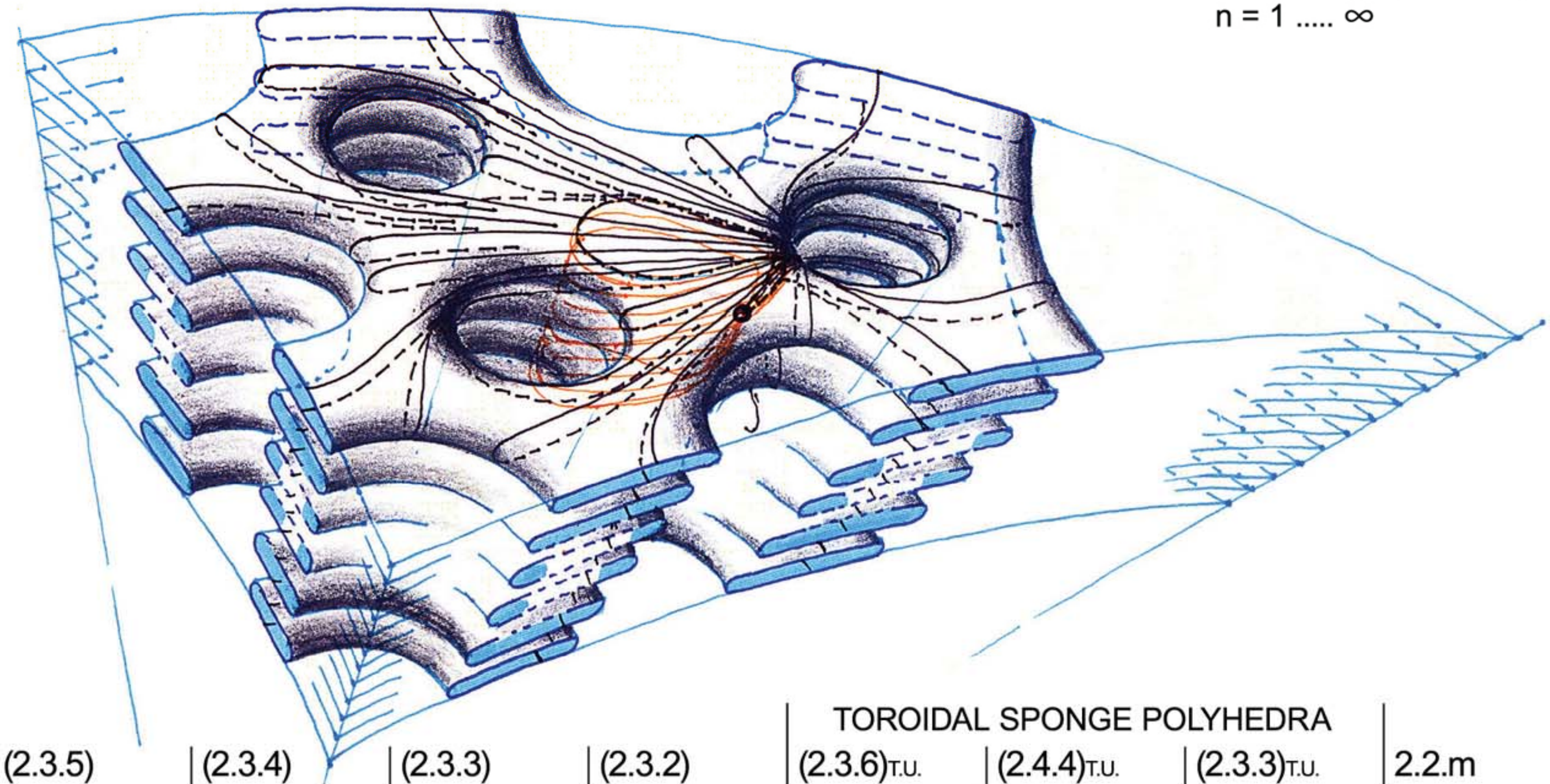
The millennia long accumulated heritage of uniform plane faceted polyhedral shapes, partitions and solids account just for a tiny fraction of the 'theoretically imaginable' universe, even within the g = 0 (spherical – finite) domain and the g = 1 (toroidal) domain.


UNEXPLORED
POLYHEDRAL DOMAIN
FILLED WITH
SPONGE POLYHEDRA
Σα
BURT - 1996
BURT, WACHMAN,
KLEINMAN - 1974
COXETER-PETRIE - 1932
KNOWN TO THE ANCIENTS,
UP TO THE 20-TH CENTURY
10π
6π
5π
4π
3π
2π
3_4^{12}
6_3^6
4_3^6
6_3^4
3_1^6
6_1^3
5_0^3
3_0^5
4_0^3
3_0^4
TOROIDAL POLYHEDRA
POLY-DIGONAL POLYHEDRA
Val.
g
15
20

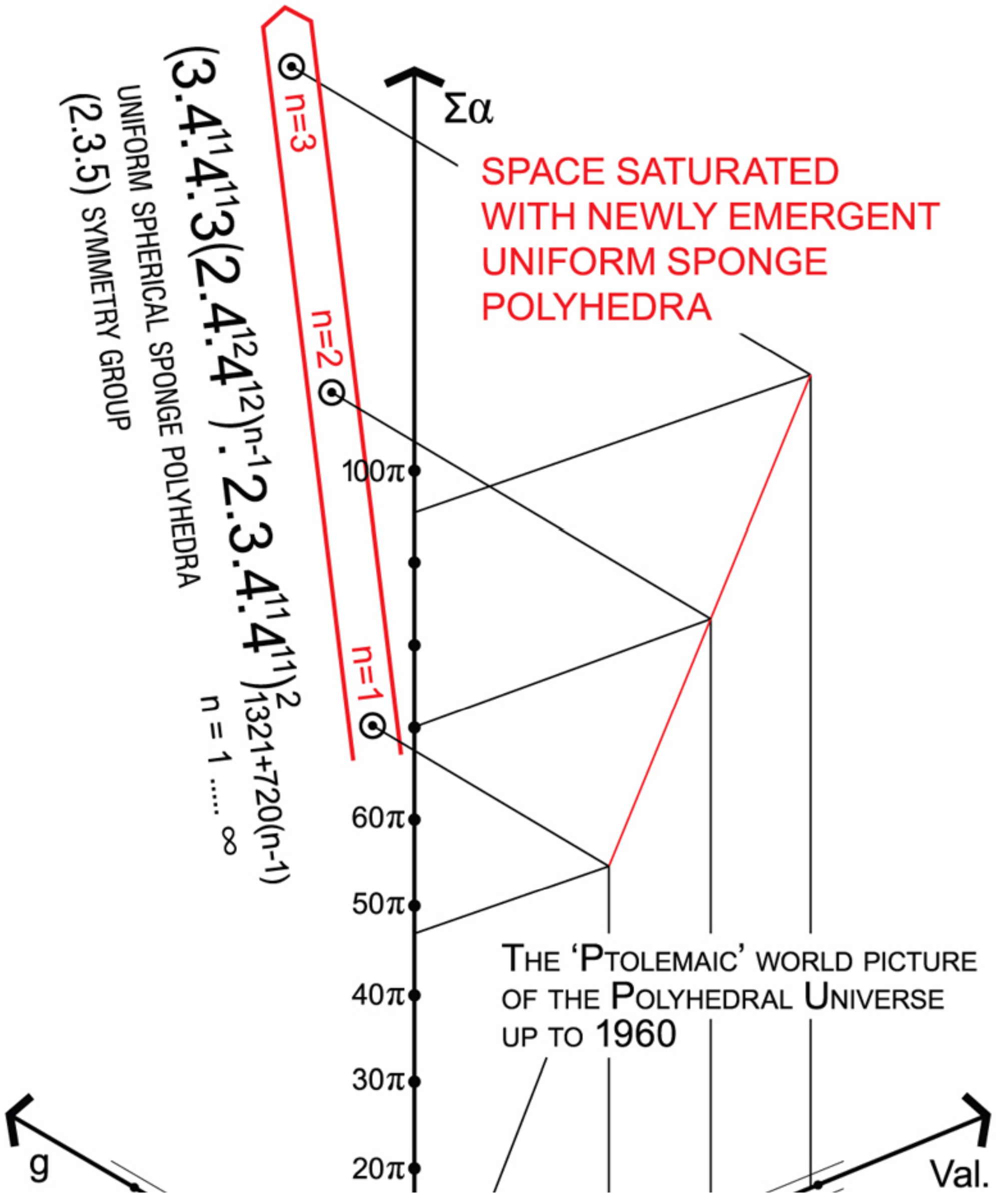
It transpires that hyperbolical sponge surfaces and their related (polyhedral) tessellations are not just the most abundant forms in nature but, also, are overwhelmingly the most abundant and most dominant features of the 'theoretically imaginable' configurations within the Periodic Table's space of the polyhedral universe.

$$(3.4^{11}.4^{11}.3(2.4^{12}.4^{12})^{n-1}.2.3.4^{11}.4^{11})^{2}_{1321+720(n-1)} \quad ;(2.3.5)$$

$$n = 1 \; ..... \; \infty$$



				TOROIDAL SPONGE POLYHEDRA			
(2.3.5)	(2.3.4)	(2.3.3)	(2.3.2)	(2.3.6)T.U.	(2.4.4)T.U.	(2.3.3)T.U.	2.2.m
1321+720(n-1)	529+288(n-1)	265+144(n-1)	133+72(n-1)	133+72(n-1)	89+48(n-1)	67+36(n-1)	(22+12(n-1))m+1
46π+24π(n-1)	⟶	⟶	⟶	⟶	⟶	⟶	⟶
96+50(n-1)	⟶	⟶	⟶	⟶	⟶	⟶	⟶
120	48	24	12	12	8	6	2m



$(3.4.4^{11}4^{11}3(2.4.4^{12}4^{12})^{n-1}.2.3.4.4^{11}4^{11})^2_{1321+720(n-1)}$
n = 1 .... ∞
UNIFORM SPHERICAL SPONGE POLYHEDRA
(2.3.5) SYMMETRY GROUP
n=3
n=2
n=1
Σα
SPACE SATURATED WITH NEWLY EMERGENT UNIFORM SPONGE POLYHEDRA
100π
60π
50π
40π
30π
20π
THE 'PTOLEMAIC' WORLD PICTURE OF THE POLYHEDRAL UNIVERSE UP TO 1960
g
Val.

Any significant venture into the field of periodic sponge surfaces and polyhedra dictates a systematic exploration of the uniform space lattice domain.

It came as a shocking surprise to realize that in spite of the great efforts of the last three centuries or so, in the exploration of the structure of matter and space (crystallography included), **no systematic effort was committed to exhaustively explore the network domain in the "abstract realm of the theoretically imaginable"**

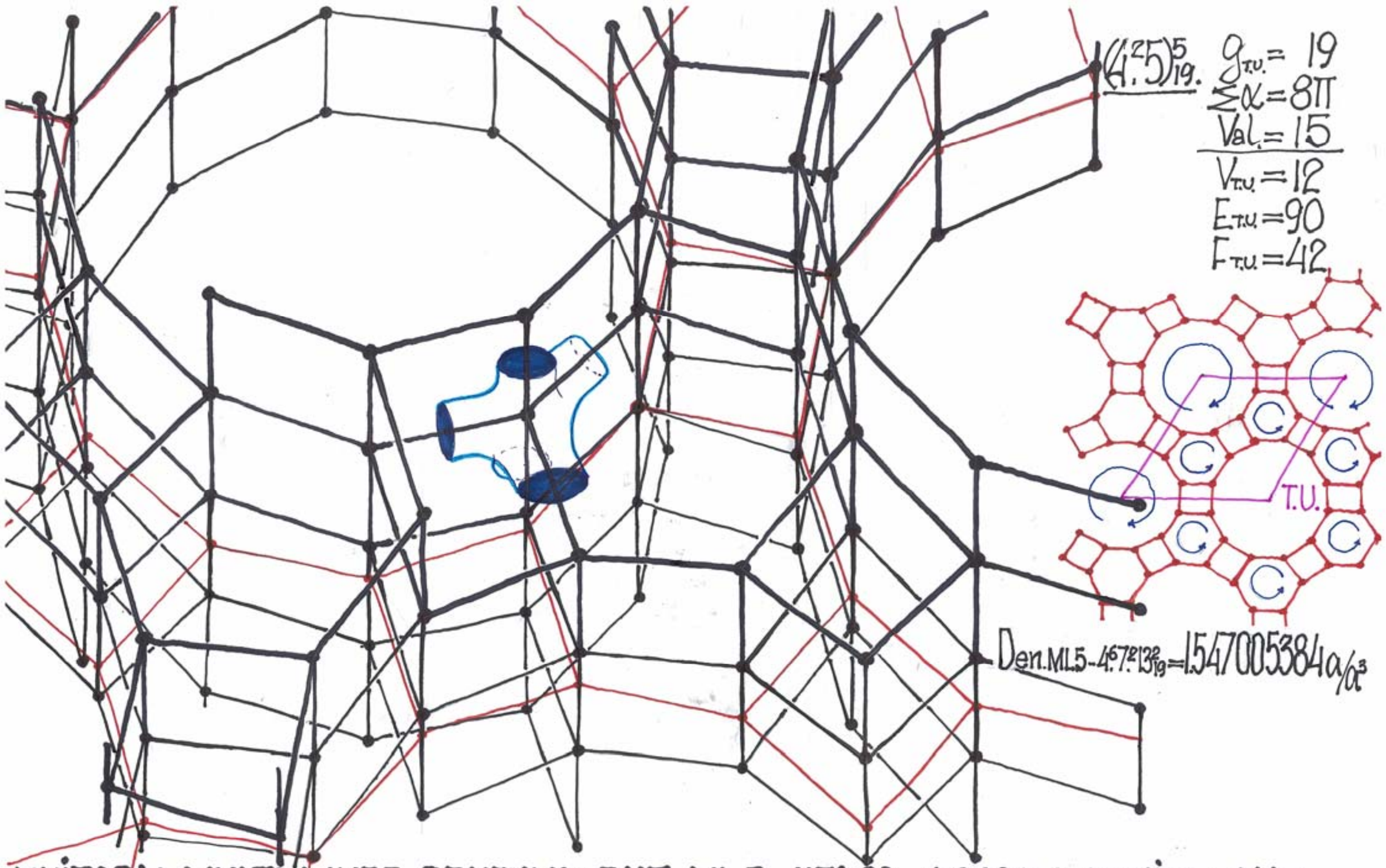
In his monumental publication on ‘Structural inorganic chemistry’ **A.F. Wells** makes a startling observation: “**The theory of these nets does not appear to be known, and in fact no attempt to derive them systematically seems to have been made** until comparatively recently”.

A periodic ordered space network may be formed-generated through one of the following processes:

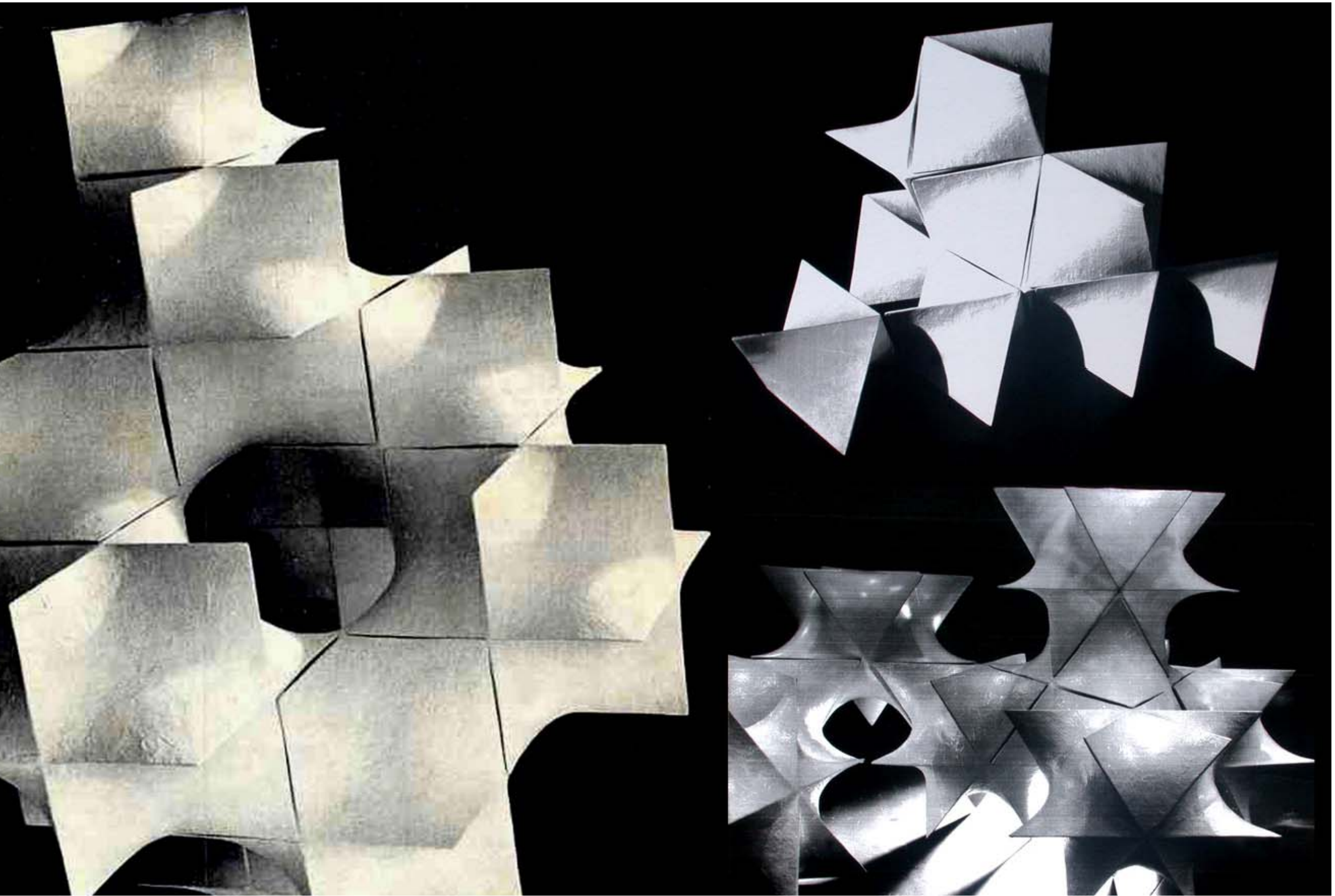
- By an extended repetition of a locally and globally symmetrical association of vertex figures. It amounts nearly to the same as planting a vertex-edge motif –form in an Elementary Periodic Region (E.P.R) of the network, characteristic of a given symmetry group, and symmetrically replicating it ..

- As a result of a close (compact) packing of polyhedral cells, the vertex-edge array of which combine to form the network .

- As a result of a tessellation-mapping process of an unbounded periodic (2d-manifold) surface, spherical, toroidal or hyperbolical, leading eventually to a connected 3D network.



UNIFORM MULTI-LAYER PENTAVALENT ML5-$4^6.7^2.13^2_{19}$ SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON



= 6

$=2\frac{5}{6}\pi$

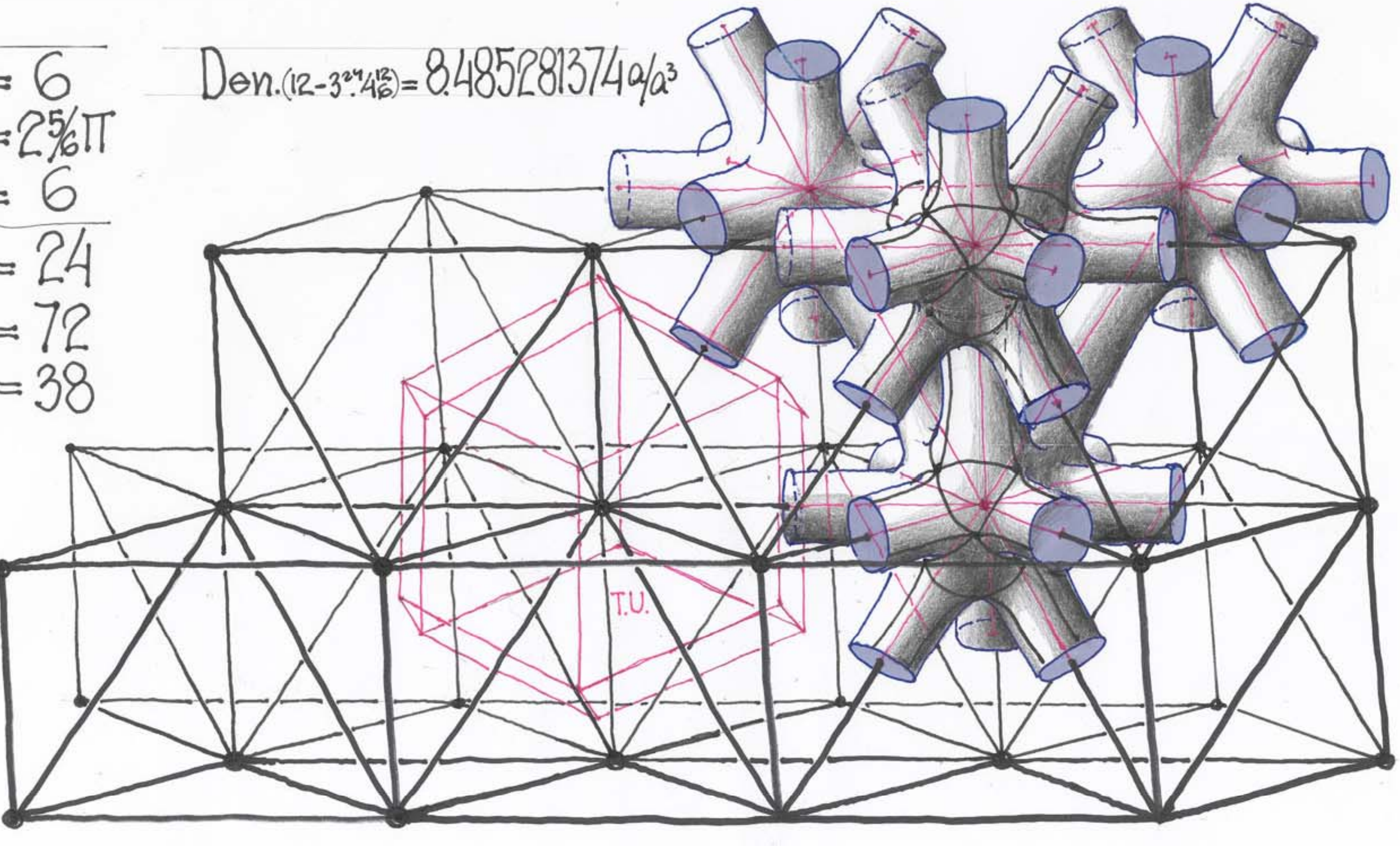
= 6

= 24

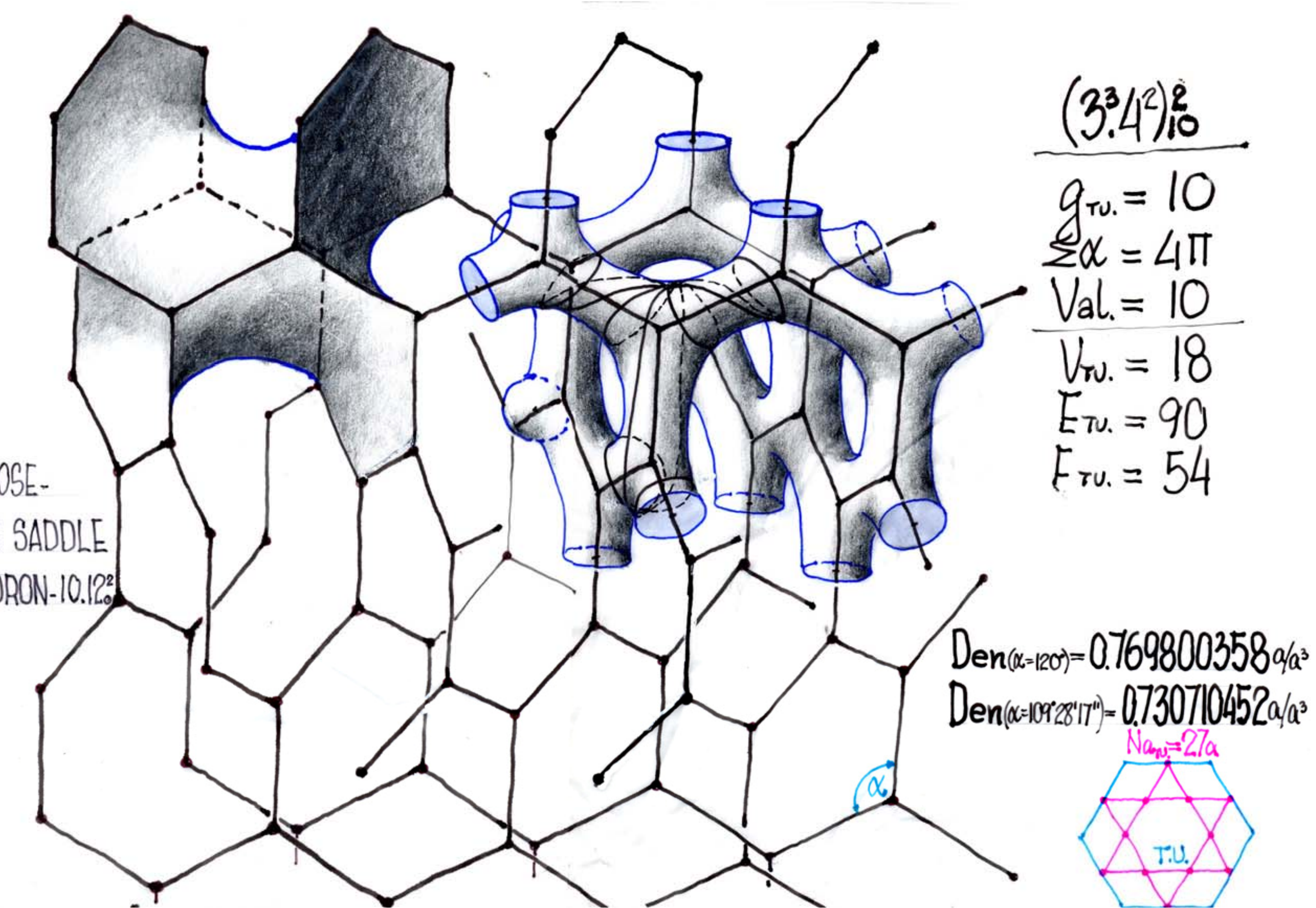
= 72

= 38

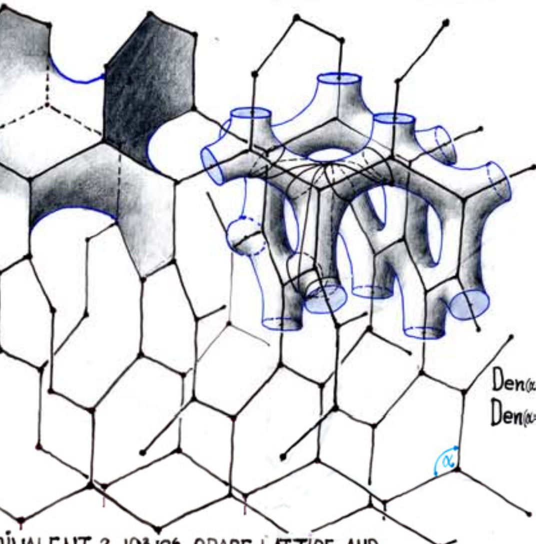
$Den._{(12-3^{24}.4^{12}_{6})} = 8.485281374\, a/a^3$



ORM POREQUIVALENT IS $3^{24}4^{12}$ SPACE LATTICE (OCTET LATTICE)



OSE-
SADDLE
DRON-10.12
(3³.4²)²₁₀
g T.U. = 10
Σα = 4π
Val. = 10
V T.U. = 18
E T.U. = 90
F T.U. = 54
Den(α=120°) = 0.769800358 a/a³
Den(α=109°28'17'') = 0.730710452 a/a³
Na T.U. = 27a
α
T.U.



$(3^3 4^2)^{2}_{10}$

$g_{TU.} = 10$

$\Sigma\alpha = 4\pi$

Val. = 10

$V_{TU.} = 18$

$E_{TU.} = 90$

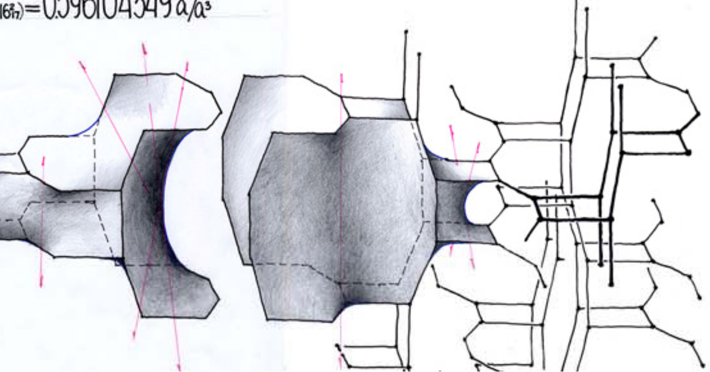
$F_{TU.} = 54$

$Den_{(\alpha=120°)} = 0.769800358\, a/a^3$

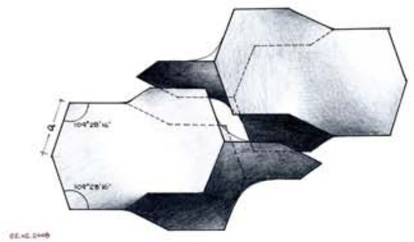
$Den_{(\alpha=109°28'17'')} = 0.730710452\, a/a^3$

...IVALENT 3-10³.12⁴₁₀ SPACE LATTICE AND

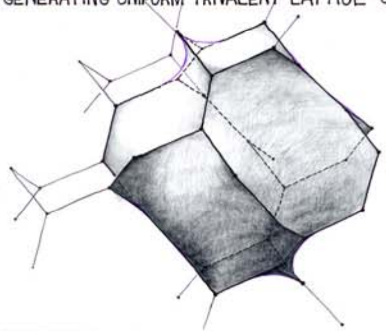
...$= 0.596104349\, a/a^3$

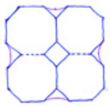


DECA-TETRAHEDRON, A SELF CLOSE-PACKING SADDLE-POLYHEDRON, GENERATING THE UNIFORM TRIVALENT SPACE LATTICE-10³.

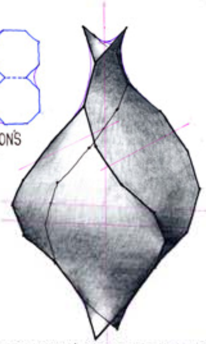


SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE- 6.10⁵₄

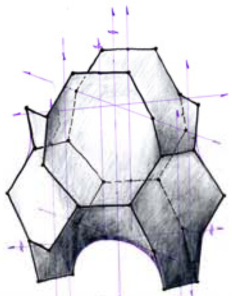




POLYHEDRONS OVERVIEW

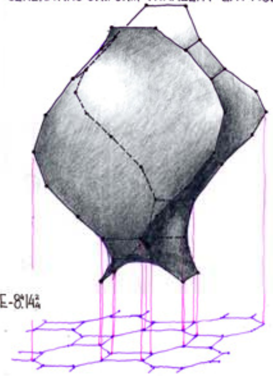


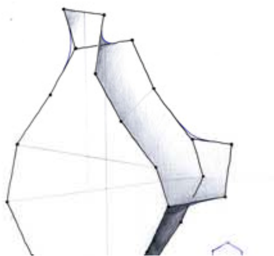
SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE-4.14...



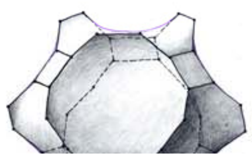
SELF CLOSE PACKING SADDLE POLYHEDRON GENERATING UNIFORM TRIVALENT SPACE LATTICE-8.14...

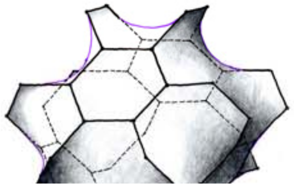
SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE-4.8.16...

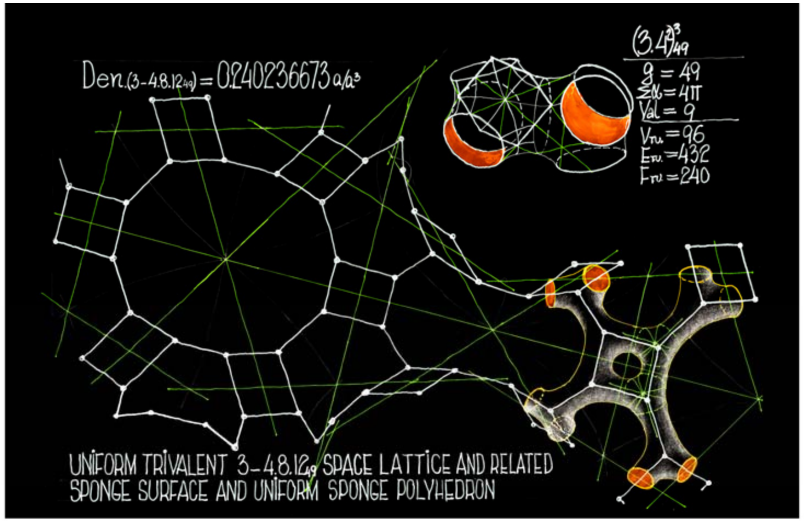


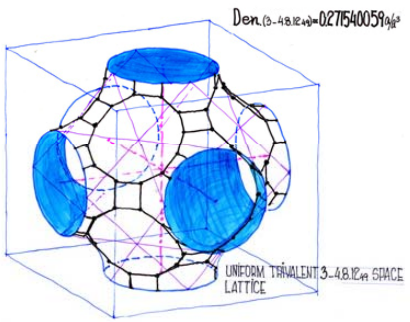


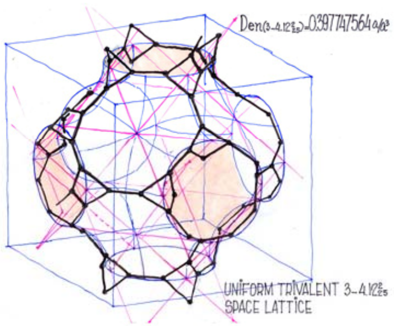
SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE - THE 4.12...

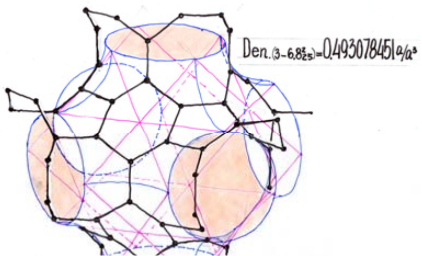


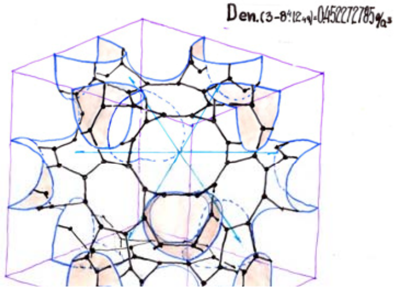



Den.(3-4.8.12₄₉) = 0.240236673 a/a³
(3.4²)³₄₉
g = 49
Σα = 4π
Val = 9
Vru = 96
Eru = 432
Fru = 240
UNIFORM TRIVALENT 3-4.8.12₄₉ SPACE LATTICE AND RELATED
SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON


Den.(3-4.8.12₄₉)=0.271540059 a/a³
UNIFORM TRIVALENT 3-4.8.12₄₉ SPACE
LATTICE


Den(3-4.12)=0.397747564 a/a³
UNIFORM TRIVALENT 3-4.12
SPACE LATTICE


Den.(3-6.8)=0.493078451 a/a³


Den.(3-8.12)=0.452272785 a/a³

$3\text{-}6.8^2_{49}$ Den.$_{(3\text{-}6.8^2_{49})}$ $= 0.47140452\ a/a^3$

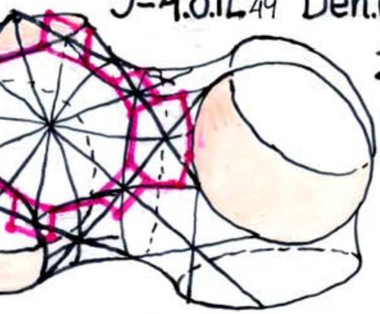
$\Sigma\alpha = 390°$

$Con_{T.U.} \equiv (g_{T.U.}) = 49$

$3\text{-}4.8.12_{49}$ Den.$_{(3\text{-}4.8.12_{49})}$ $= 0.240236673\ a/a^3$

$\Sigma\alpha = 375°$

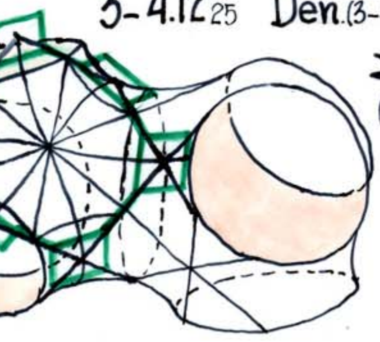
$Con_{T.U.} \equiv (g_{T.U.}) = 49$

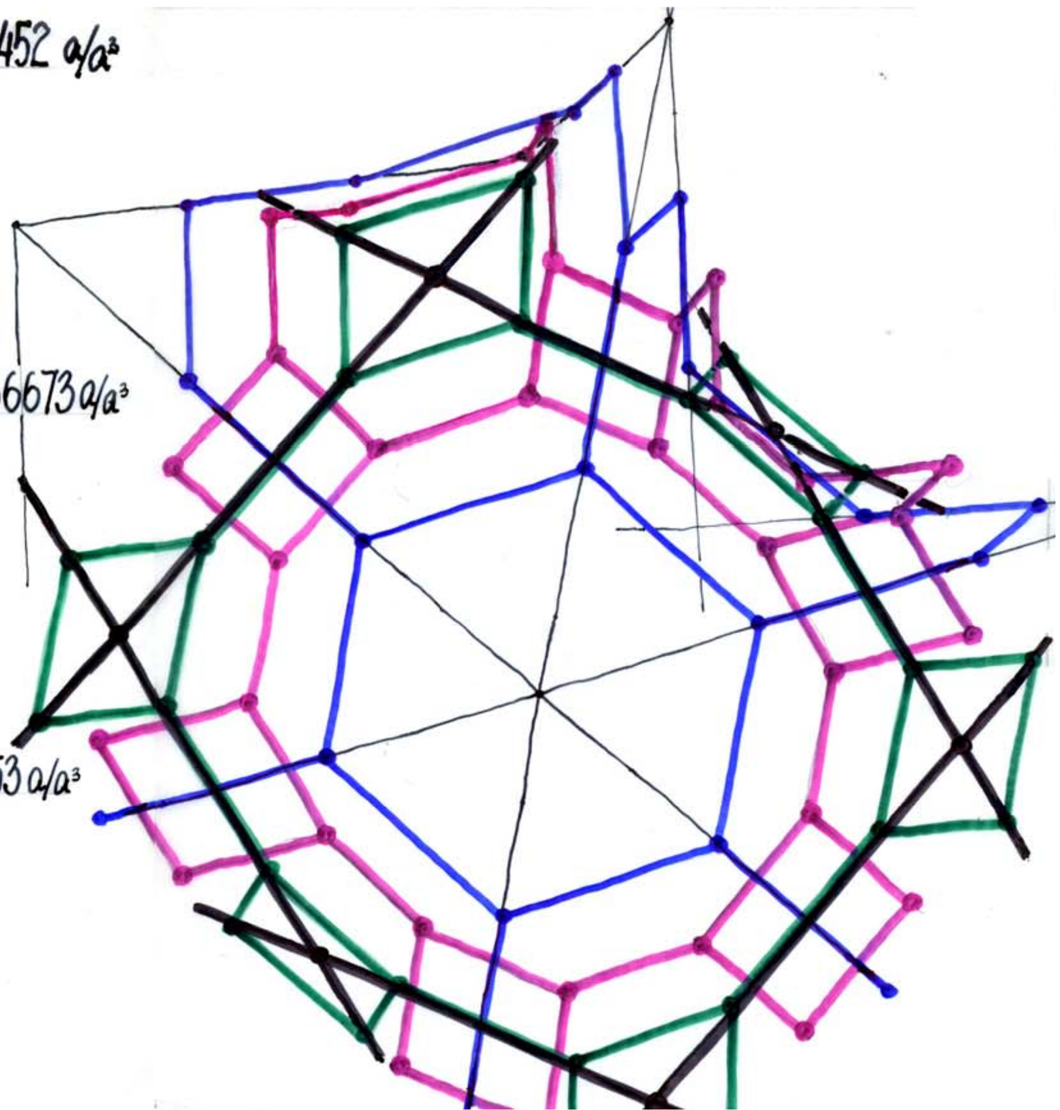


$3\text{-}4.12^2_{25}$ Den.$_{(3\text{-}4.12^2_{25})}$ $= 0.319805153\ a/a^3$

$\Sigma\alpha = 390°$

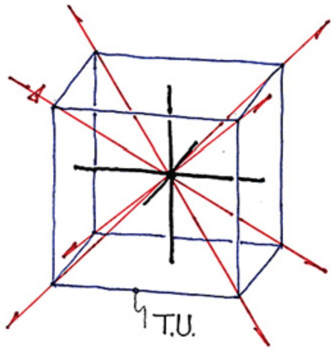
$Con_{T.U.} \equiv (g_{T.U.}) = 25$





UNIFORM CUBIC
PV.6-4_3^{12} SPACE LATTICE (6;10π;3)

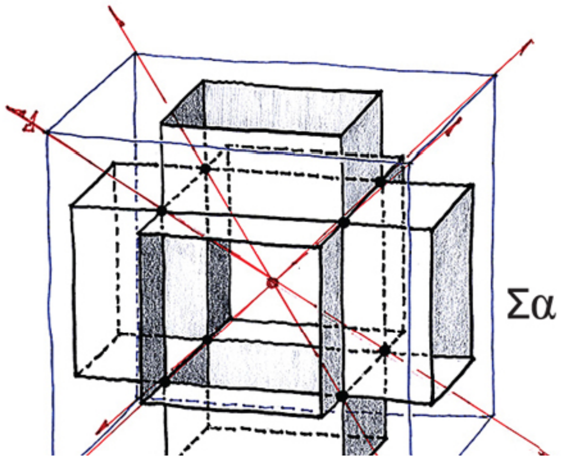
(Val.;Σα;g≡Con.)



$$\Sigma\alpha = 2\pi\left[1 - \frac{2(1-3)}{1}\right] = 10\pi$$

UNIFORM HEXAVALENT
4_3^6 INFINITE POLYHEDRON (6;3π;3)

(Val.;Σα;g)



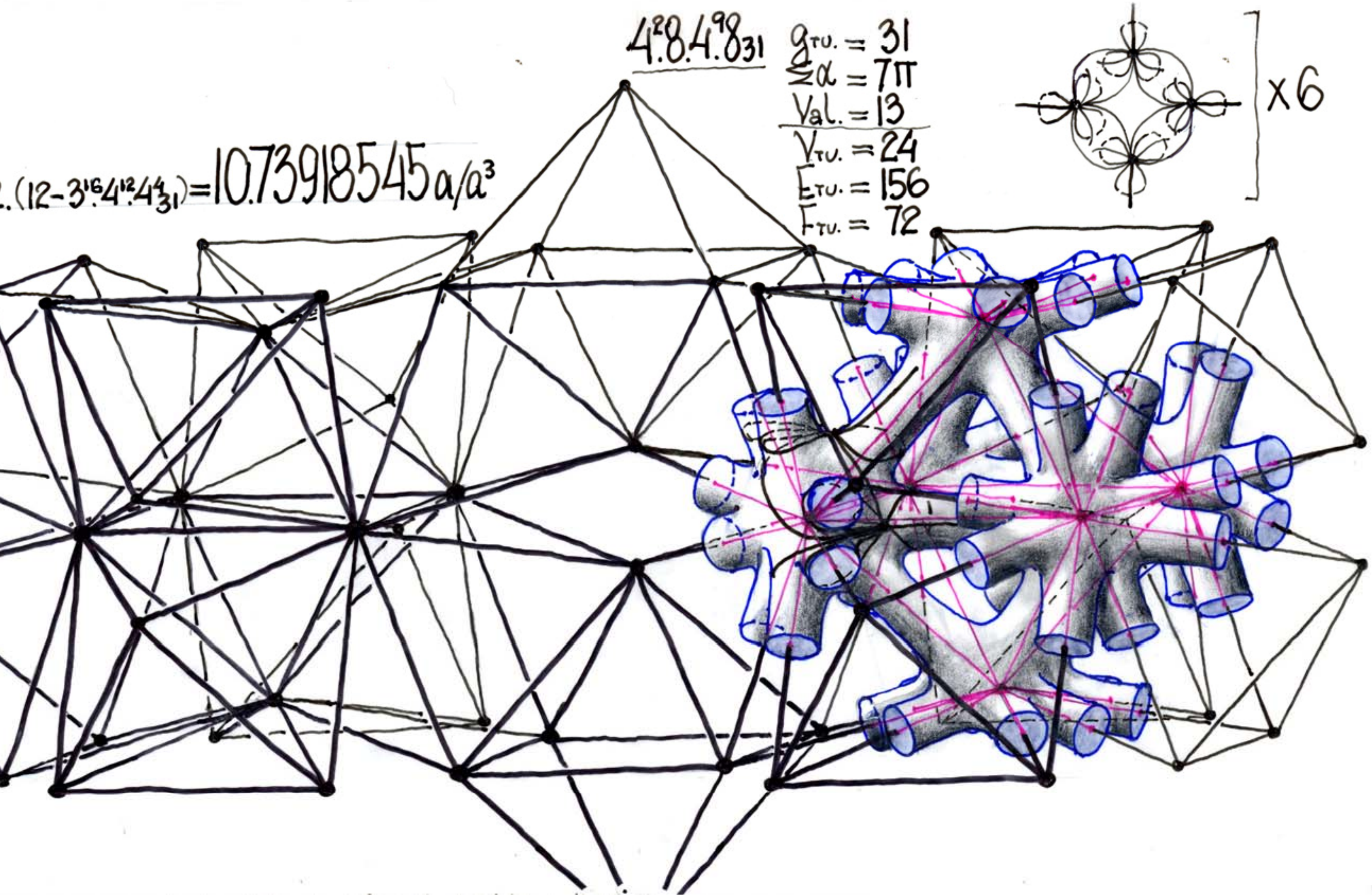
$$\Sigma\alpha = 2\pi\left[1 - \frac{2(1-3)}{8}\right] = 3\pi$$

Spatial density (in terms of a/a^3) of the space lattice (the number of edges of the length of **a** per cubic volume of $\mathbf{a^3}$). As a referential basis we should have in mind that the spatial density of the tetravalent diamond lattice is ~ 1,299a/a^3; that of the hexavalent cubic lattice is 3,000 a/a^3 and that of the dodecavalent octet lattice is ~ 8,485 a/a^3 (By density we refer to the lowest possible value for a distinct topology). **It is of great theoretical interest and probably even of practical importance how far down and up can the density values descend or aspire.**

Uniform Dodecavalent and higher valency Space Lattices or: how far valency and spatial density values can go.

Uniform dodecavalent 'octet' based space lattices exist in more than one topological version, but all come to same spatial density of **8,485281374 a/a^3.**

The infinite sponge polyhedron **$3^{12}{}_4$** gives rise to a uniform dodecavalent **(PV.12-$3^{12}.4^{20}{}_{31}$)** space lattice, the density of which **is 10,73918545a/a^3 (!)**



(12-3¹⁶.4¹².4⁴₃₁)=10.739185 45 a/a³
4².8.4⁹8₃₁
gTU. = 31
Σα = 7π
Val. = 13
VTU. = 24
ETU. = 156
FTU. = 72
x 6

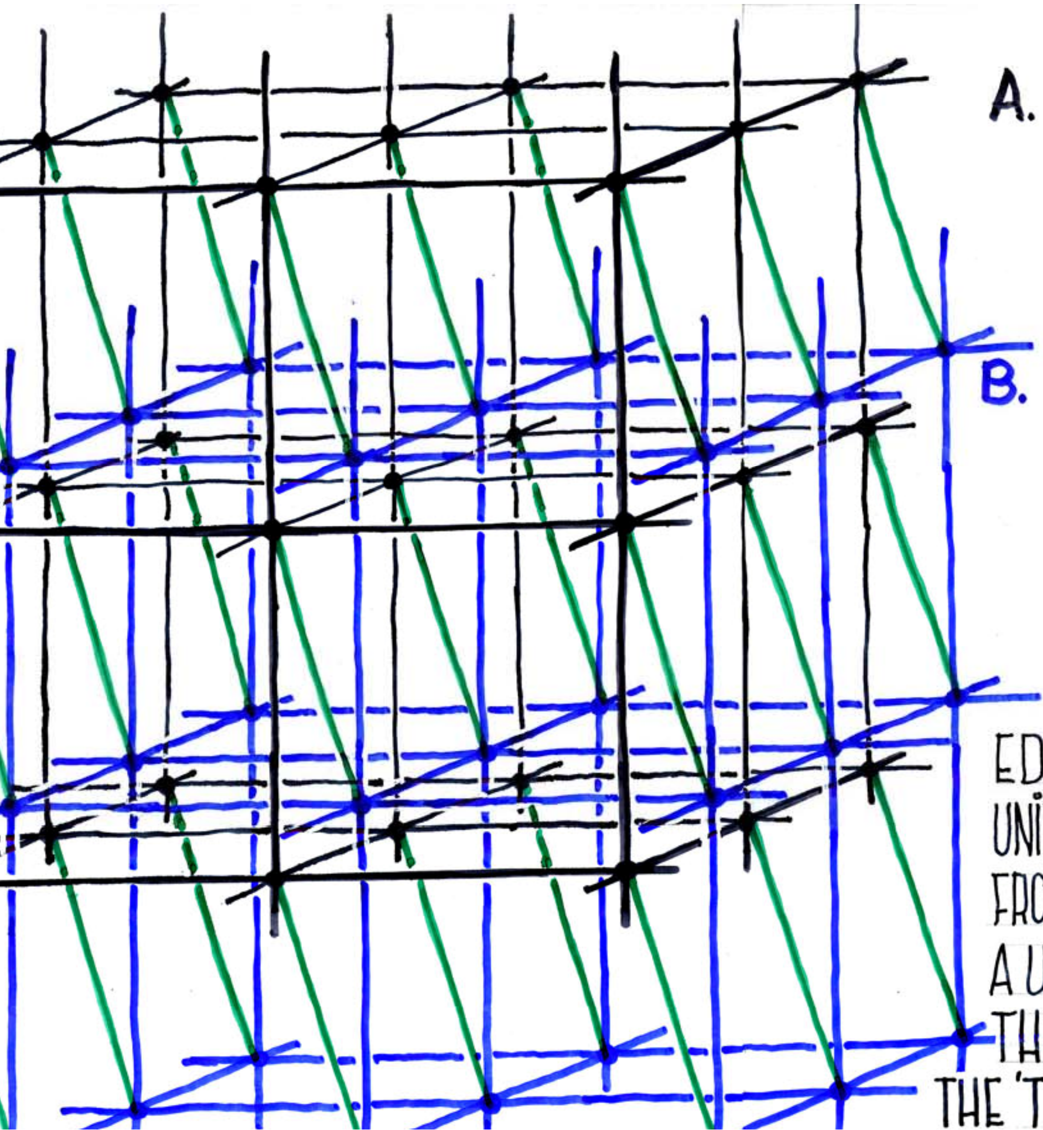
ne quest for higher density networks led to a spatial experiment as llows:

ets perform an **edge-length translation** of a given uniform n-valent ace lattice in an arbitrary direction from position A into position B. ne resulting network is a 4-dimensional feature, the 3-dimensional presentation of which displays a uniform(n+1)- valent lattice as ell.

ne spatial density of the resulting space lattice will be:

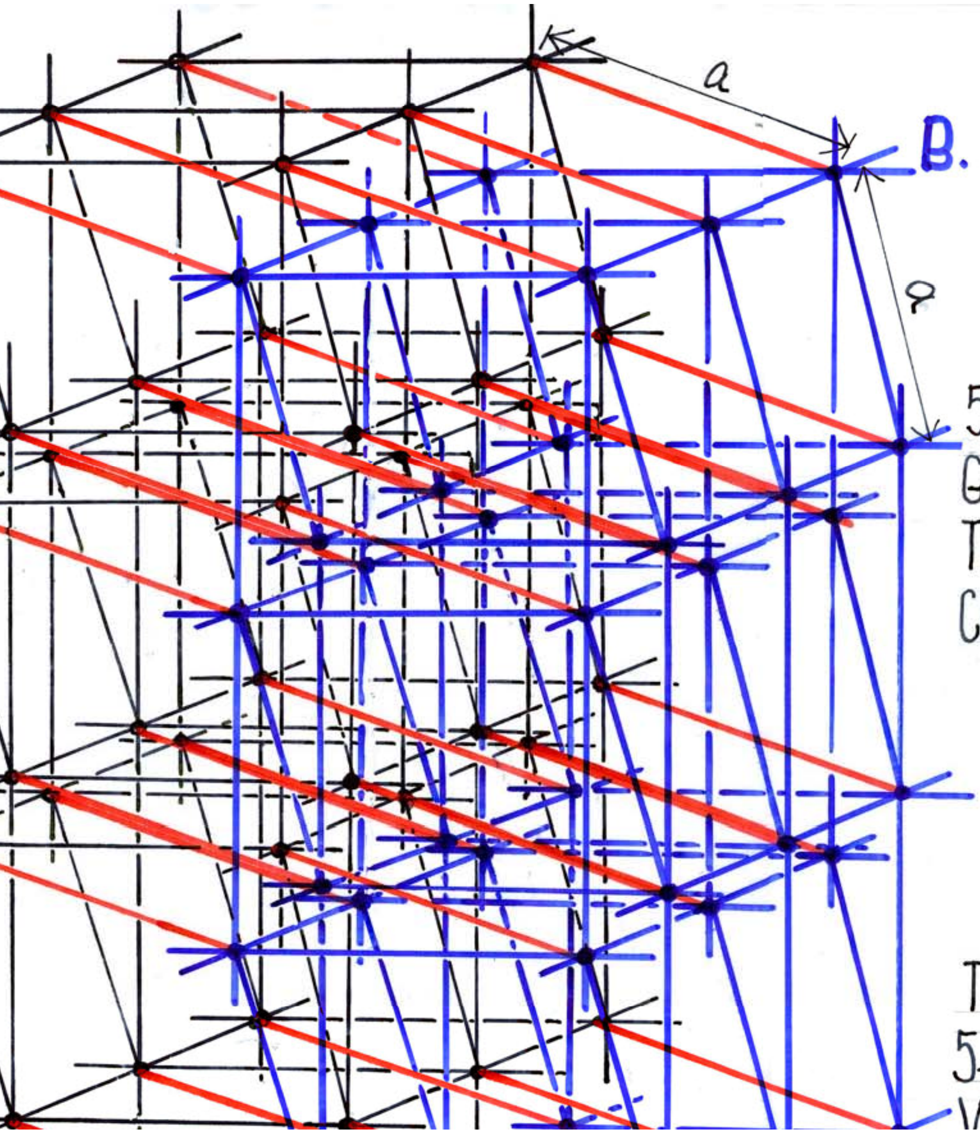
$$\mathbf{en.\ (n+1)} = \frac{\mathbf{Den.(n)}}{\mathbf{E_{T.U.}}}\mathbf{(2E_{T.U.}+1)},$$

with $\mathbf{E_{T.U.}}$ as the number of edges thin the translation unit of the lattice.



ENTANGLED NETWORKS

EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT (CUBIC) LATTICE FROM A-TO B-POSITION, RESULTING IN A UNIFORM SEPTAVALENT LATTICE, THE DENSITY OF WHICH IS 7.00 a/a^3. THE 'TRANSLATION LATTICE' IS A 3-D



ENTANGLED NETWORKS

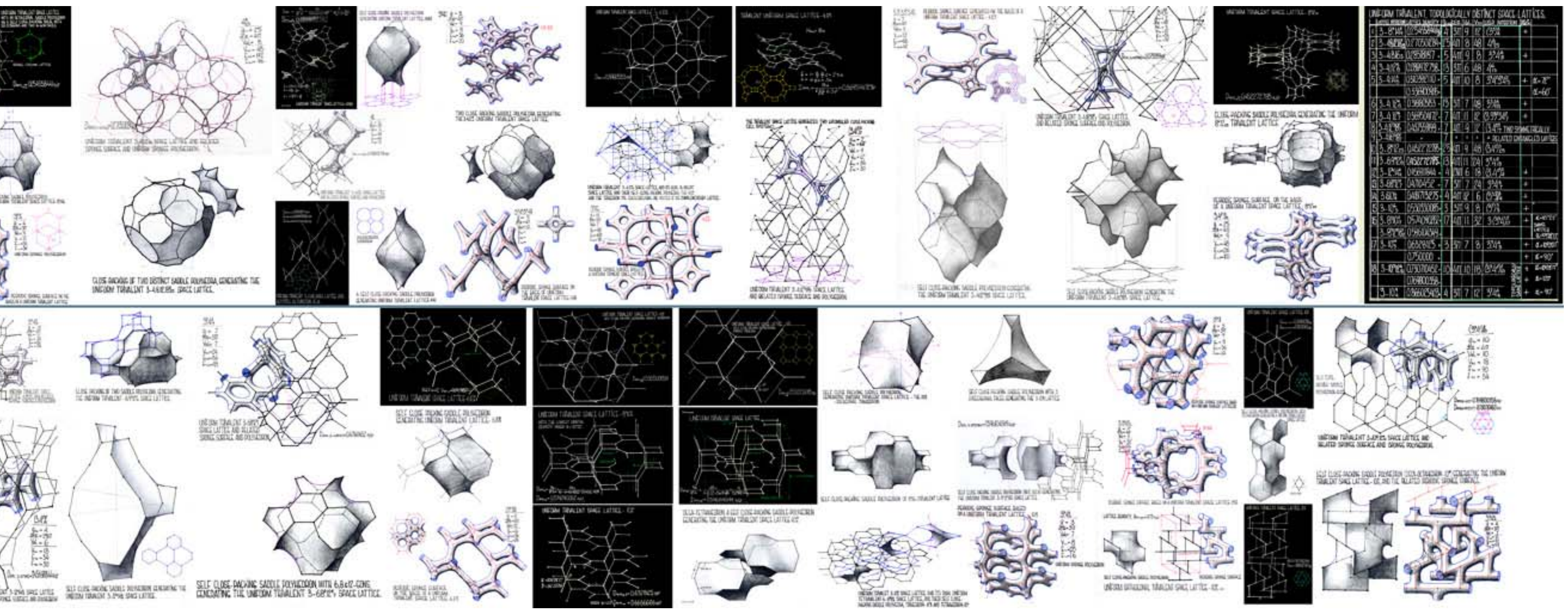
5-DIMENSIONAL OCTAVALENT NETWORK, GENERATED BY A DOUBLE EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT CUBIC SPACE-LATTICE.

THE 3-D REPRESENTASION OF THE 5-D NETWORK IS A UNIFORM OCTA-VALENT SPACE LATTICE WITH A

The edge length translation could be performed **m** times, leading to a uniform **(n+m)**-valent space lattice, the spatial density of which will amount to:

$$\mathbf{Den.(n+m)} = \frac{\mathbf{Den.(n)}}{\mathbf{E_{T.U.}}}\left[\,\mathbf{2^{m}.\,E_{T.U.}} + \frac{\mathbf{(1+m)m}}{\mathbf{2}}\,\right]$$

In fact <u>m</u> and the spatial density values can reach to infinity (!) and that, at least theoretically, without causing any edge intersections. These lattices represent a novel class, that of the **Uniform Entangled Space Lattices.**

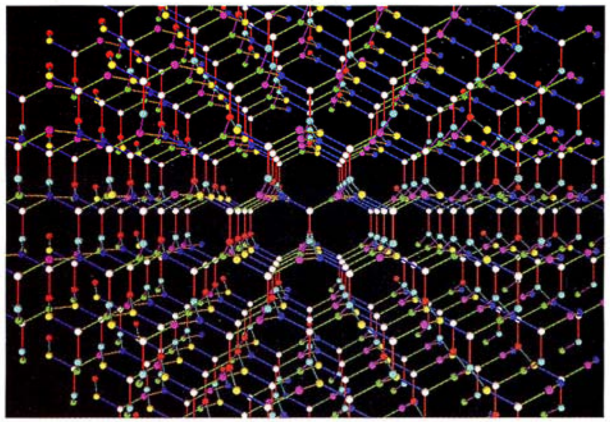


Crystal Math

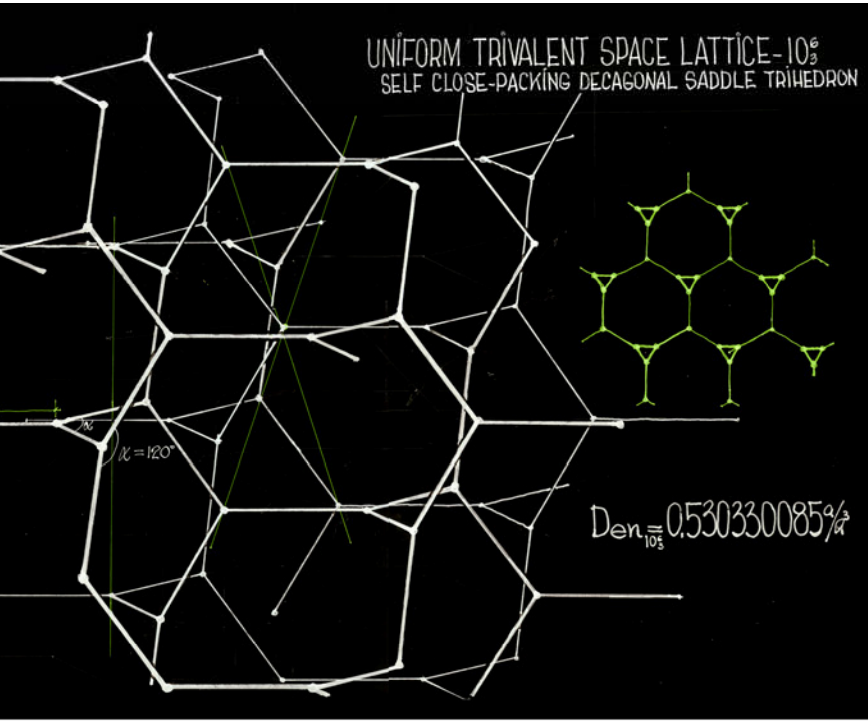
Diamonds are rarities not just on earth but also mathematically. The crystal structure of diamond has two key distinguishing properties, notes mathematician Toshikazu Sunada of Meiji University in Japan. It has maximal symmetry, which means that its components cannot be rearranged to make it any more symmetrical than it is, and a strong isotropic property, which means that it looks the same when viewed from the direction of any edge. In the February *Notices of the American Mathematical Society,* Sunada finds that out of an infinite universe of crystals that can exist mathematically, just one other shares these properties with diamond. Whereas diamond is a web of hexagonal rings, its cousin is made of 10-sided rings.

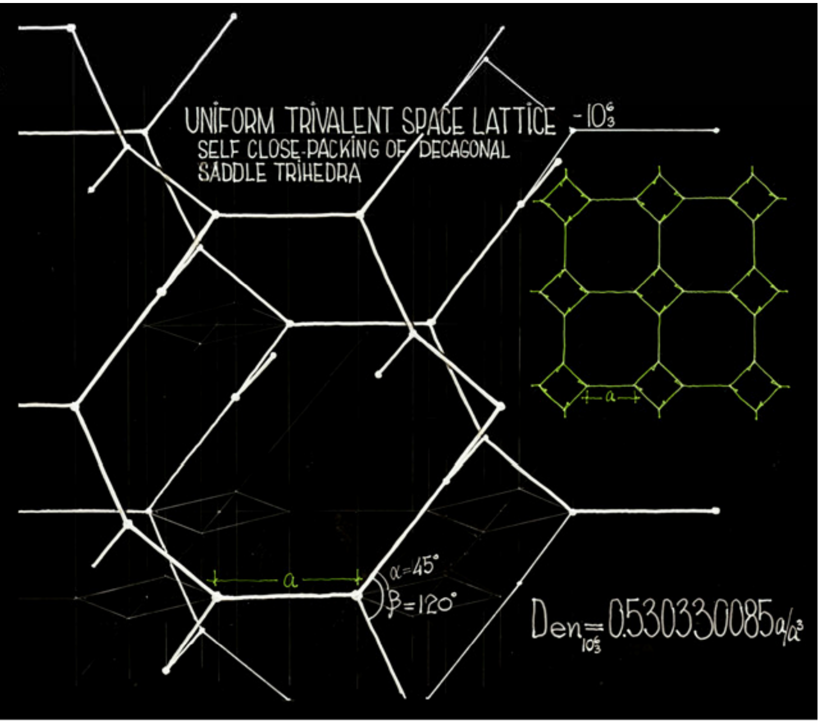
Sunada had originally thought that no one had described this object be-

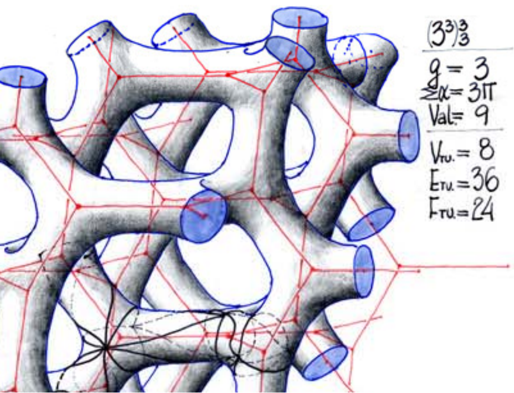
"I rediscovered the crystal structure mathematically in rather an accidental way" while working on another problem, Sunada says. After his paper was published, chemists and crystallographers informed him that they had long known about the crystal, which was called (10,3)-a by A. F. Wells in 1977. Diamond's mathematical twin can exist in a slightly distorted form as an arrangement of silicon atoms in strontium silicide. —*Charles Q. Choi*

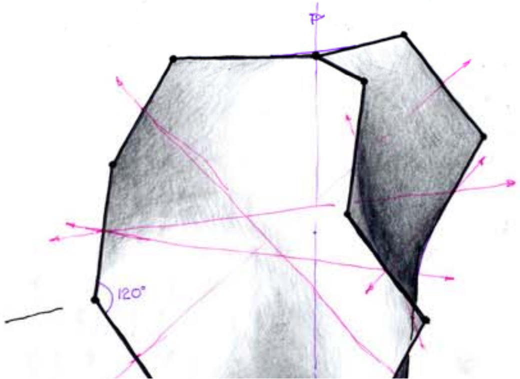


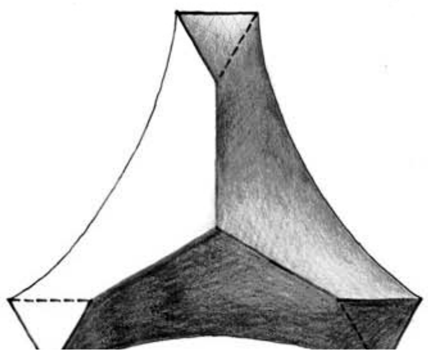
TWO OUT OF INFINITY: **Diamond and the K4, or (10,3)-a,**

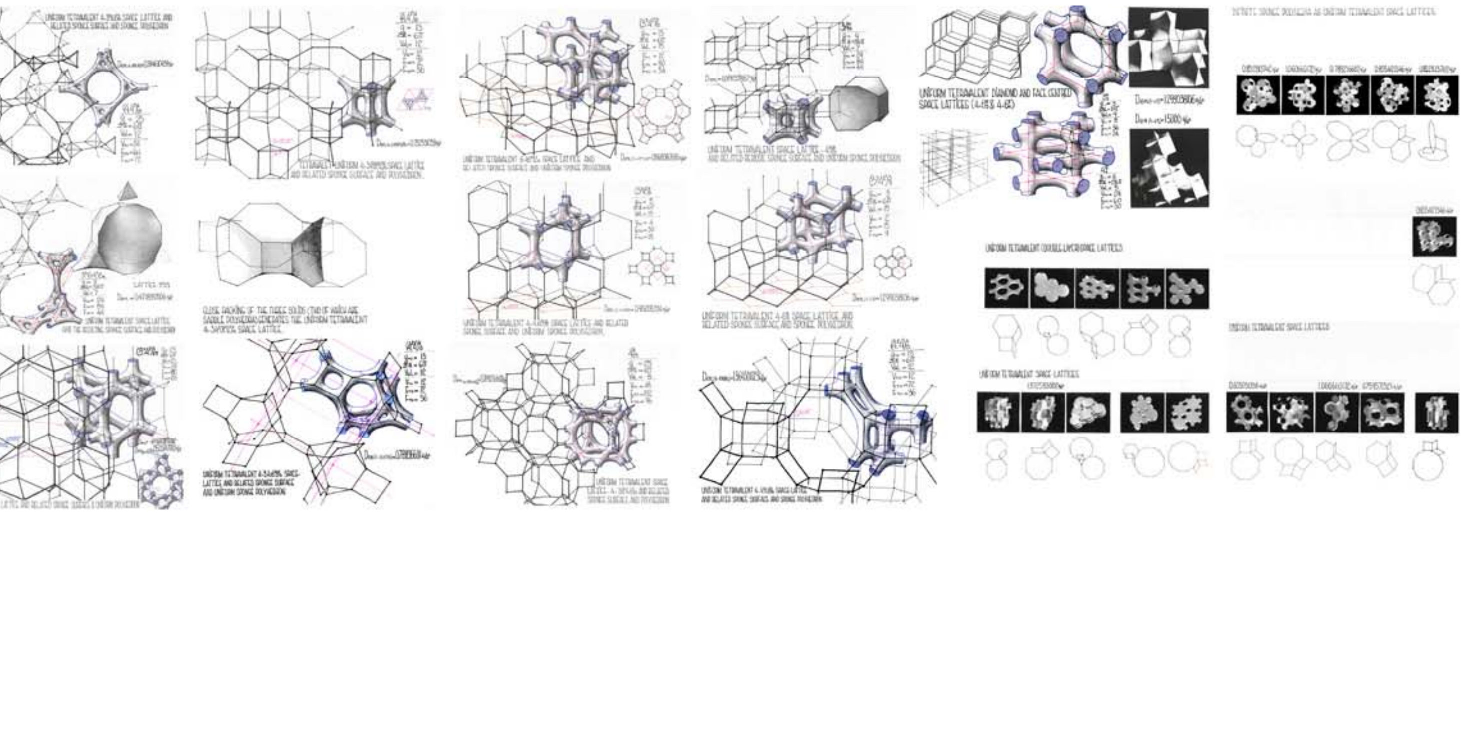

UNIFORM TRIVALENT SPACE LATTICE - 10^6_3
SELF CLOSE-PACKING DECAGONAL SADDLE TRIHEDRON
α=120°
Den_10^6_3 = 0.530330085 a/α³


UNIFORM TRIVALENT SPACE LATTICE - 10^6_3
SELF CLOSE-PACKING OF DECAGONAL
SADDLE TRIHEDRA
a
α=45°
β=120°
Den_10^6_3 = 0.530330085 a/α³

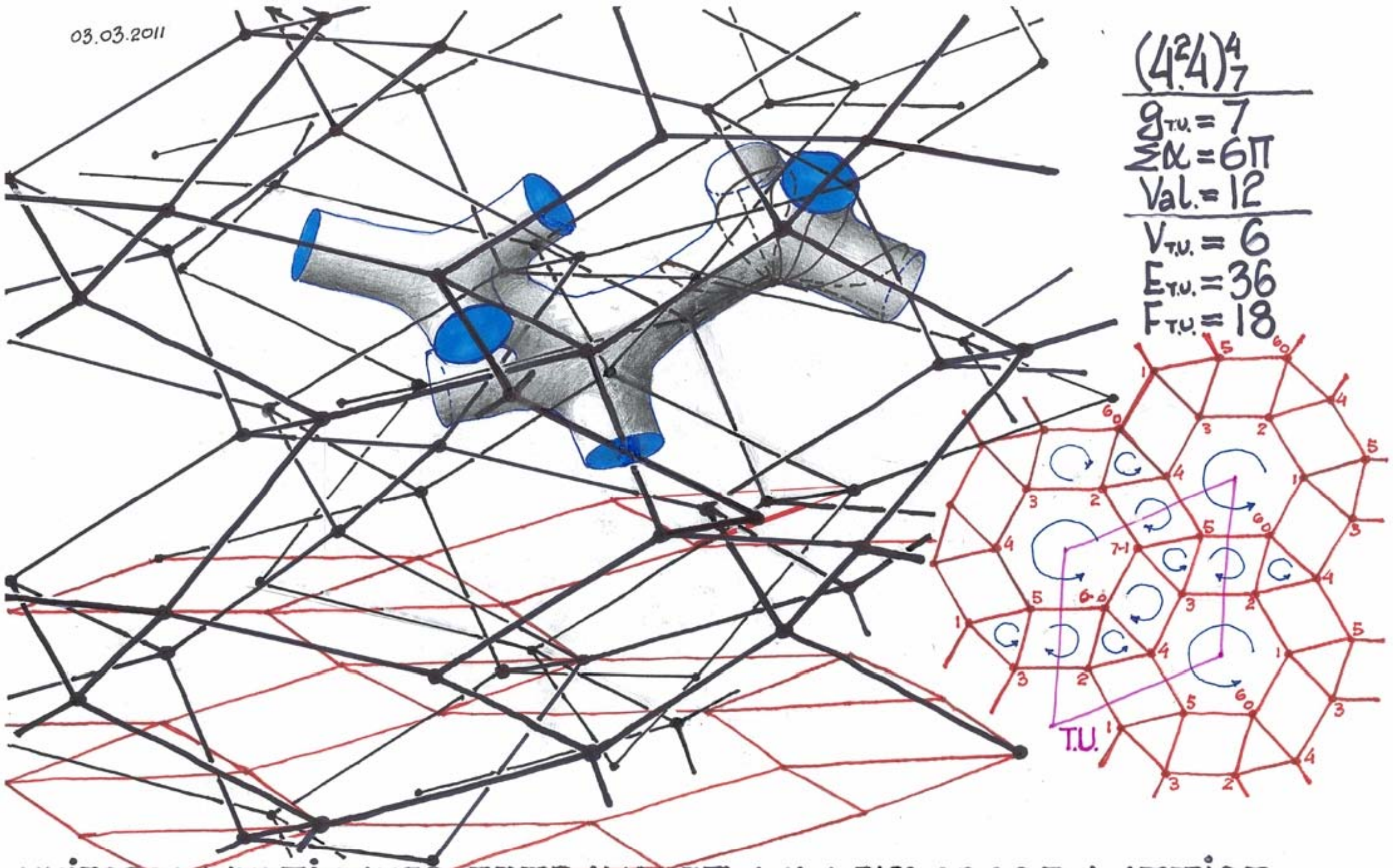

(3³)³/3
g = 3
Σα = 3π
Val = 9
Vru = 8
Eru = 36
Fru = 24


120°

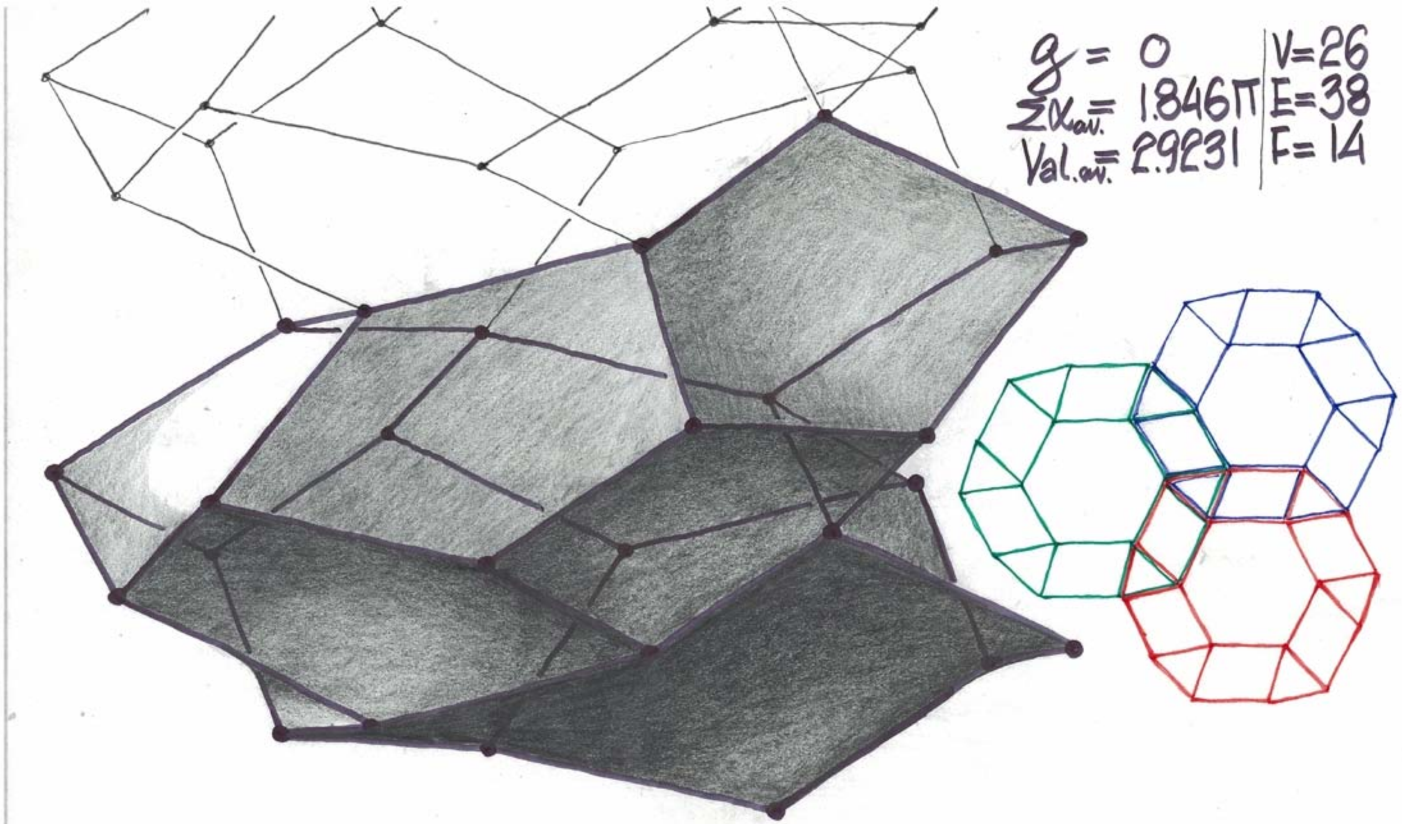




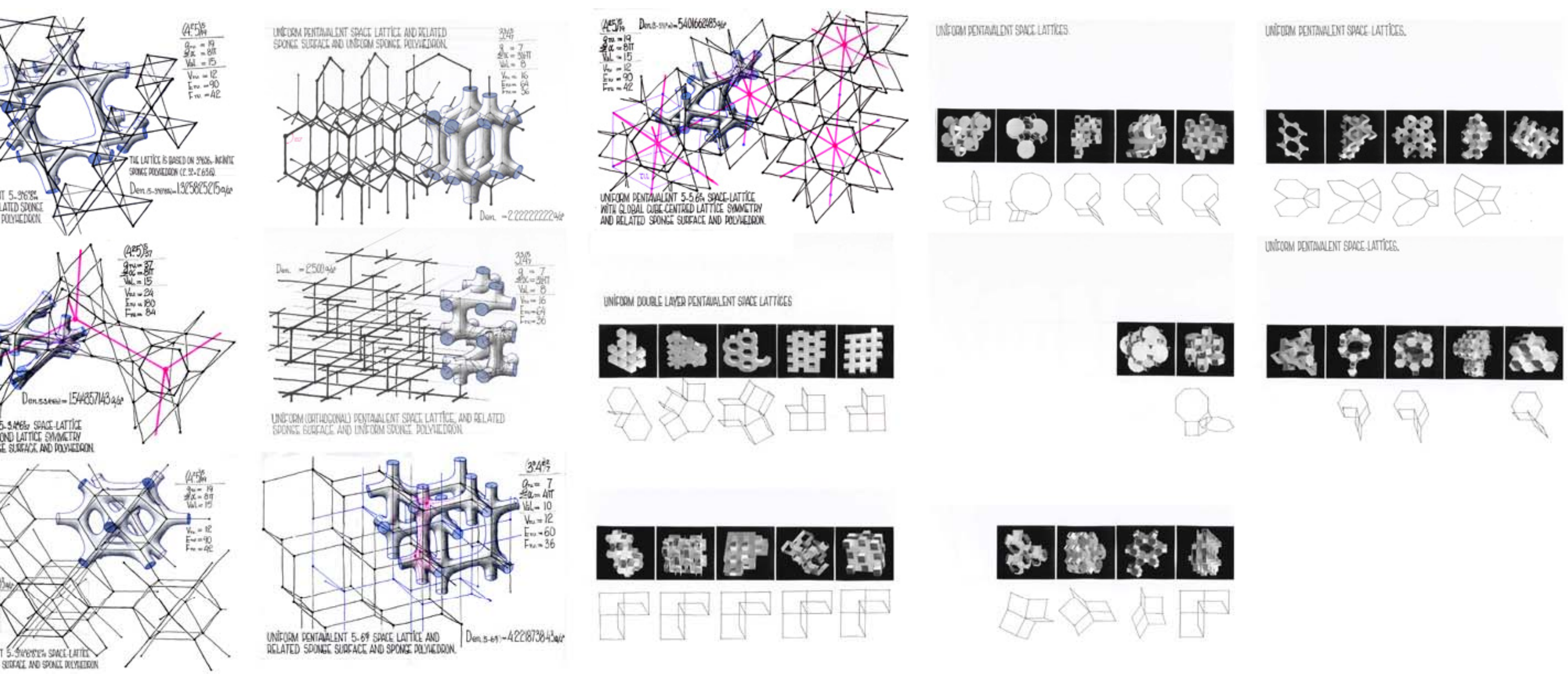
UNIFORM TETRAVALENT DIAMOND AND FACE CENTRED SPACE LATTICES
UNIFORM TETRAVALENT SPACE LATTICES

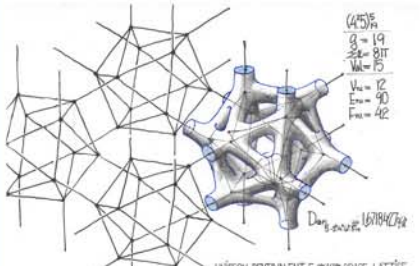


UNIFORM MULTI-LAYER TETRAVALET ML4-$5^{4}8^{3}_{7}$ SPACE LATTICE
AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON



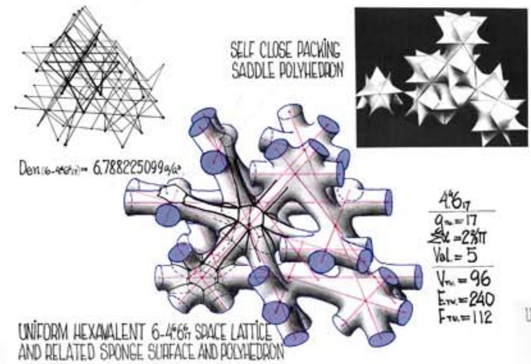
SELF-PACK SADDLE POLYHEDRON GENERATING THE UNIFORM MULTI-LAYER TETRAVALENT ML.$4\text{-}5^4.8^4$ SPACE LATTICE.


UNIFORM PENTAVALENT SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.
UNIFORM PENTAVALENT 5-5 SPACE-LATTICE WITH GLOBAL CUBE-CENTRED LATTICE SYMMETRY AND RELATED SPONGE SURFACE AND POLYHEDRON.
UNIFORM PENTAVALENT SPACE LATTICES.
UNIFORM PENTAVALENT SPACE LATTICES.
UNIFORM DOUBLE LAYER PENTAVALENT SPACE LATTICES
UNIFORM PENTAVALENT SPACE LATTICES.
UNIFORM (ORTHOGONAL) PENTAVALENT SPACE LATTICE, AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.
UNIFORM PENTAVALENT 5-6 SPACE LATTICE AND RELATED SPONGE SURFACE AND SPONGE POLYHEDRON.



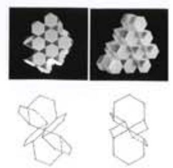
UNIFORM PENTAVALENT 5- SPACE LATTICE AND RELATED SPONGE SURFACE AND SPONGE POLYHEDRON

THREE SOLIDS, CLOSE PACKING OF WHICH GENERATES THE UNIFORM HEXAVALENT 6- SPACE LATTICE.

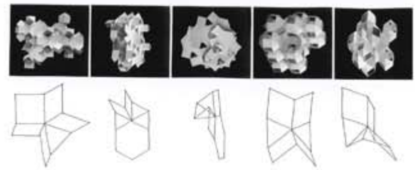


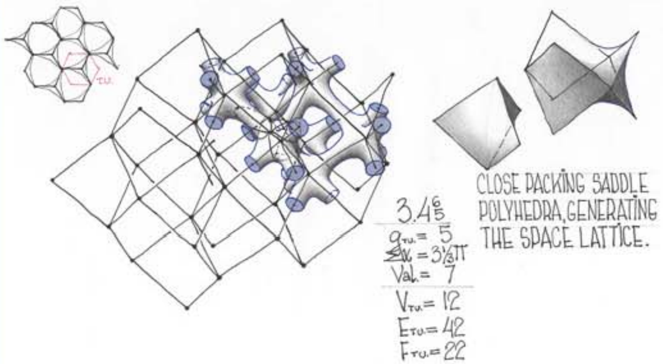
UNIFORM HEXAVALENT 6- SPACE LATTICE AND RELATED SPONGE SURFACE AND POLYHEDRON

UNIFORM HEXAVALENT SPACE-LATTICES.



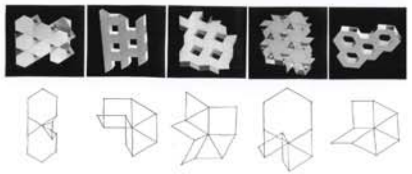
UNIFORM HEXAVALENT SPACE LATTICES.



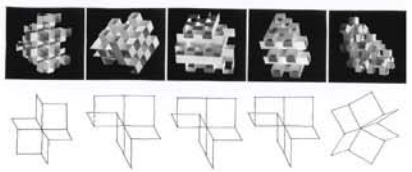


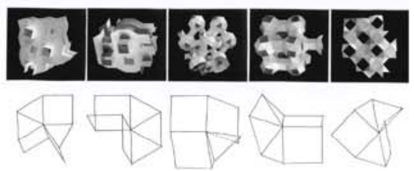
UNIFORM HEXAVALENT 6-(4.6) SPACE LATTICE AND RELATED PERIODIC SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

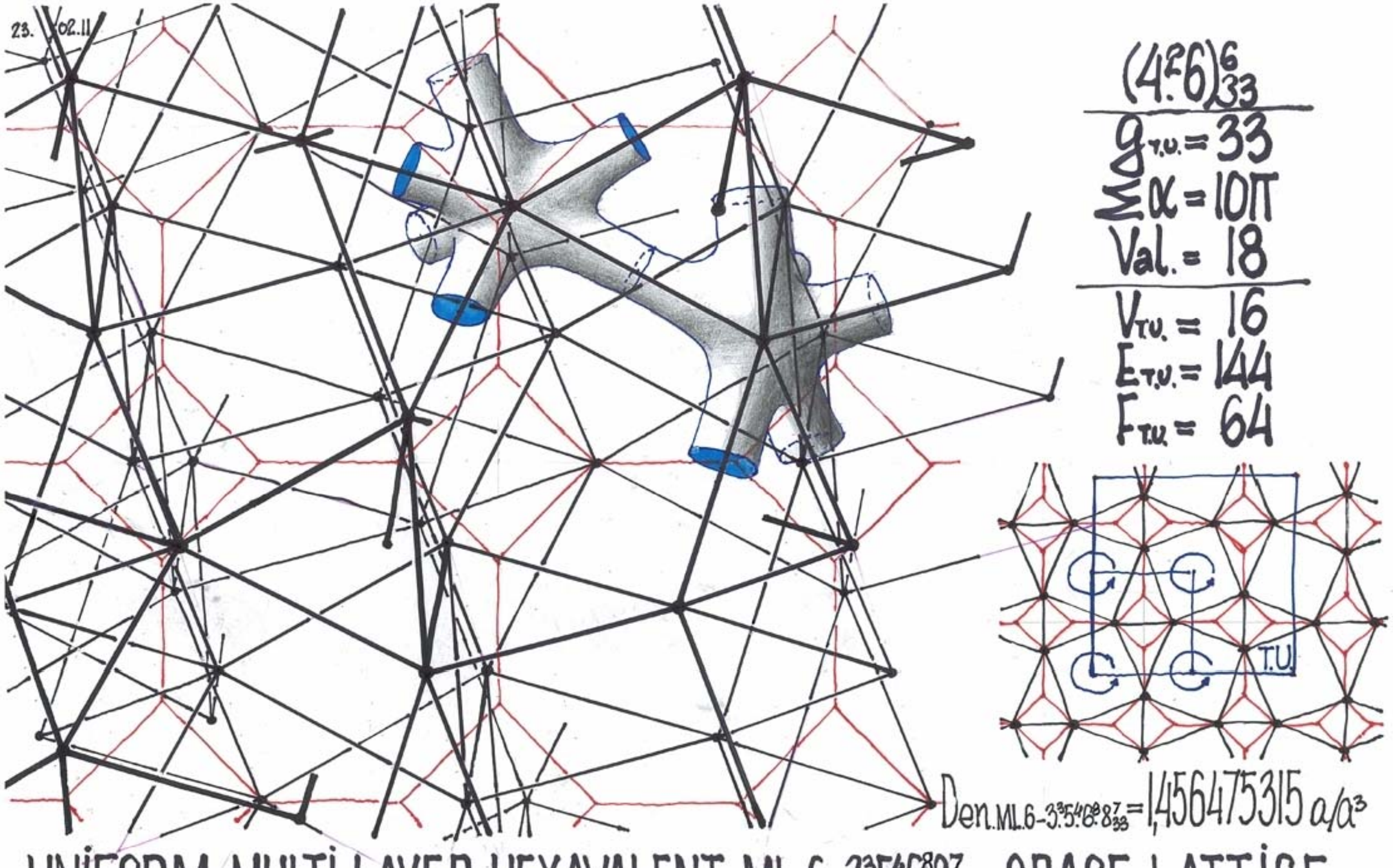
UNIFORM DOUBLE LAYER HEXAVALENT SPACE-LATTICES.



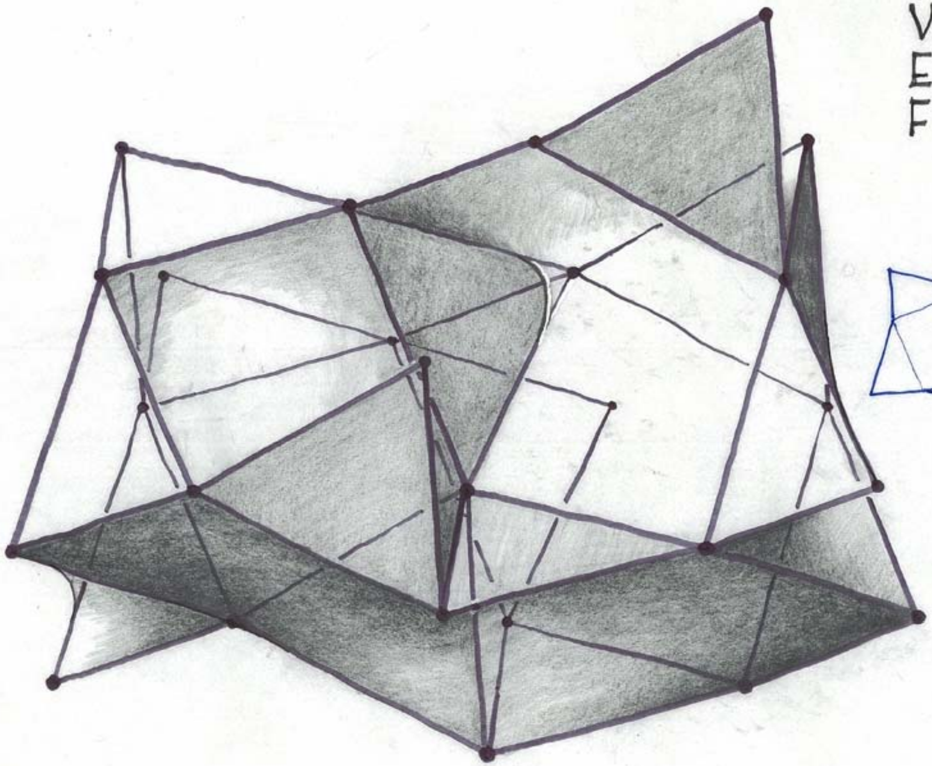
UNIFORM HEXAVALENT SPACE-LATTICES.



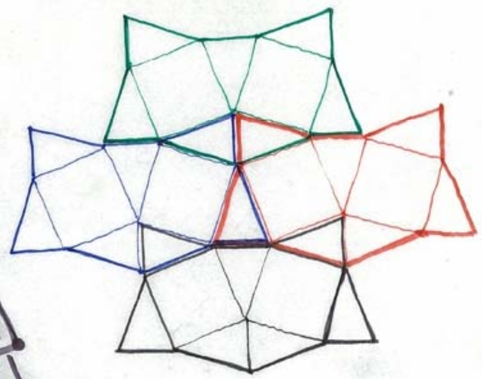




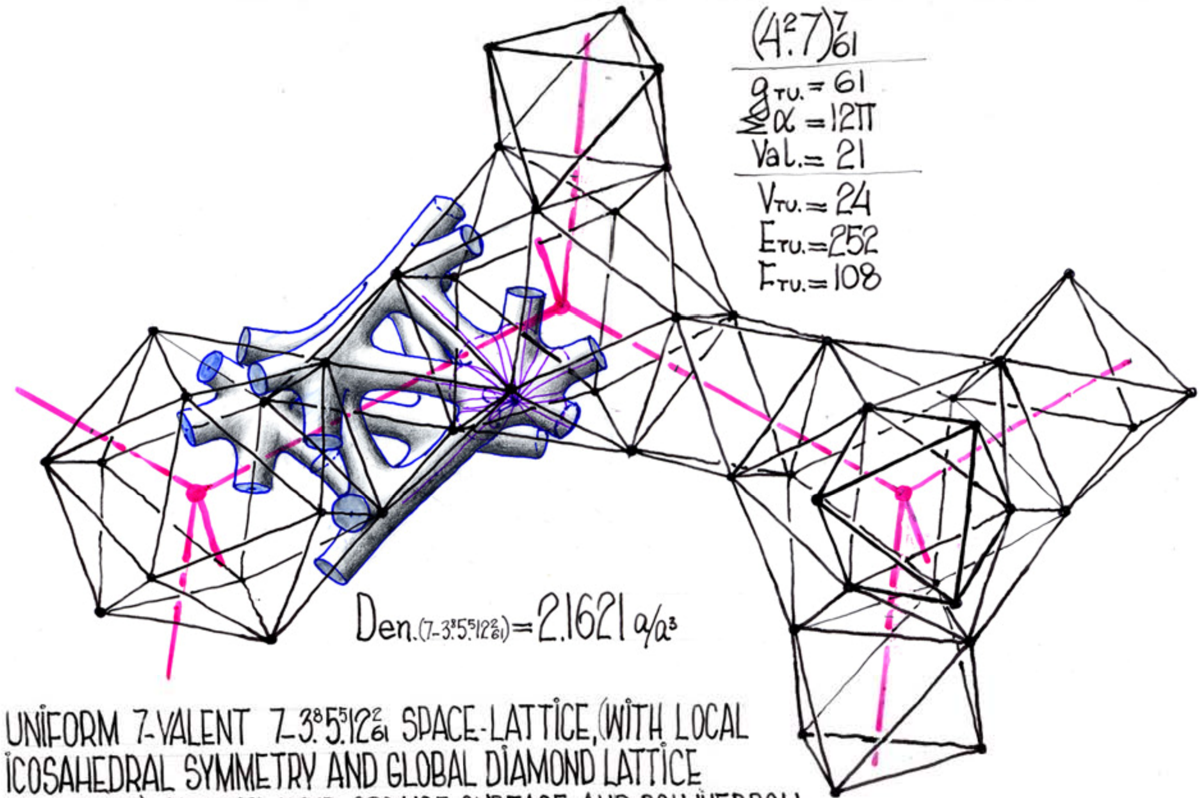
UNİFORM MULTİ-LAYER HEXAVALENT ML.6-$3^3.5^4.6^8.8^7_{33}$ SPACE LATTİCE
AND RELATED SPONGE SURFACE AND UNİFORM SPONGE POLYHEDRON



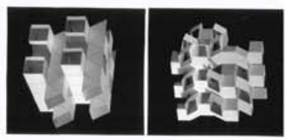
V = 26	g = 0
E = 48	$\Sigma\alpha_{av.} = 1.8462\pi$
F = 24	$Val._{av.} = 3.7308$

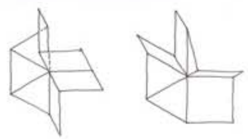


SELF-PACK SADDLE POLYHEDRON GENERATING THE UNIFORM MULTI-LAYER HEXAVALENT $ML.6\text{-}3^{3}.5^{4}.6^{8}.8^{7}_{33}$ SPACE LATTICE.

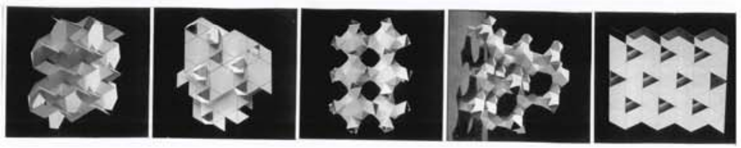


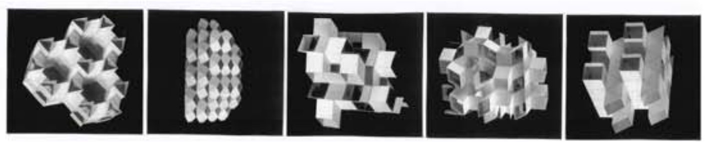
UNIFORM 7-VALENT $7\text{-}3^8.5^5.12^2_{61}$ SPACE-LATTICE, (WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL DIAMOND LATTICE SYMMETRY) AND RELATED SPONGE SURFACE AND POLYHEDRON.

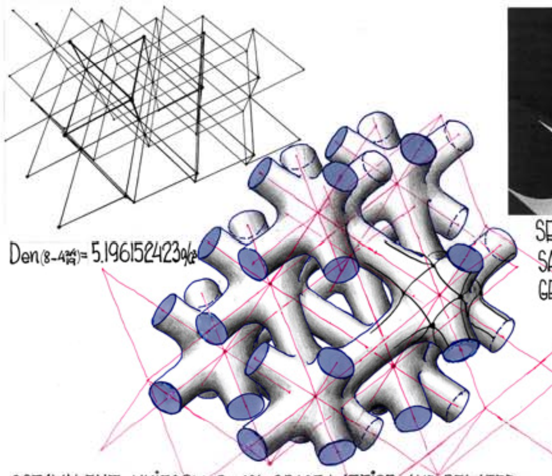


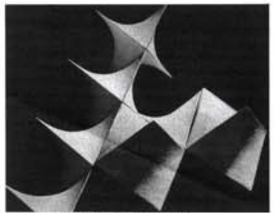


UNIFORM SEPTAVALENT SPACE-LATTICES.









SELF CLOSE PACKING
SADDLE POLYHEDRON,
GENERATING THE LATTICE.

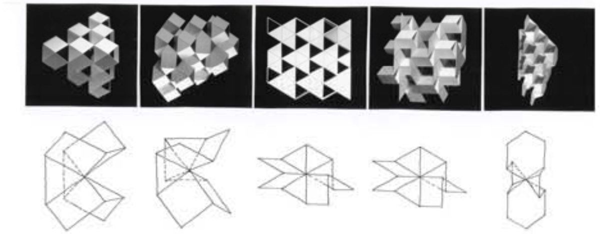
4^{5}_{19}

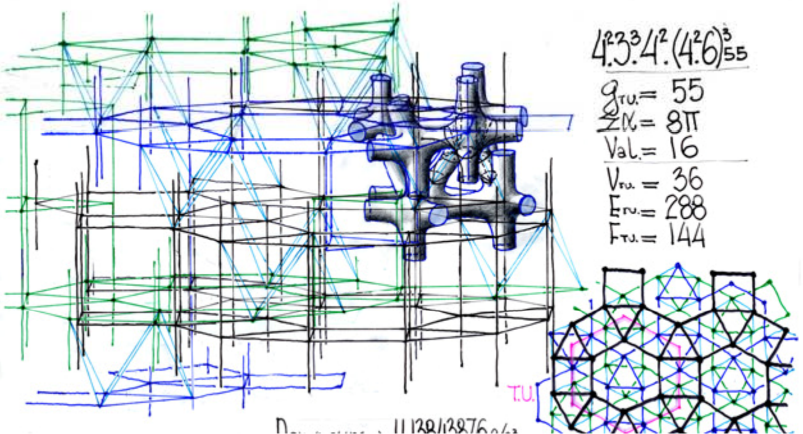
$g_{T.U.} = 19$
$\Sigma\alpha = 2\frac{1}{2}\pi$
$Val. = 5$

$V_{T.U.} = 144$
$E_{T.U.} = 360$
$F_{T.U.} = 180$

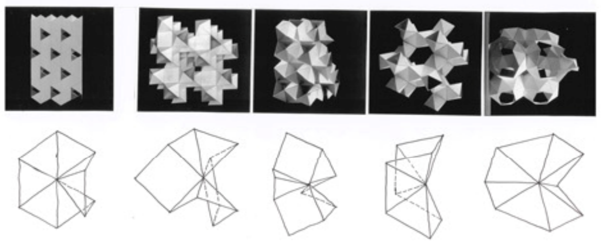
OCTAVALENT UNIFORM 8–4^{24}_{19} SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON

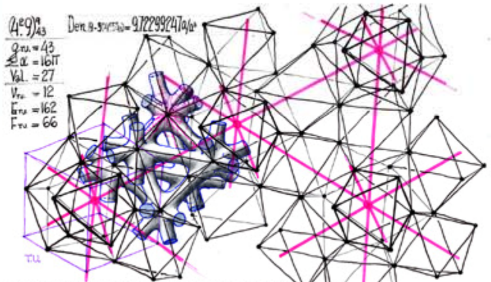
UNIFORM OCTAVALENT SPACE-LATTICES.



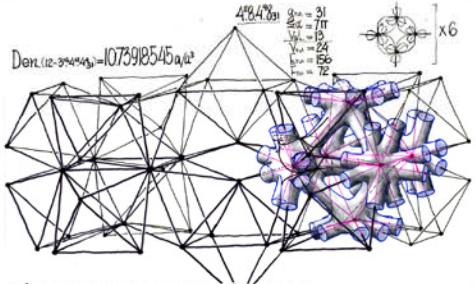


UNIFORM OCTAVALENT SPACE-LATTICES.

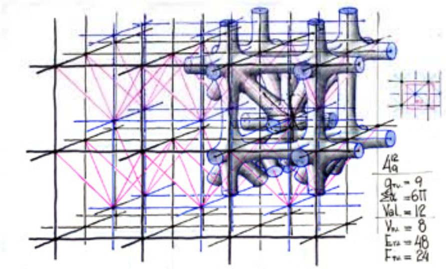




UNIFORM 9-VALENT 9-$3^{8}4^{6}5^{3}_{43}$ SPACE-LATTICE (WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL CUBE-CENTRED LATTICE SYMMETRY) AND REL.



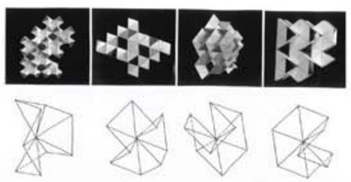
UNIFORM DODECAVALENT 12-$3^{8}4^{6}_{31}$ SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

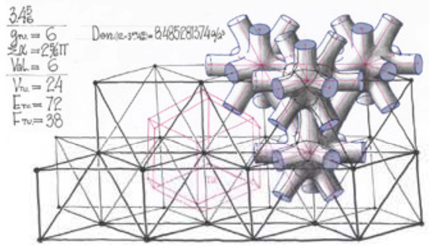


UNIFORM DECAVALENT 10-$3^{8}4^{6}_{9}$ SPACE LATTICE AND RELATED SPONGE SURFACE AND SPONGE POLYHEDRA

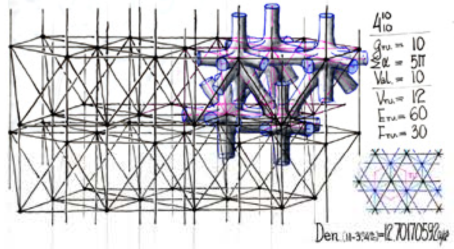
Den.(10-$3^{8}4^{6}_{9}$) = 10.000 a/a³

UNIFORM 9-VALENT SPACE-LATTICES.



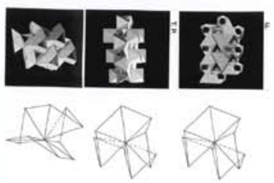


UNIFORM DODECAVALENT 12-$3^{8}4^{6}$ SPACE LATTICE (OCTET LATTICE) AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

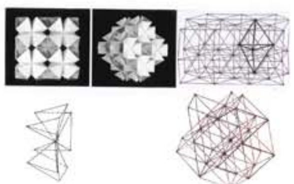


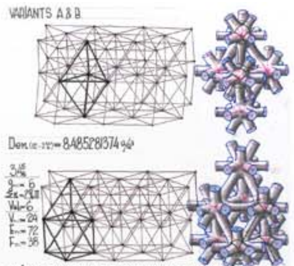
UNIFORM 11-VALENT 11-$3^{8}4^{6}_{10}$ SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON

UNIFORM DECAVALENT SPACE-LATTICES.

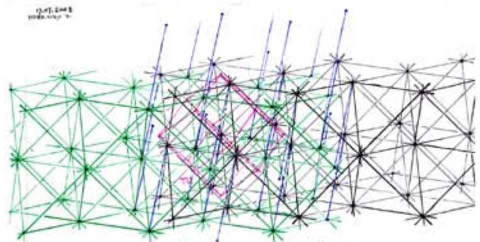


UNIFORM DODECAVALENT SPACE-LATTICES.



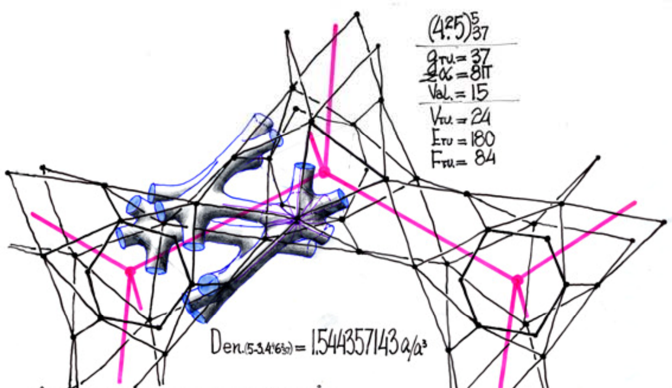


UNIFORM DODECAVALENT 12-$3^{8}4^{6}$ SPACE LATTICE (OCTET) AND RELATED SPONGE SURFACES AND SPONGE POLYHEDRA.

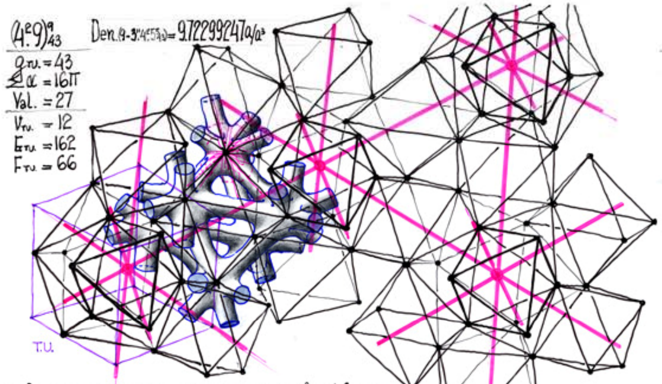


DENSITY = 18.38477631 a/a³

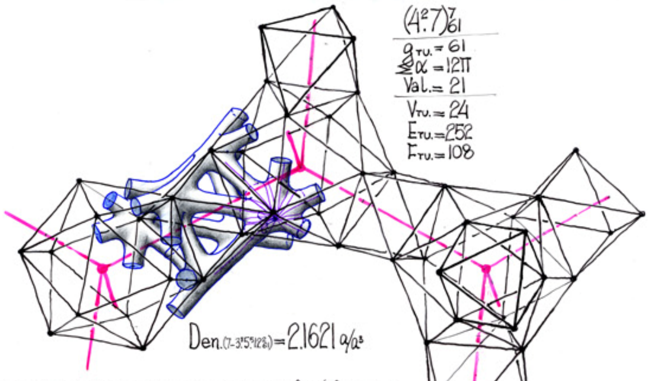
TWO INTER-PENETRATING UNIFORM OCTET SPACE LATTICES, WITH EDGE-a, WHEN JOINED TOGETHER WITH A SET OF PARALLEL a-EDGES, GENERATE A UNIFORM 13-VALENT SPACE LATTICE.



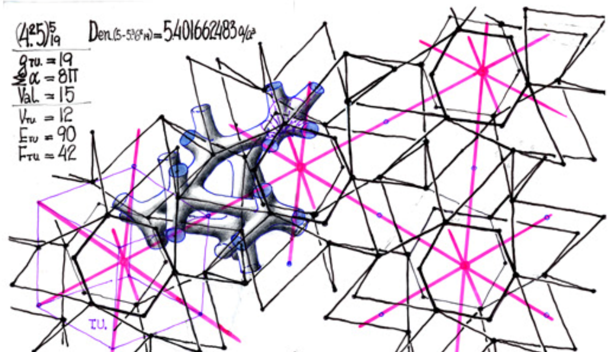
UNIFORM 5-VALENT 5-3.4⁴6²₃₇ SPACE-LATTICE WITH GLOBAL DIAMOND LATTICE SYMMETRY AND RELATED SPONGE SURFACE AND POLYHEDRON.



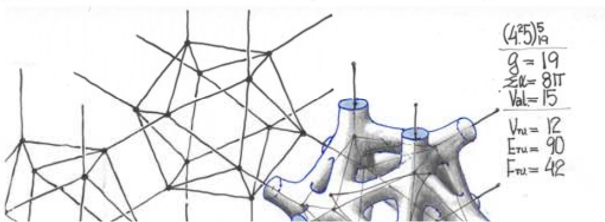
UNIFORM 9-VALENT 9-3¹¹4¹²5⁵₄₃ SPACE-LATTICE,(WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL CUBE-CENTRED LATTICE SYMMETRY), AND RELATED SPONGE SURFACE AND POLYHEDRON.

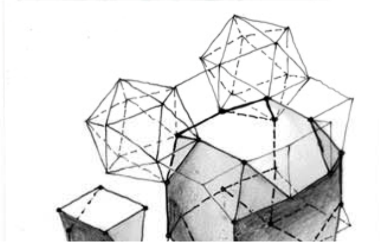


UNIFORM 7-VALENT 7-3⁸5⁴12²₆₁ SPACE-LATTICE,(WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL DIAMOND LATTICE SYMMETRY) AND RELATED SPONGE SURFACE AND POLYHEDRON.



UNIFORM PENTAVALENT 5-5.6²₁₉ SPACE-LATTICE WITH GLOBAL CUBE-CENTRED LATTICE SYMMETRY AND RELATED SPONGE SURFACE AND POLYHEDRON.





Considering all **uniform network configurations** in 3D space, we can categorize and classify them as follows:

Centroid related networks (Centroidal), (denoted as C.Val. ma.nb…$g_{T.U.}$), the vertices of which are equi-distant from a fixed point in space .

Axis related networks (Axial), (denoted as A.Val. - ma.nb…$g_{T.U.}$), the vertices of which are equi-distant from a fixed axis .

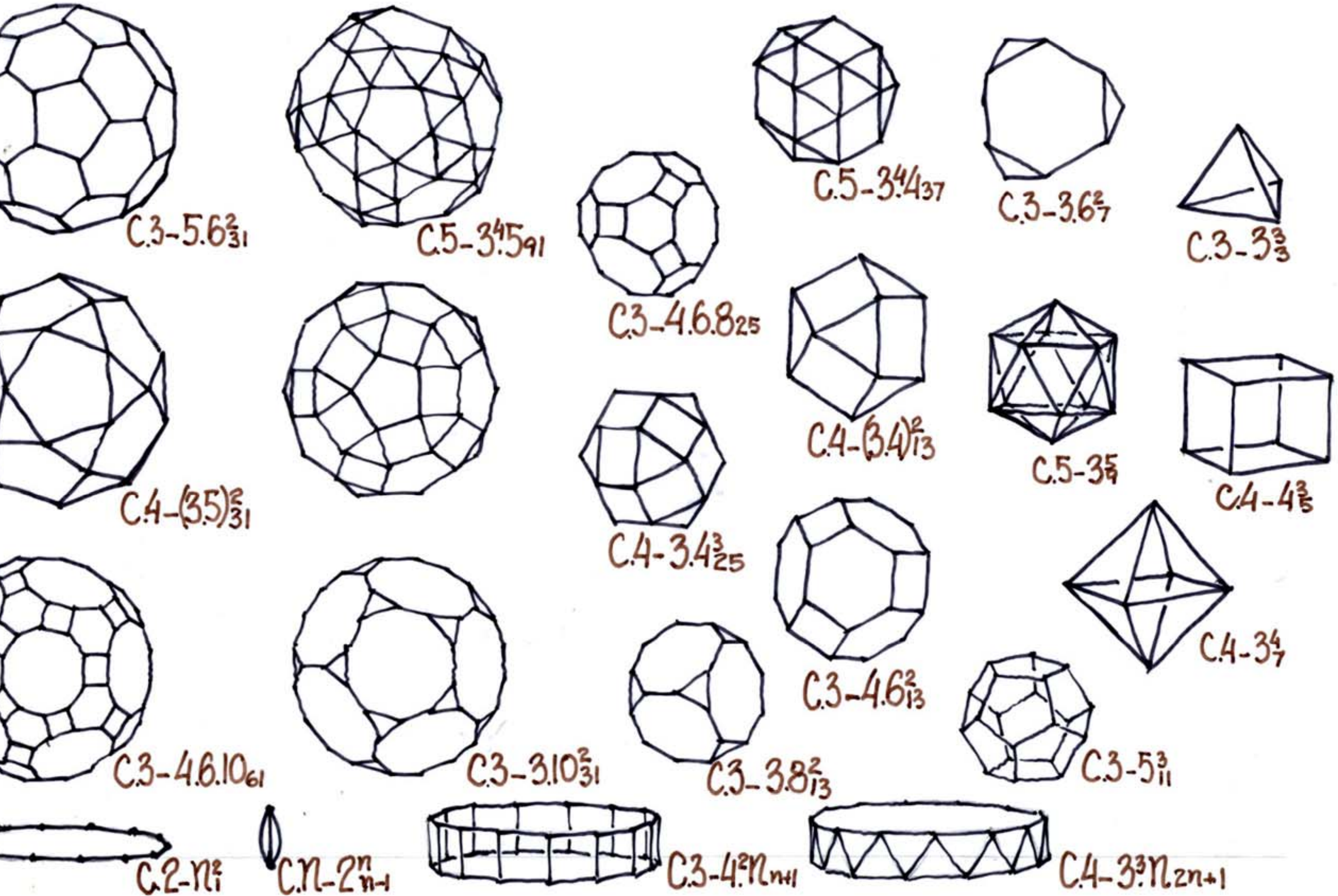
Plane related networks, (Double layer) (denoted as D.Val. - ma.nb…$g_{T.U.}$), the vertices of which are equi-distant form a fixed plane .

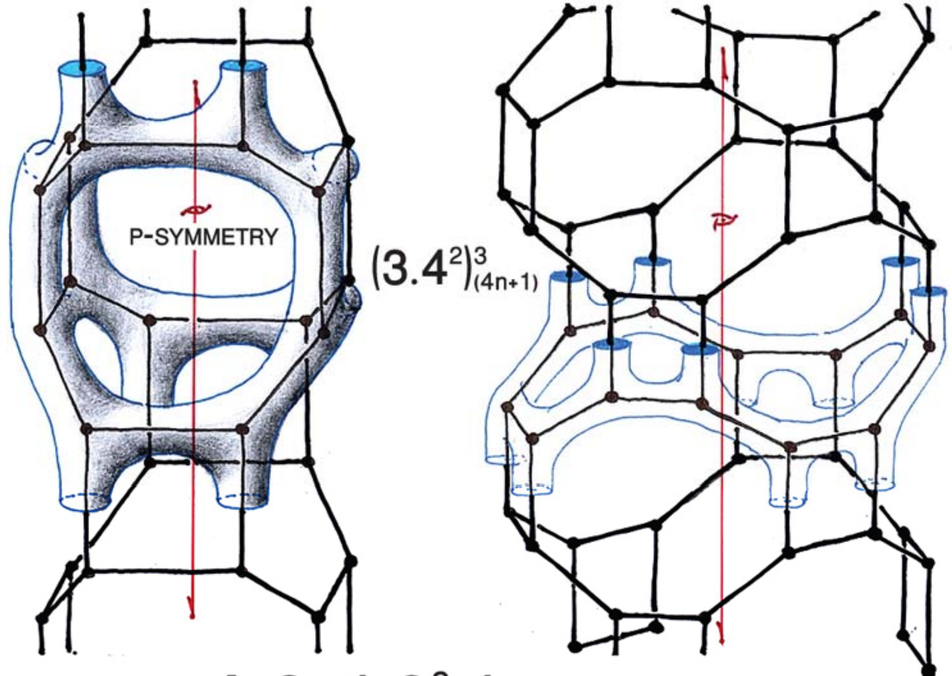
Centroid-Axial-Plane related networks (denoted as CAP. Val. - ma.nb…$g_{T.U.}$), the vertices of which are equi-distant from a fixed

Multi-Layered Space Lattices (ML.Val. - ma.nb…$g_{T.U.}$), the vertices of which are distributed on an infinite set off parallel planes.

Poly-Vectorial Space Lattices (PV. Val. - ma.nb…$g_{T.U.}$), the symmetry group of which is not dominated by only one set of same principal axes of symmetry.

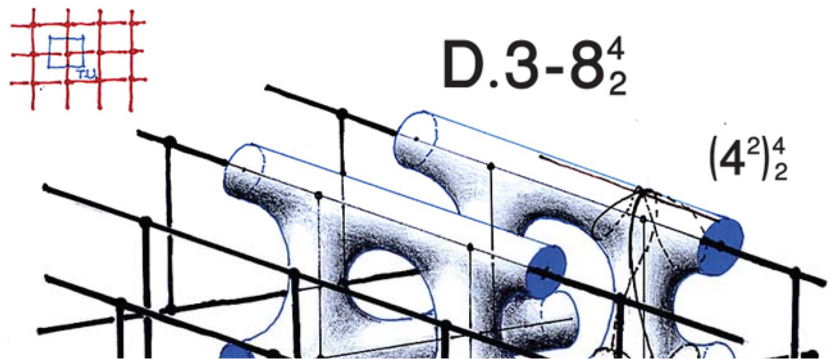
Translation Space Lattices (T.Val. - ma.nb…$g_{T.U.}$), generated by a repeated edge-length translation in a given direction, of a distinct uniform Multi-layered or Poly-Vectorial space lattice. The

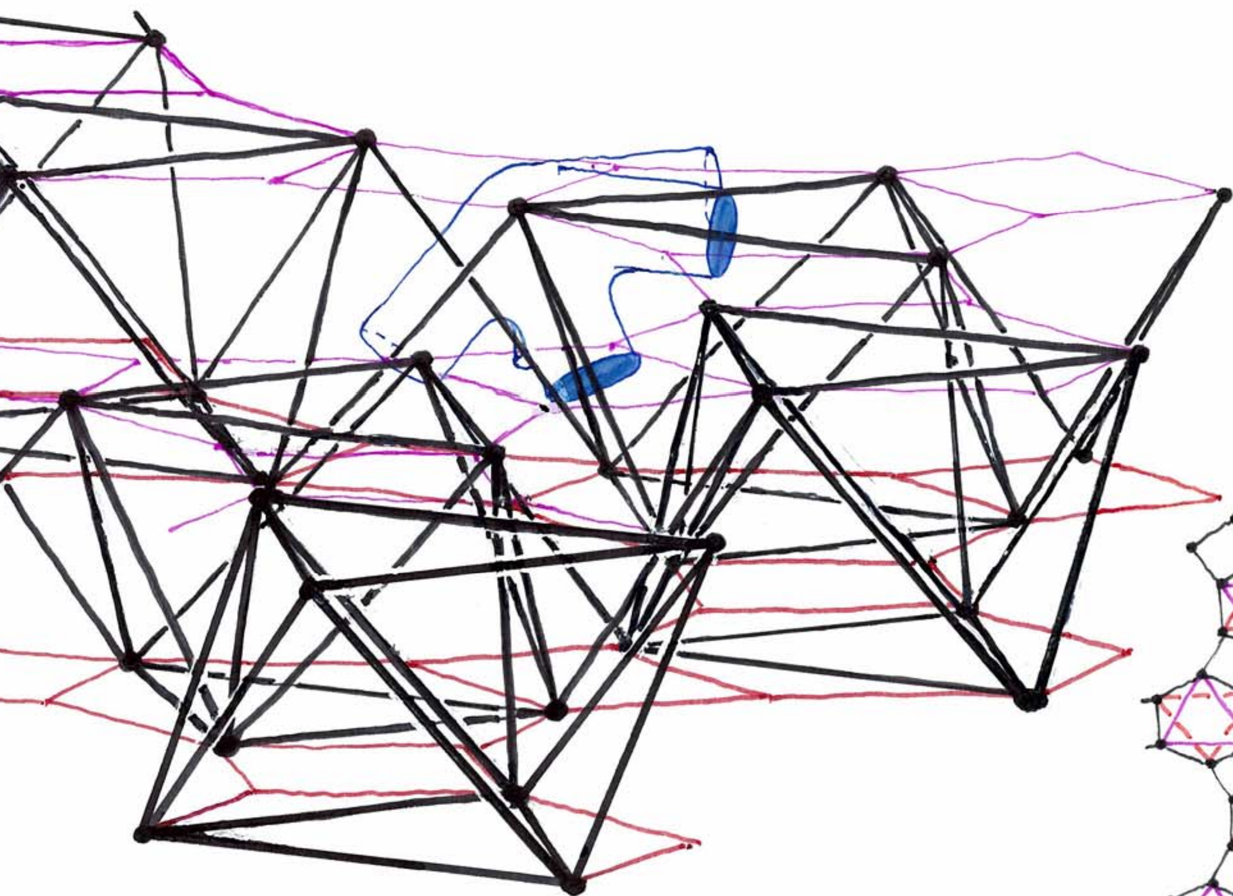

$C.3-5.6^2_{31}$
$C.5-3^4.5_{91}$
$C.3-4.6.8_{25}$
$C.5-3^4.4_{37}$
$C.3-3.6^2_{7}$
$C.3-3^3_{3}$
$C.4-(3.5)^2_{31}$
$C.4-(3.4)^2_{13}$
$C.5-3^5$
$C.4-4^3_{5}$
$C.4-3.4^3_{25}$
$C.3-4.6^2_{13}$
$C.4-3^4_{7}$
$C.3-4.6.10_{61}$
$C.3-3.10^2_{31}$
$C.3-3.8^2_{13}$
$C.3-5^3_{11}$
$C.2-n^2_{1}$
$C.n-2^n_{n-1}$
$C.3-4^2.n_{n+1}$
$C.4-3^3.n_{2n+1}$



$A.3\text{-}4.8^2.4n_{(4n+1)}$

$D.3\text{-}8^4_2$

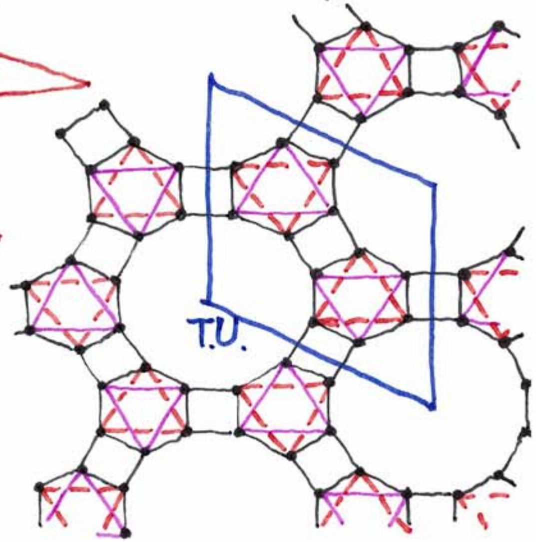




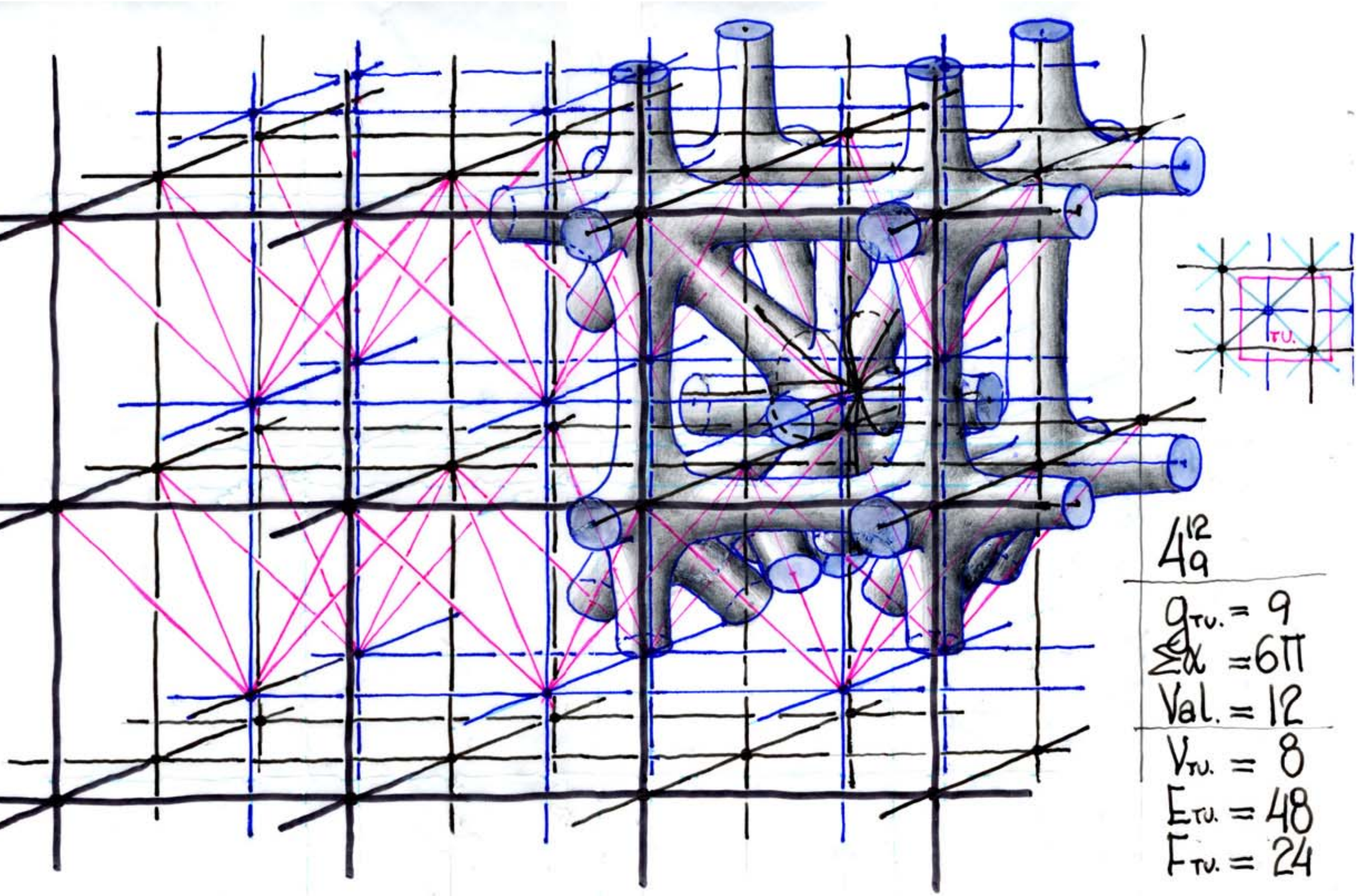
$(4^2.5)^5_{19}$

$g_{T.U.} = 19$
$\Sigma\alpha = 8\pi$
$Val. = 15$

$V_{T.U.} = 12$
$E_{T.U.} = 90$
$F_{T.U.} = 42$

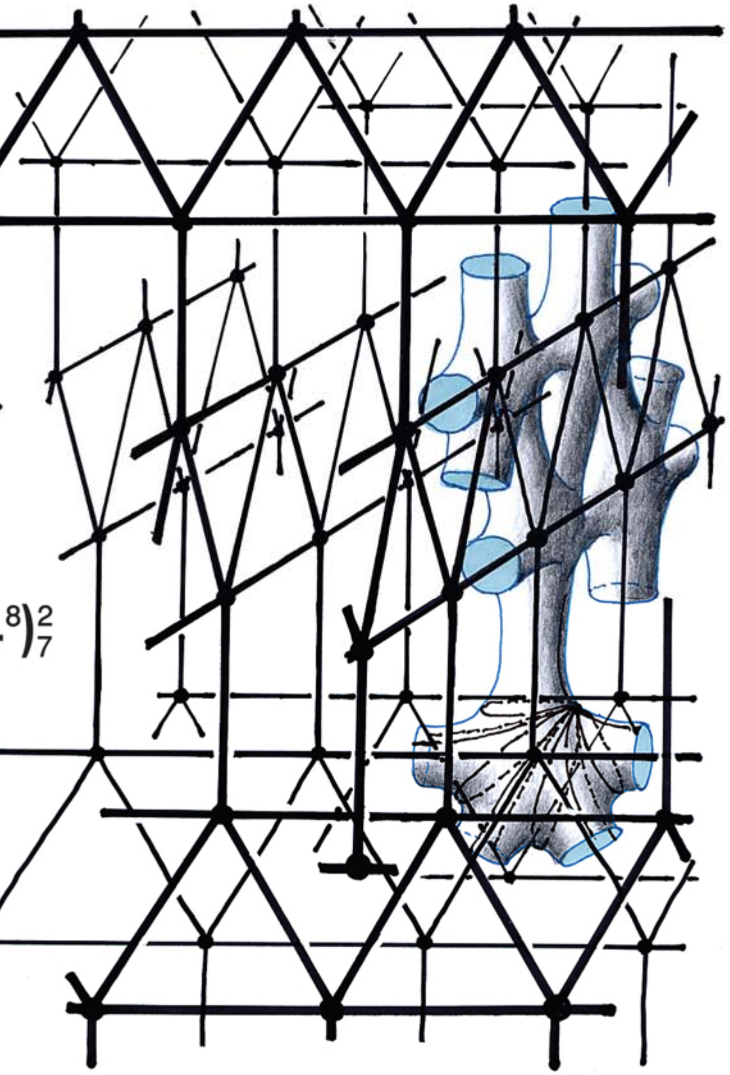


FORM DOUBLE LAYER PENTAVALENT $D5 - 3^4 4\ 12_{19}$ SPACE LATTICE

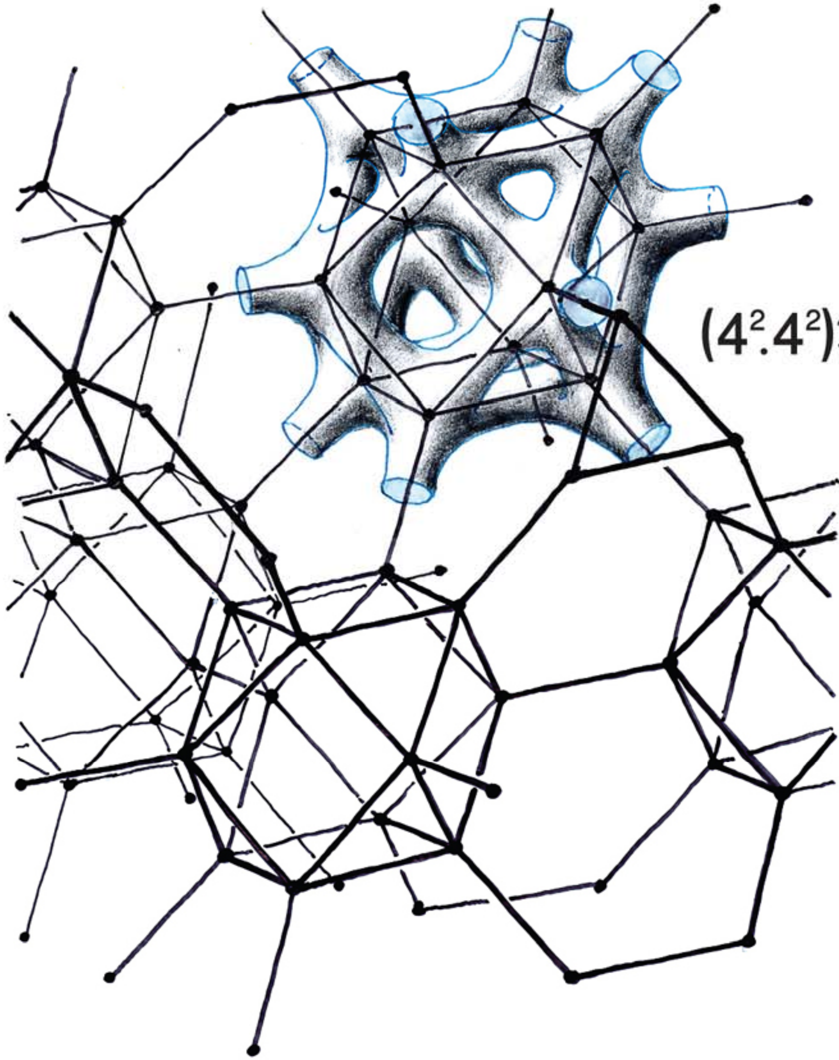


ORM DECAVALENT 10-$3^{12}4^{12}$ SPACE LATTICE

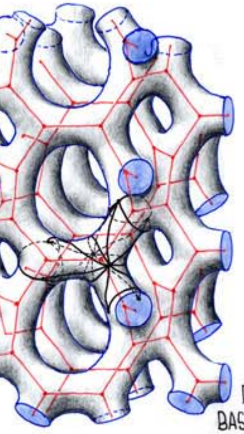
Den(10-$3^{12}4^{12}$)=10,000



$^{8})^{2}_{7}$



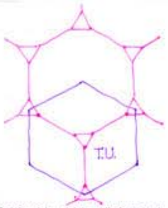
$(4^{2}.4^{2})^{4}_{19}$



$(3^3)^3_4$

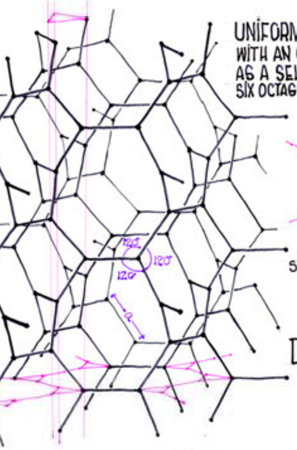
$g = 4$
$\sum\alpha = 3\pi$
Val. = 9

$V_{TU} = 12$
$E_{TU} = 54$
$F_{TU} = 36$

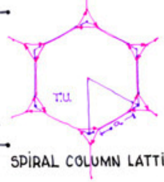


UNIFORM SPONGE POLYHEDRON

PERIODIC SPONGE SURFACE ON THE BASIS OF A UNIFORM TRIVALENT LATTICE

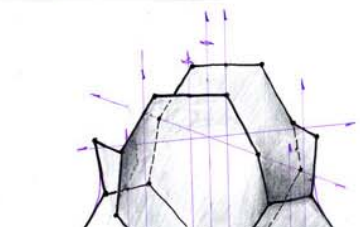


UNIFORM TRIVALENT SPACE LATTICE WITH AN OCTAHEDRAL SADDLE POLYHEDRON AS A SELF CLOSE PACKING SOLID, WITH SIX OCTAGONS AND TWO 14-GON FACES.



SPIRAL COLUMN LATTICE

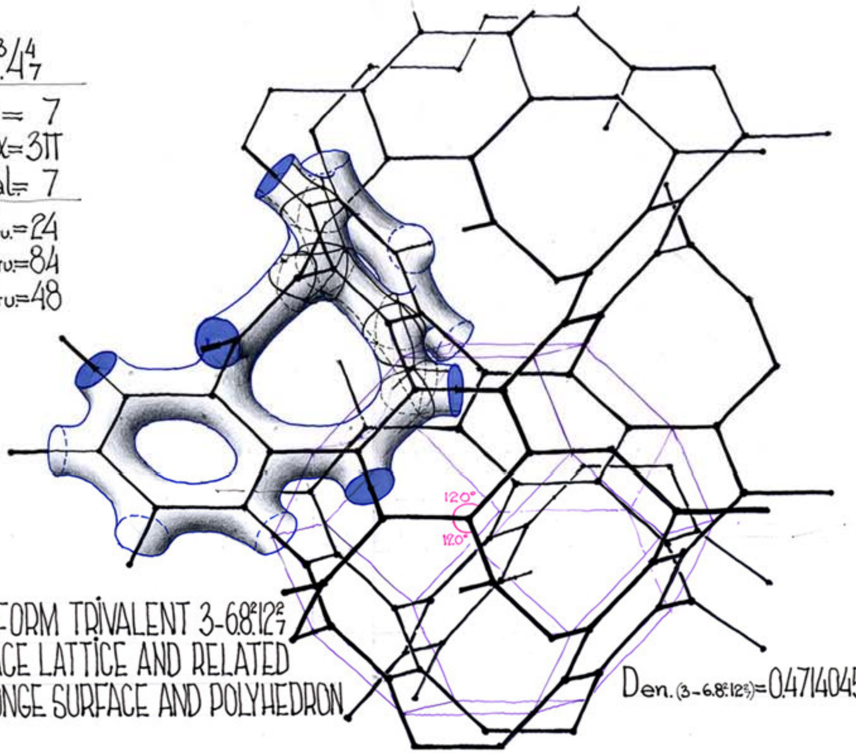
$Den_{8^4 14^2} = 0.2545584412\ a/a^3$



$3^3.4^4_7$

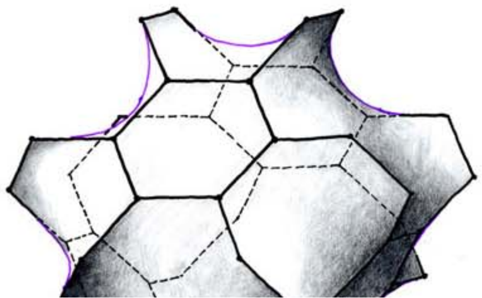
$g = 7$
$\sum\alpha = 3\pi$
Val. = 7

$V_{TU} = 24$
$E_{TU} = 84$
$F_{TU} = 48$



UNIFORM TRIVALENT $3\text{-}6.8^2.12^2_7$ SPACE LATTICE AND RELATED SPONGE SURFACE AND POLYHEDRON

$Den._{(3-6.8^2.12^2_7)} = 0.47140452\ a/a^3$



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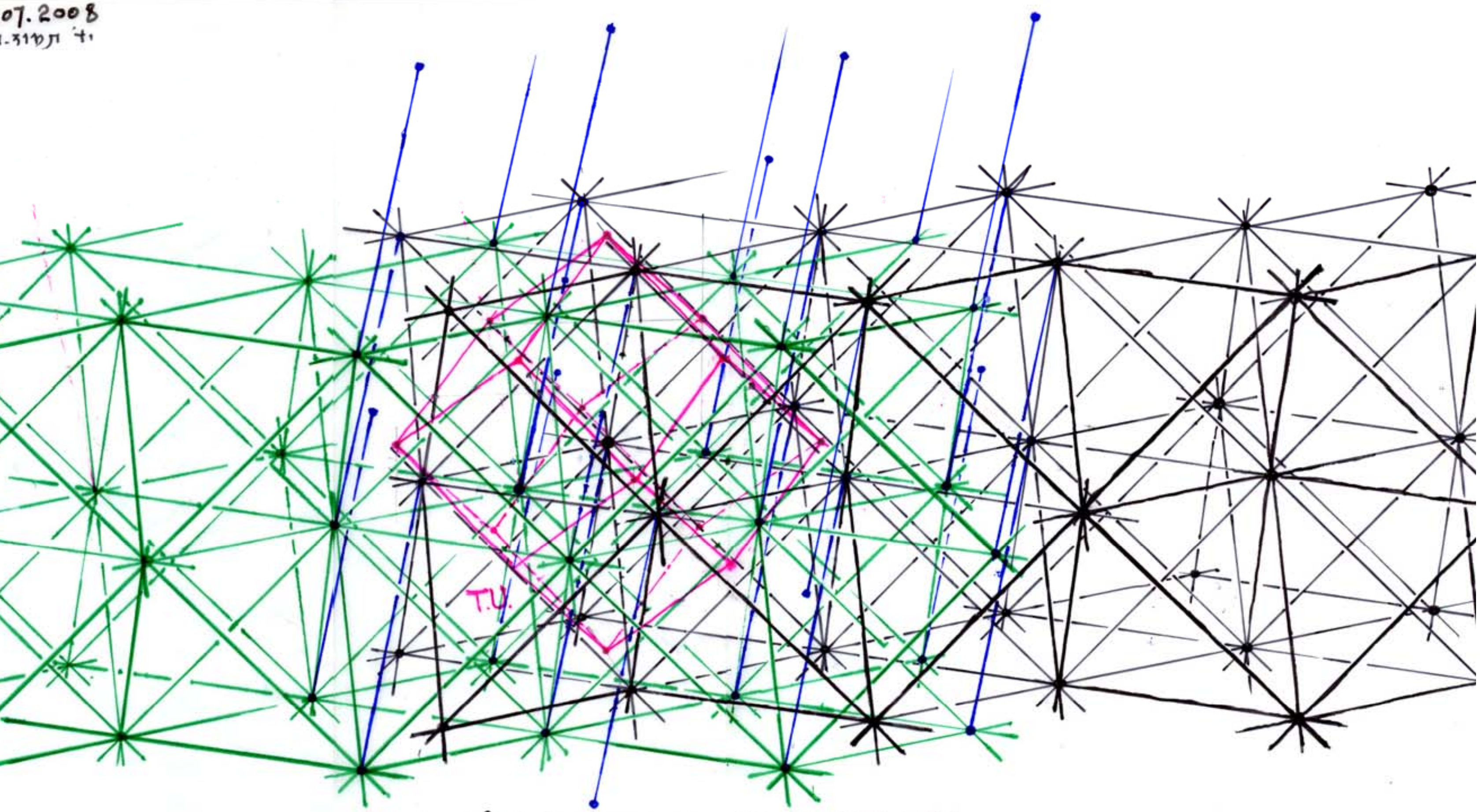


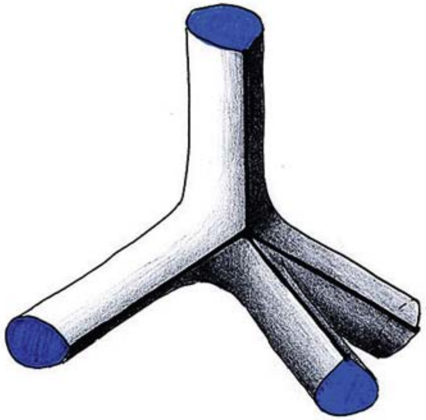
$$\text{DENSITY}_{(13-3^{24}.4^{12}_{12})} = 18.38477631\, a/a^3$$

TWO INTER-PENETRATING UNIFORM OCTET SPACE LATTICES WITH EDGE-a, It seems that the 7-th category of the **Uniform Translation Lattice Networks** (named after the mode of their generation) with a membership list which is reaching to infinity, is a novel feature, not discussed yet in the morphological-mathematical literature. **Their most intriguing characteristic is that (theoretically) their valency and consequently their spatial density values (expressed in terms of a/a^3 with a as the edge-length), can reach to**

A all lattices are found to be embeddable in genetically related sponge surfaces and thus perceived as their polyhedral tessellations.
The resulting sponge configuration of the embedded lattices with all their vertices and edges, stretching to infinity, possess one continuous unbounded face-surface and therefore should be aptly called: 'uni-hedron'. It is a novel feature, representing a new blend of the lattice and the polyhedral phenomenon

DIAMOND PV.4-6_{3}^{12} LATTICE

AND THE RELATED DIAMOND **UNI-HEDRON**



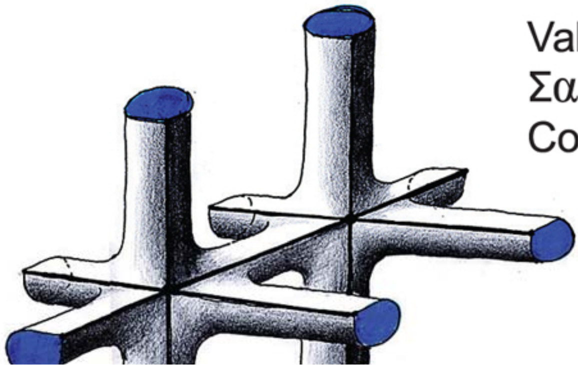
Val. = 4

$\Sigma\alpha = 6\pi$

Con.$_{T.U.}$ = 3 ≡ $g_{T.U.}$

CUBIC PV.6-4_{3}^{12} LATTICE

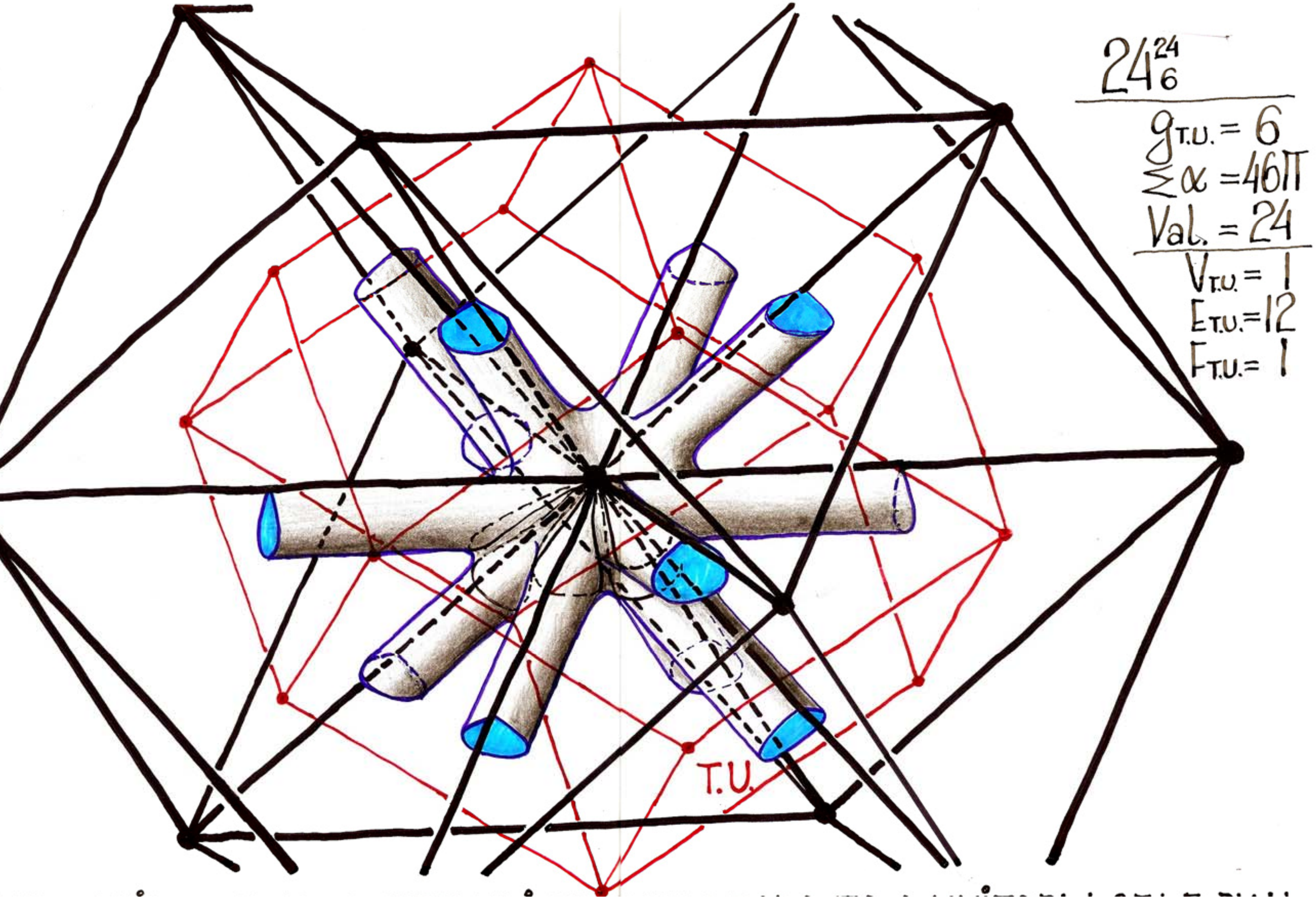
AND THE RELATED CUBIC **UNI-HEDRON**



Val. = 6

$\Sigma\alpha = 10\pi$

Con.$_{T.U.}$ = 3 ≡ $g_{T.U.}$

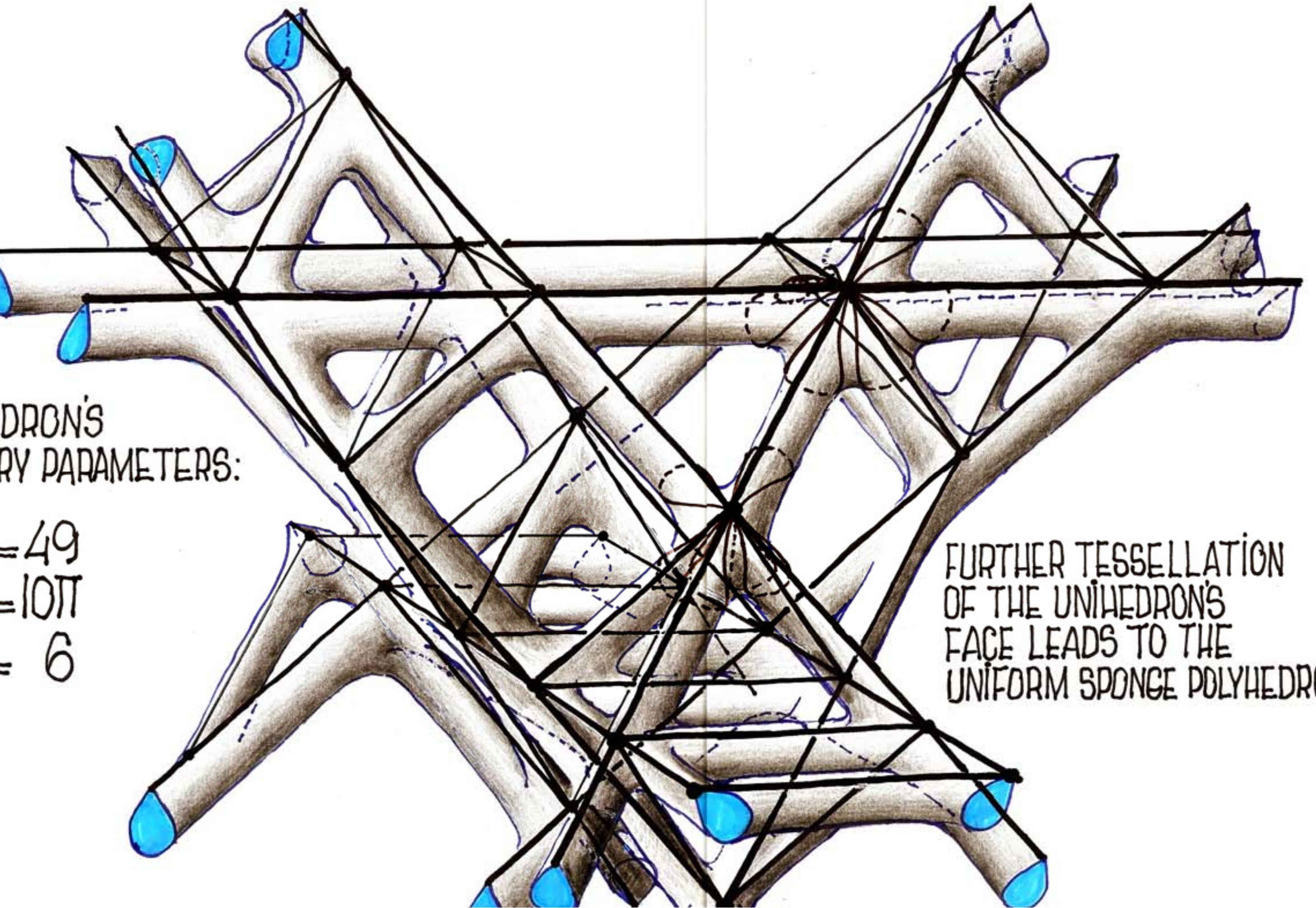

24^{24}_{6}
$g_{T.U.} = 6$
$\sum \alpha = 46\pi$
$Val. = 24$
$V_{T.U.} = 1$
$E_{T.U.} = 12$
$F_{T.U.} = 1$
T.U.

DRON'S
RY PARAMETERS:

=49

=10π

= 6



FURTHER TESSELLATION OF THE UNIHEDRON'S FACE LEADS TO THE UNIFORM SPONGE POLYHEDRON:

$(4^2.6)^6_{49}$

$g_{T.U.}=49$

$\sum\alpha=10\pi$

$Val.=18$

$V_{T.U.}=24$

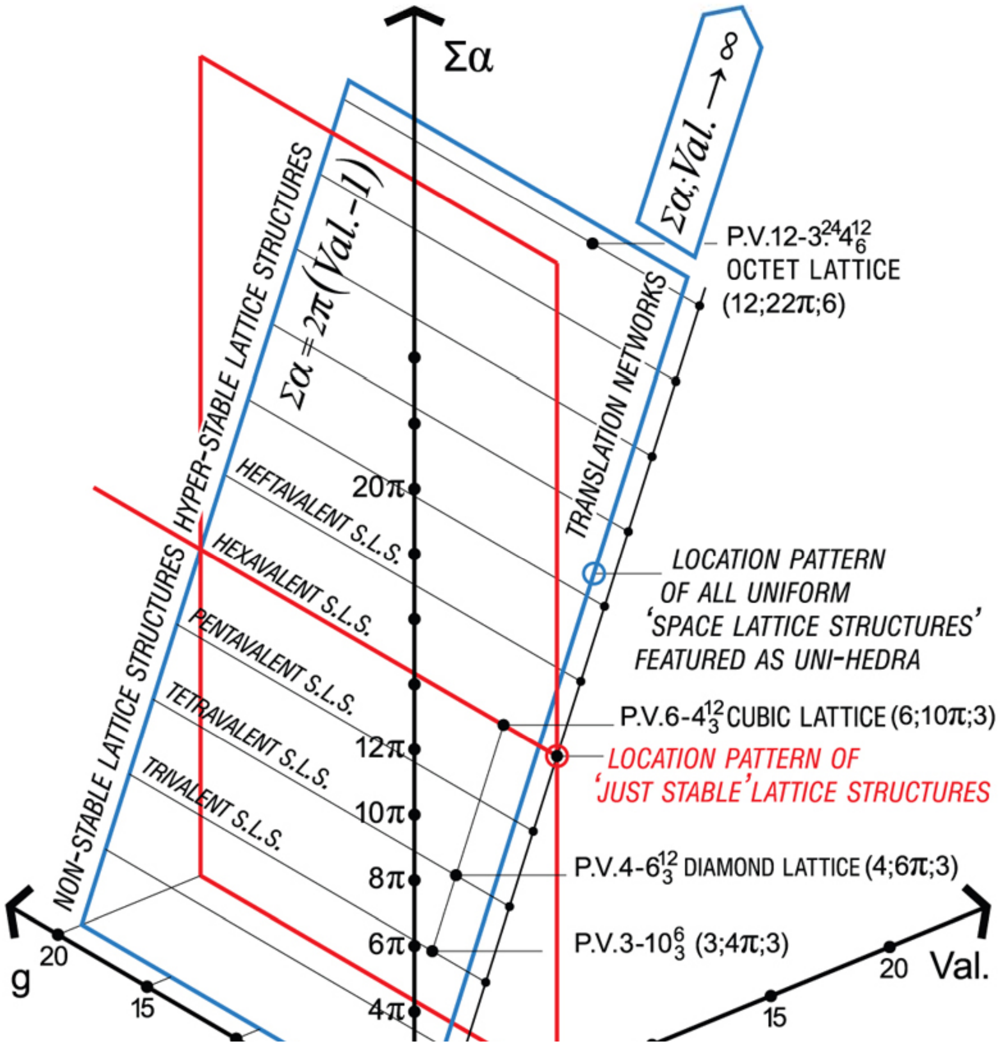
$E_{T.U.}=216$

$F_{T.U.}=96$

Featuring Uniform Space Lattice configurations as embeddable in genetically associated sponge surfaces and perceiving them as tessellations of such surfaces and therefore legitimate polyhedral structures, corresponding to same primary parameters (*Val*; ***Σα* & **Con. ≡ *g***) justifies their sharing in the theoretical umbrella of 'The Periodic Table of the Polyhedral Universe'.**

All conceivable Uniform Space Lattice configurations featuring as uni-hedra, with all their multitude of point-representations within the domain of the ‘’Periodic Table’, form into a single plane location pattern, mathematically determined by the a.m. relation of

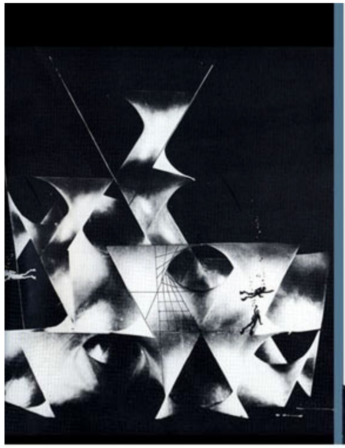
$$\Sigma\alpha = 2\Pi(\textit{Val.}-1)$$

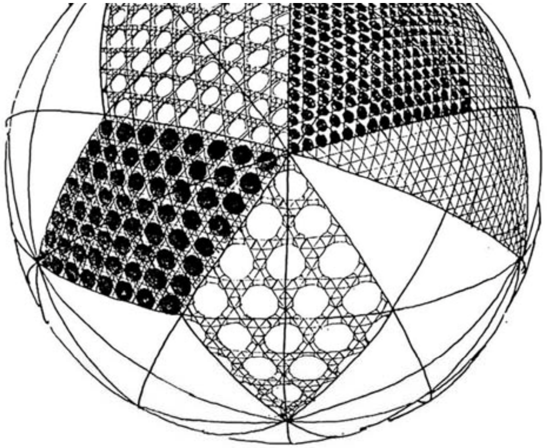


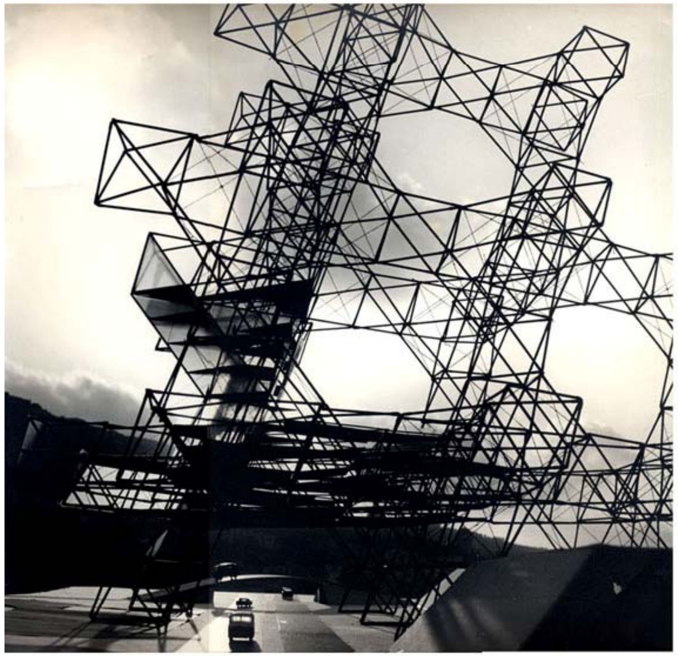
Σα
Σα;Val. → ∞
Σα = 2π(Val.-1)
HYPER-STABLE LATTICE STRUCTURES
NON-STABLE LATTICE STRUCTURES
TRANSLATION NETWORKS
HEFTAVALENT S.L.S.
HEXAVALENT S.L.S.
PENTAVALENT S.L.S.
TETRAVALENT S.L.S.
TRIVALENT S.L.S.
P.V.12-3.24 4 12 6
OCTET LATTICE
(12;22π;6)
LOCATION PATTERN
OF ALL UNIFORM
'SPACE LATTICE STRUCTURES'
FEATURED AS UNI-HEDRA
P.V.6-4 12 3 CUBIC LATTICE (6;10π;3)
LOCATION PATTERN OF
'JUST STABLE' LATTICE STRUCTURES
P.V.4-6 12 3 DIAMOND LATTICE (4;6π;3)
P.V.3-10 6 3 (3;4π;3)
20π
12π
10π
8π
6π
4π
g
20
15
Val.
20
15

IN CONCLUSION

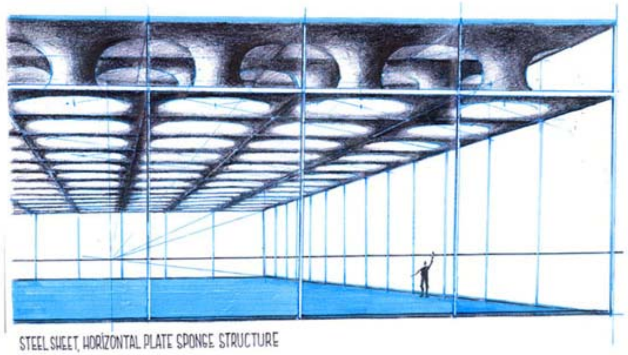
With the introduction of sponge polyhedra, spherical, toroidal and hyperbolical, and relaxation of definitions (admissibility of any polyhedral configuration that complies with the celebrated Euler's theorem), the polyhedral universe has expanded dramatically. Metaphorically speaking, it exploded from Ptolemaic- Euclidean world picture to our contemporary



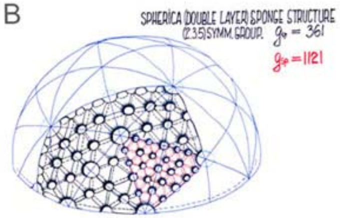




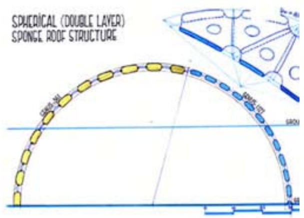
A


STEEL SHEET, HORIZONTAL PLATE SPONGE STRUCTURE

B

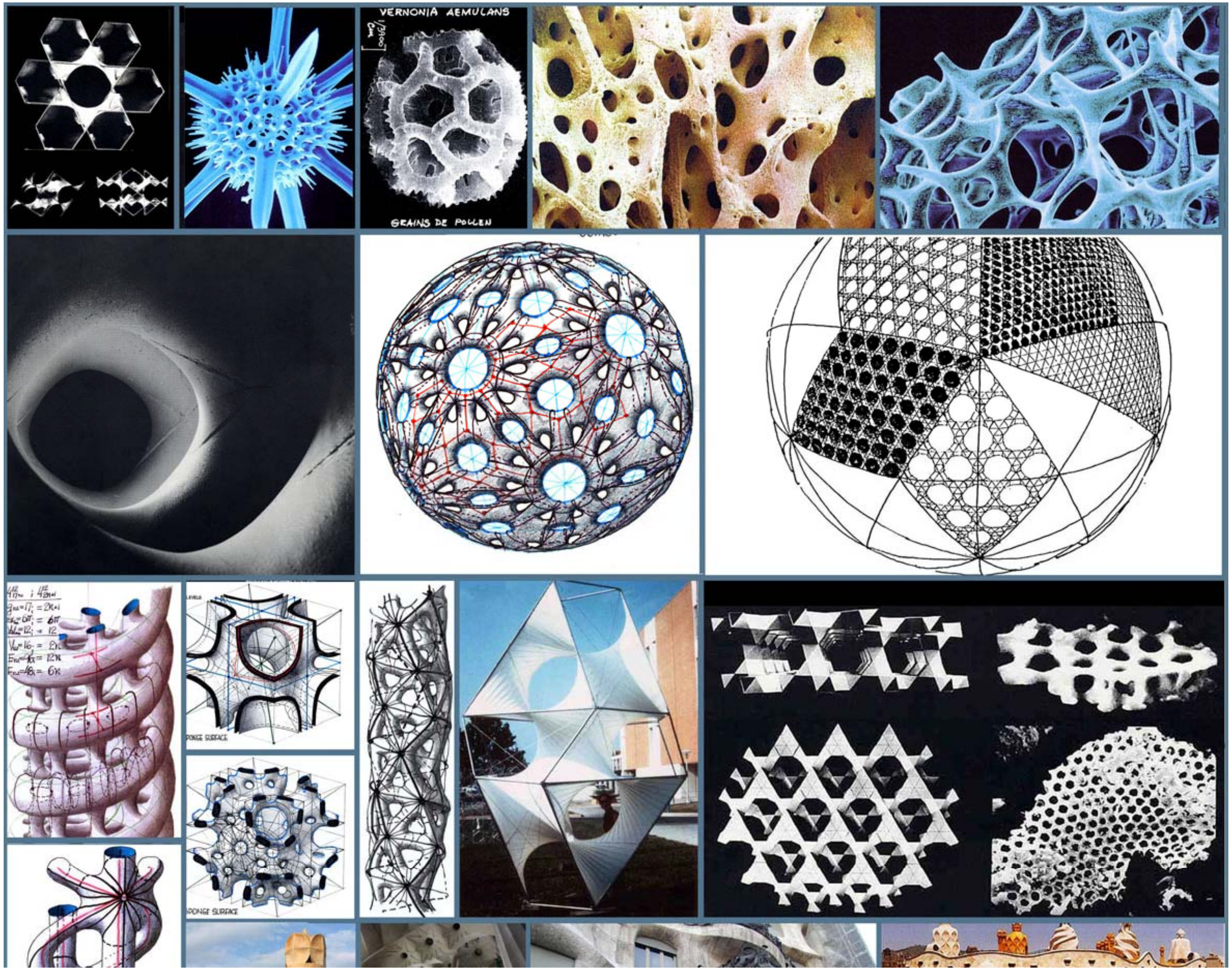

SPHERICA (DOUBLE LAYER) SPONGE STRUCTURE
(2,3,5) SYMM. GROUP. $g_0 = 361$
$g_0 = 1121$

C


SPHERICAL (DOUBLE LAYER)
SPONGE ROOF STRUCTURE

With some extrapolation of the perceiving mind it is right to claim that the sponge phenomenon, with its porosity and permeability characteristics, is central to the physical morphological nature of the human habitat, and represents its defining imagery.





VERNONIA AEMULANS
GRAINS DE POLLEN

Abstract and physical 3-D space is not a passive vacuum. It is populated with inter-relating and inter-connected entities, generating configurations represented as diagrams with a network characteristics and hyperbolic 'force fields', and surface partitions, aptly described as sponge surface configurations. Diagrams of this kind can represent the structure of almost any plurality that may exist.