The Morphologically Associated Quintuplet Phenomena of 3D Space

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Faculty of Architecture

3D space phenomenology of our habitat is mostly concerned with the morphological features of network structures, cellular polyhedral close packing agglomerations and space subdividing partition surfaces.
They form the core of our imagery of the physical and the virtual-imaginary space we live in. Their manipulation determines the structure of our habitat, provides for its architectural design and consequently for its formal evolution and development.

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ISRAEL NATIONAL LOTTERY COUNCIL FOR THE ARTS

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ent exhibition is based on an earlier one by the title of 'Conceptual Architecture' in the Haifa Museum of Art (1998); corated by Daniella Tlamor and coordinated by Lior Datz.

Special Thanks to:

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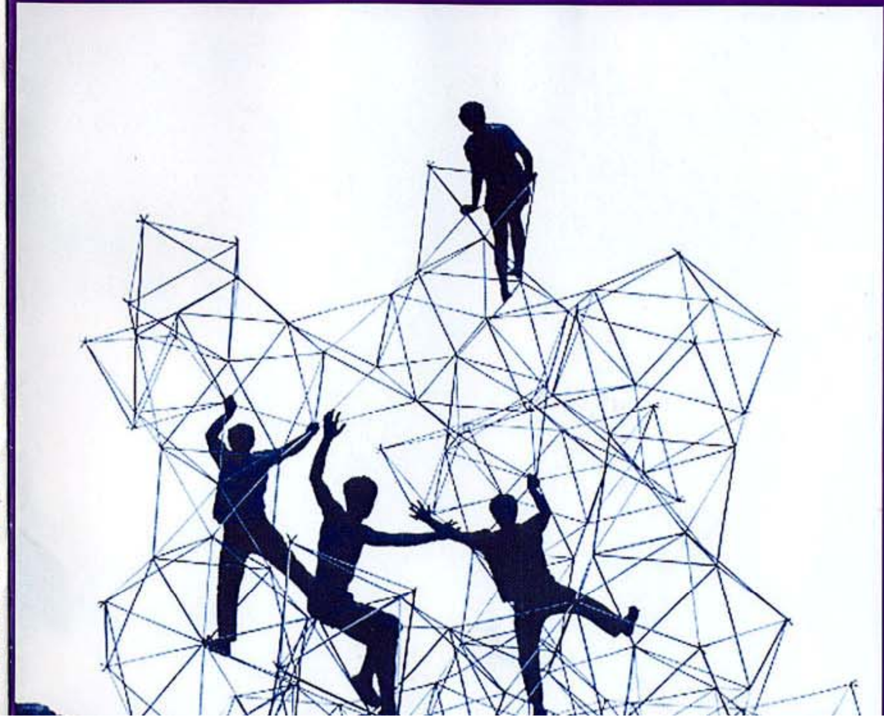
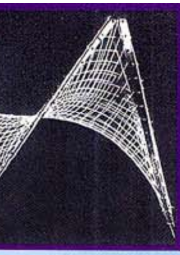
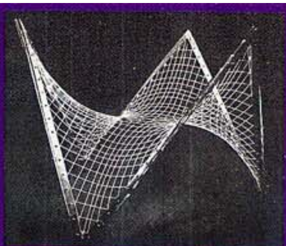
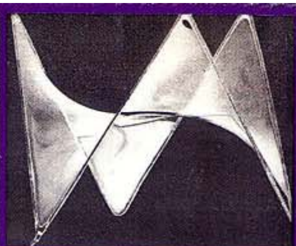
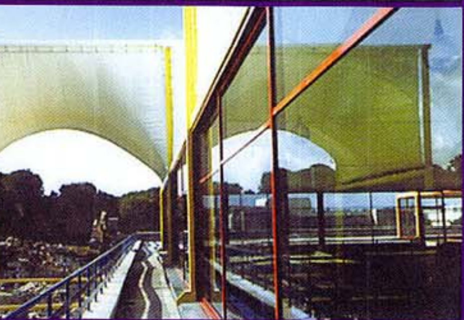
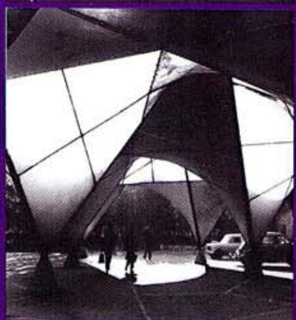
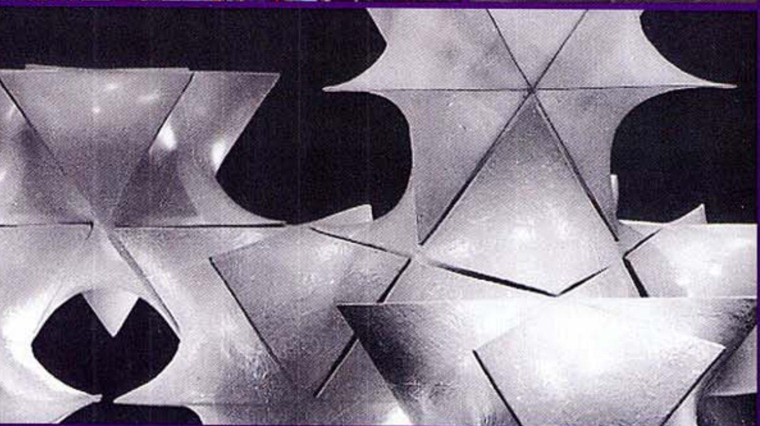
to my son Igor Bur, designer of my exhibition poster

Michael Burt - Structural Morphology and Manipulation of Space/
Conceptual Architecture/
Moscow - Summer 2003

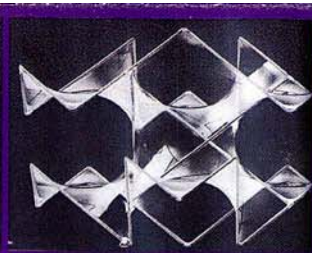
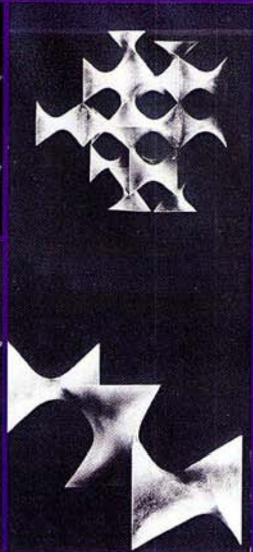
Михаель Бурт - Морфология структур и актуальное пространство/
Концептуальная архитектура/
Москва - 2003

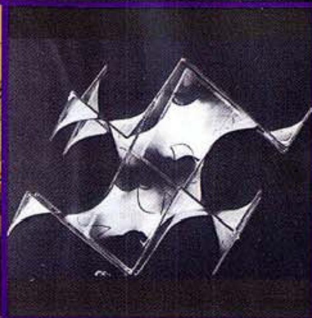


Networks, a connected assembly of vertices and edges, may represent the structure of almost any abstract or physical plurality that may exist, in the world of phenomena of the biological-physical-material-spiritual domains, on every possible scale, from the nano-molecular to the cosmological. They are the morphological essence of our built structures of products, buildings, urban sprawls, regional fabric and inter-national boundaries and all the associated transportation-communication interaction systems and installations of our living environment space

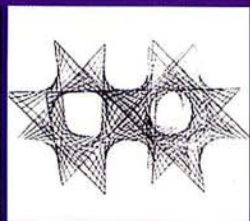
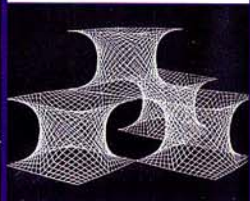









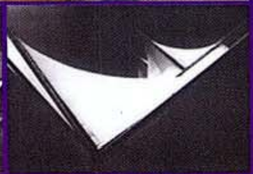



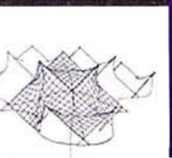


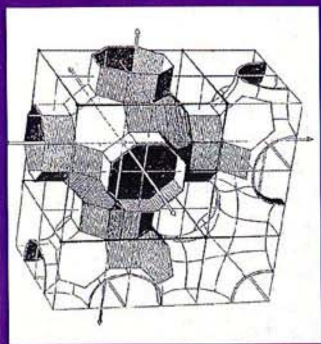
Periodic hyperbolic (saddle shaped) surfaces and their application to tensile -membrane and shell structures; membrane structures with a 'crystalline order'.

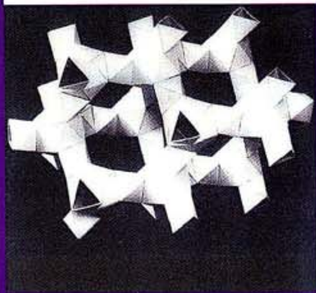




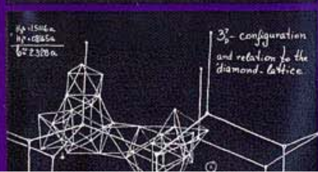
Повторяющиеся гиперболические (седловидной формы) поверхности. Их приложения в области

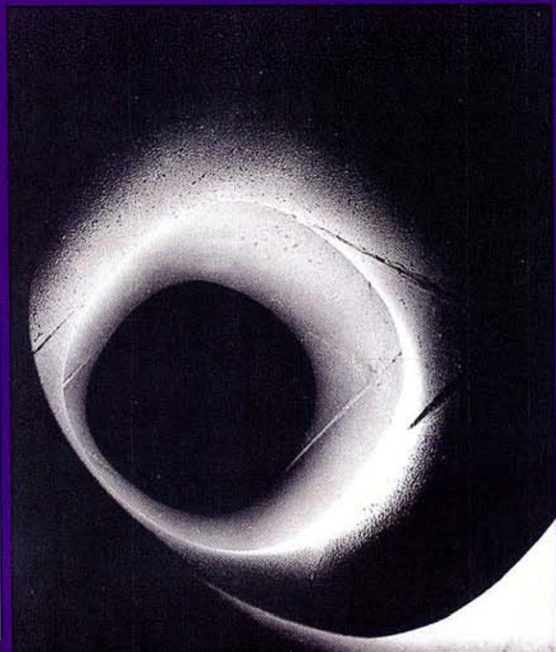



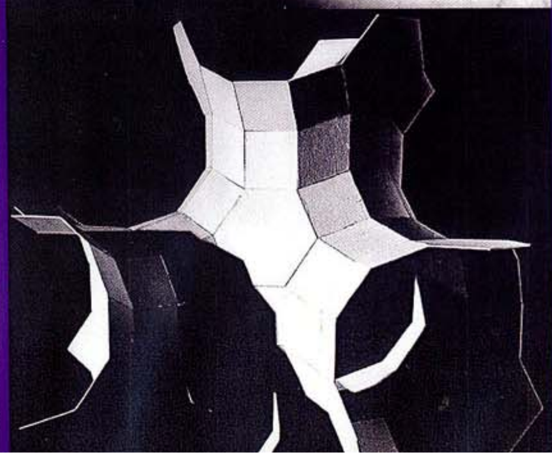




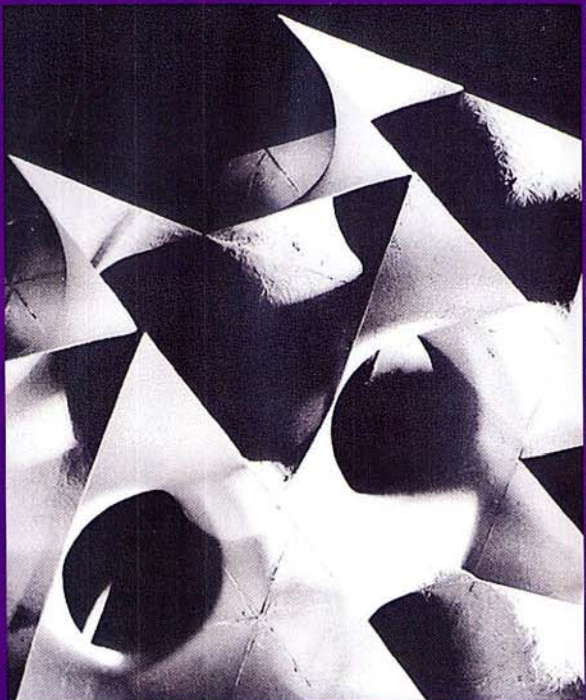
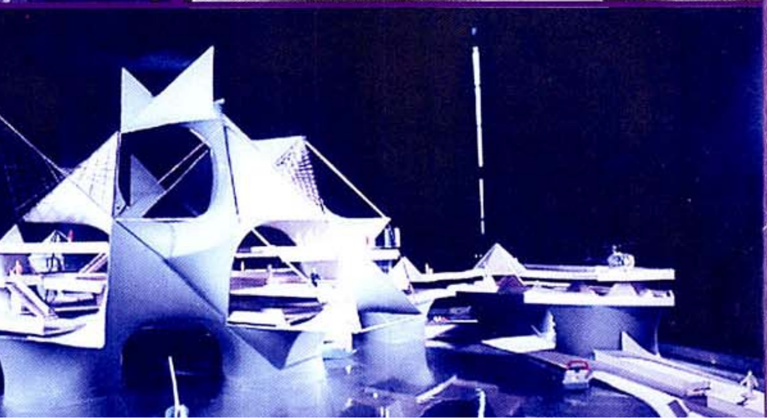



3_p^7- configuration
and relation to the
diamond-lattice

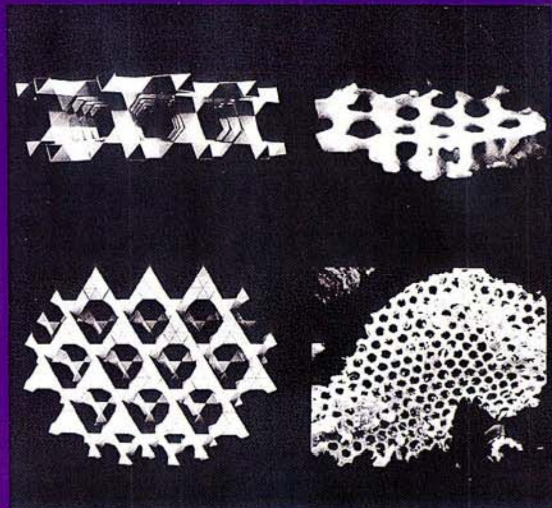


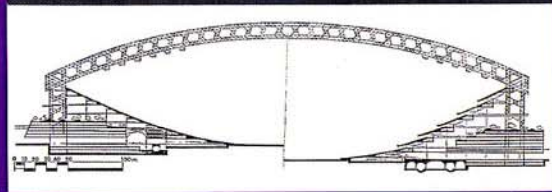
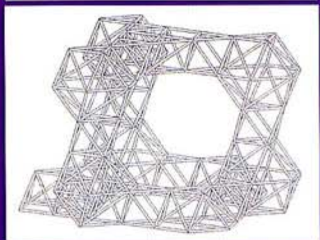
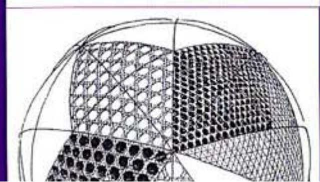
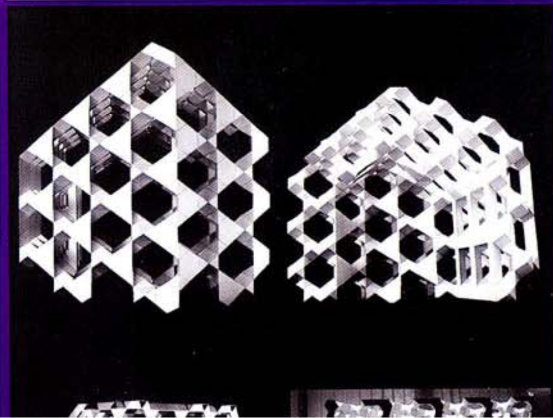


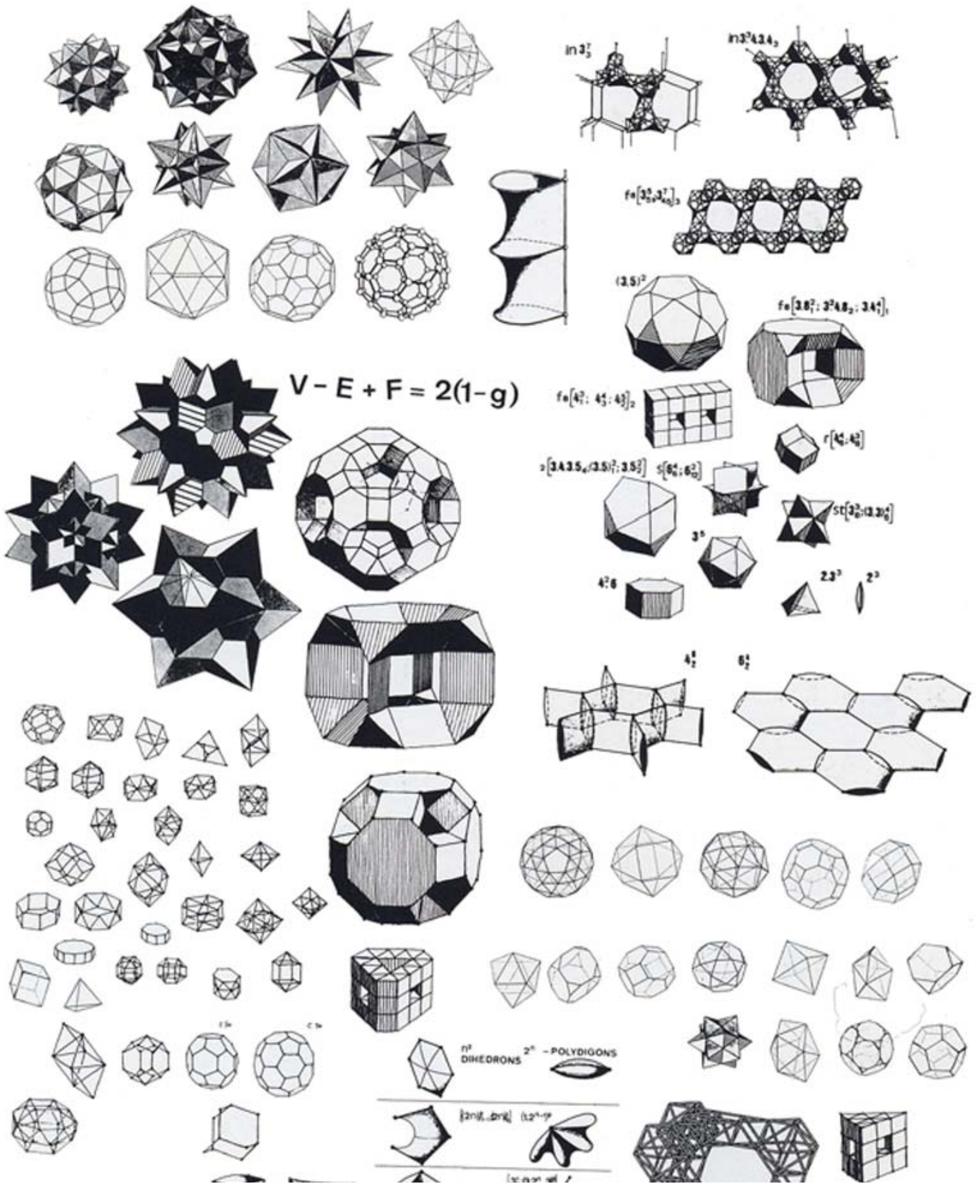


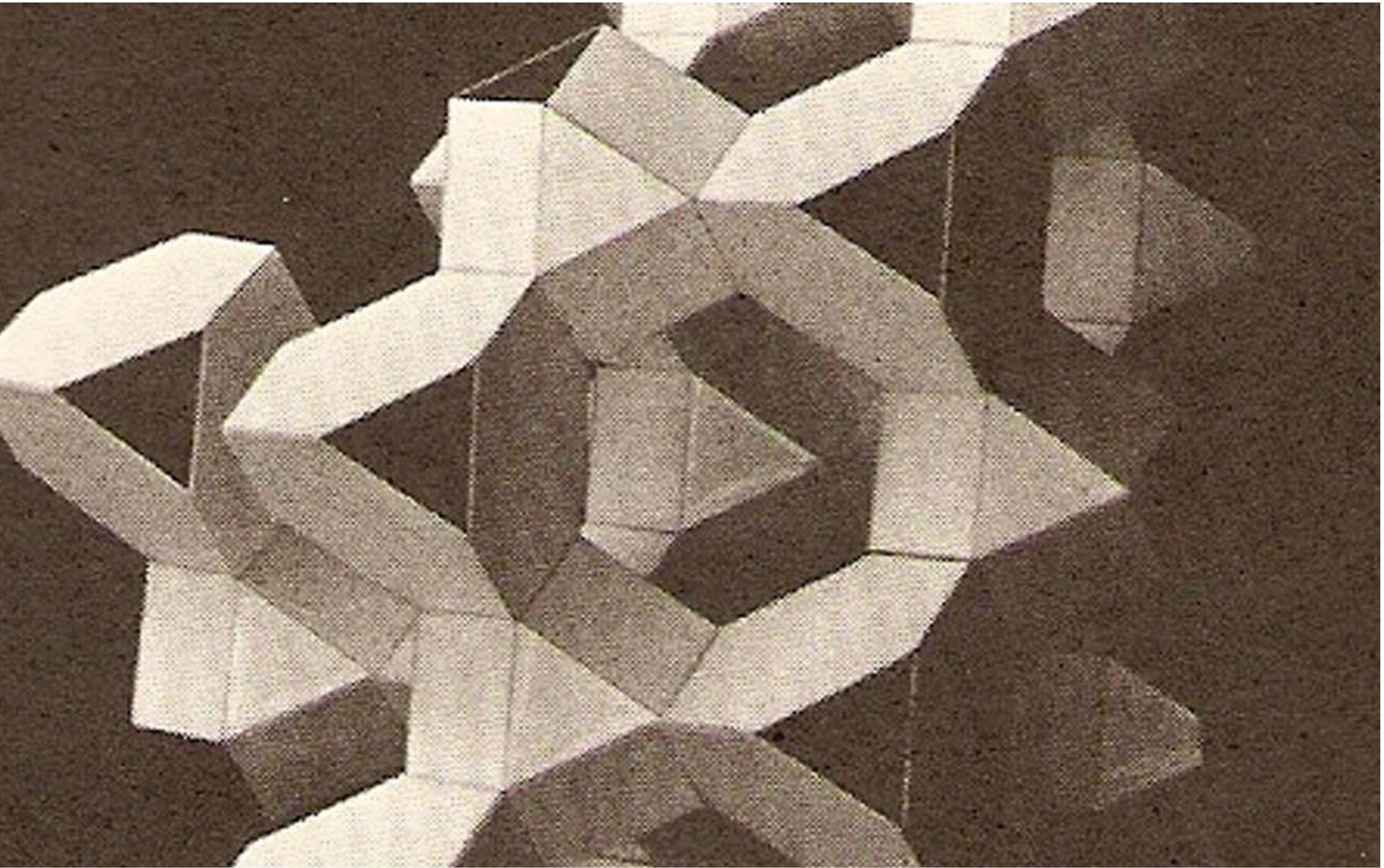
Tessellation of periodic hyperbolic surfaces,
- evolution of the Infinite Polyhedra morphology and the Infinite Polyhedra Lattice (I.P.L) space-truss concept.

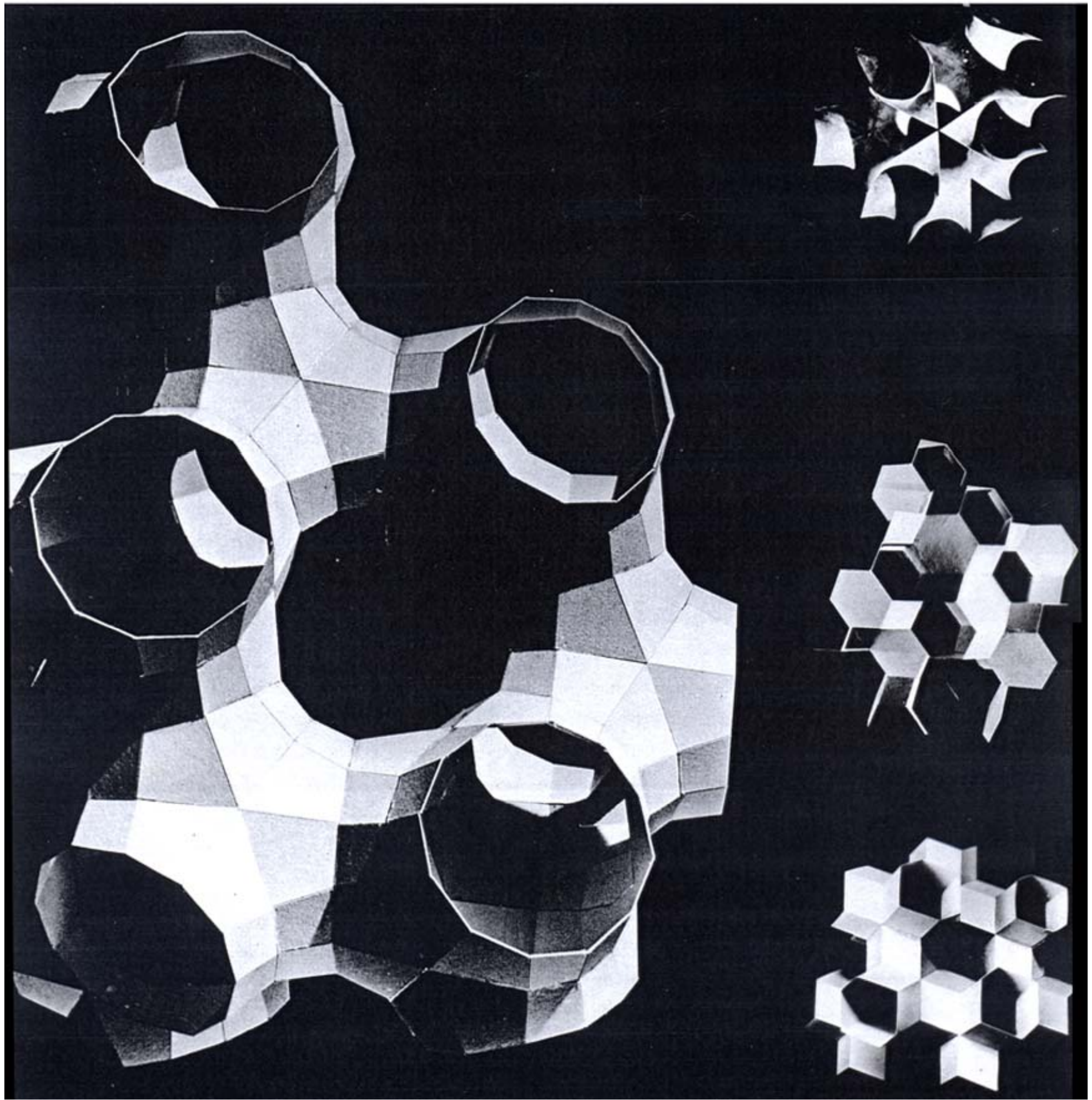


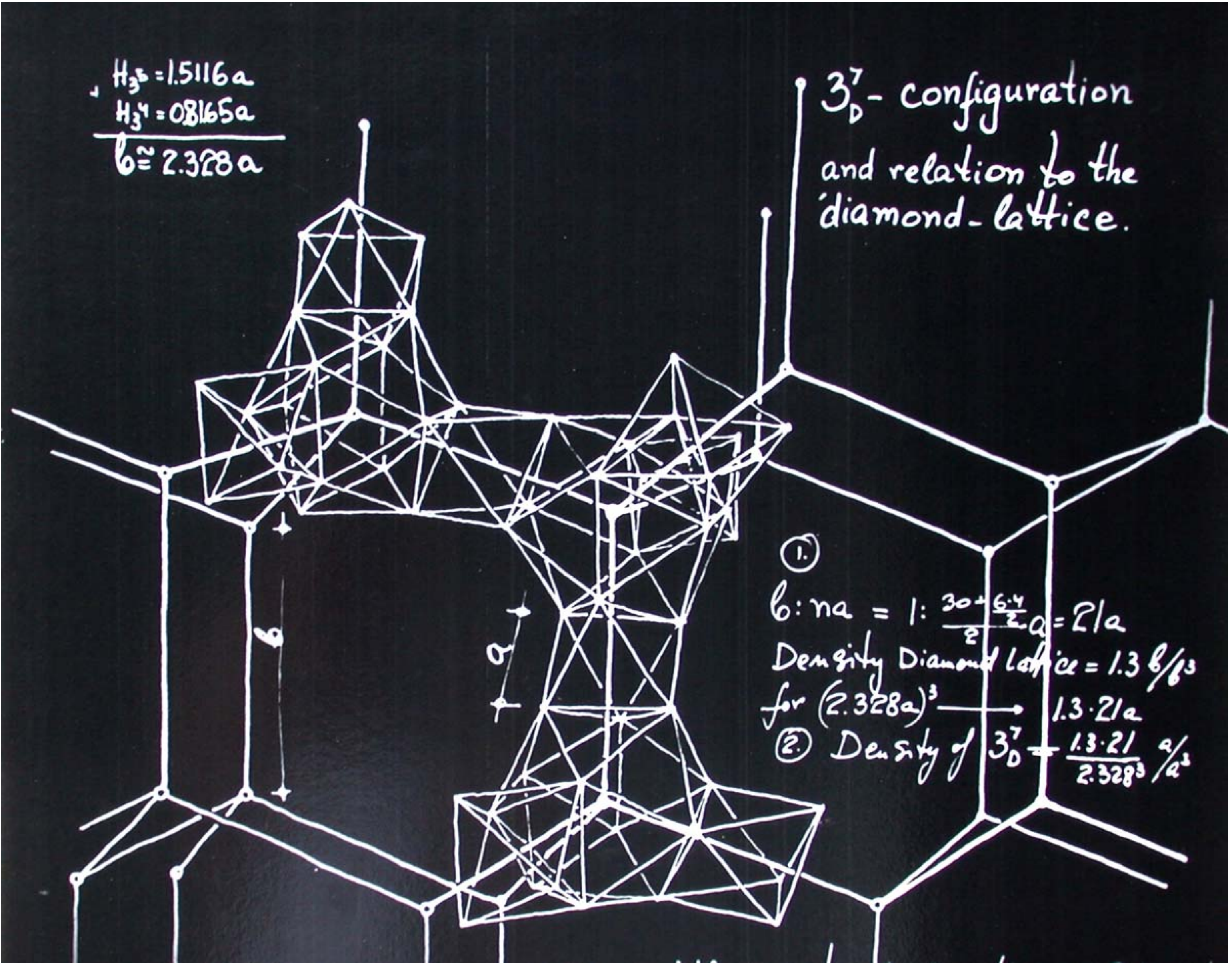







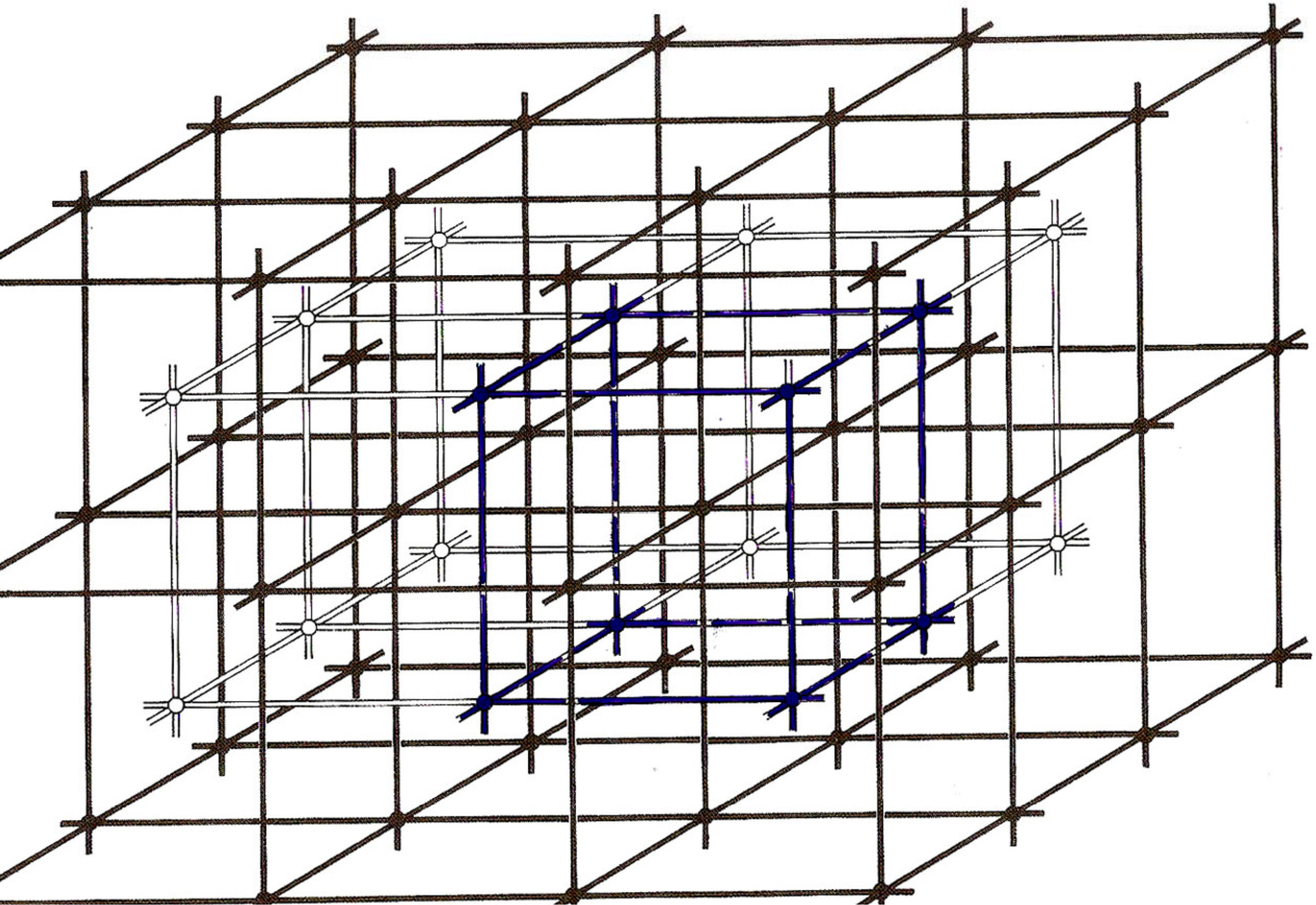
V – E + F = 2(1-g)
DIHEDRONS
– POLYDIGONS

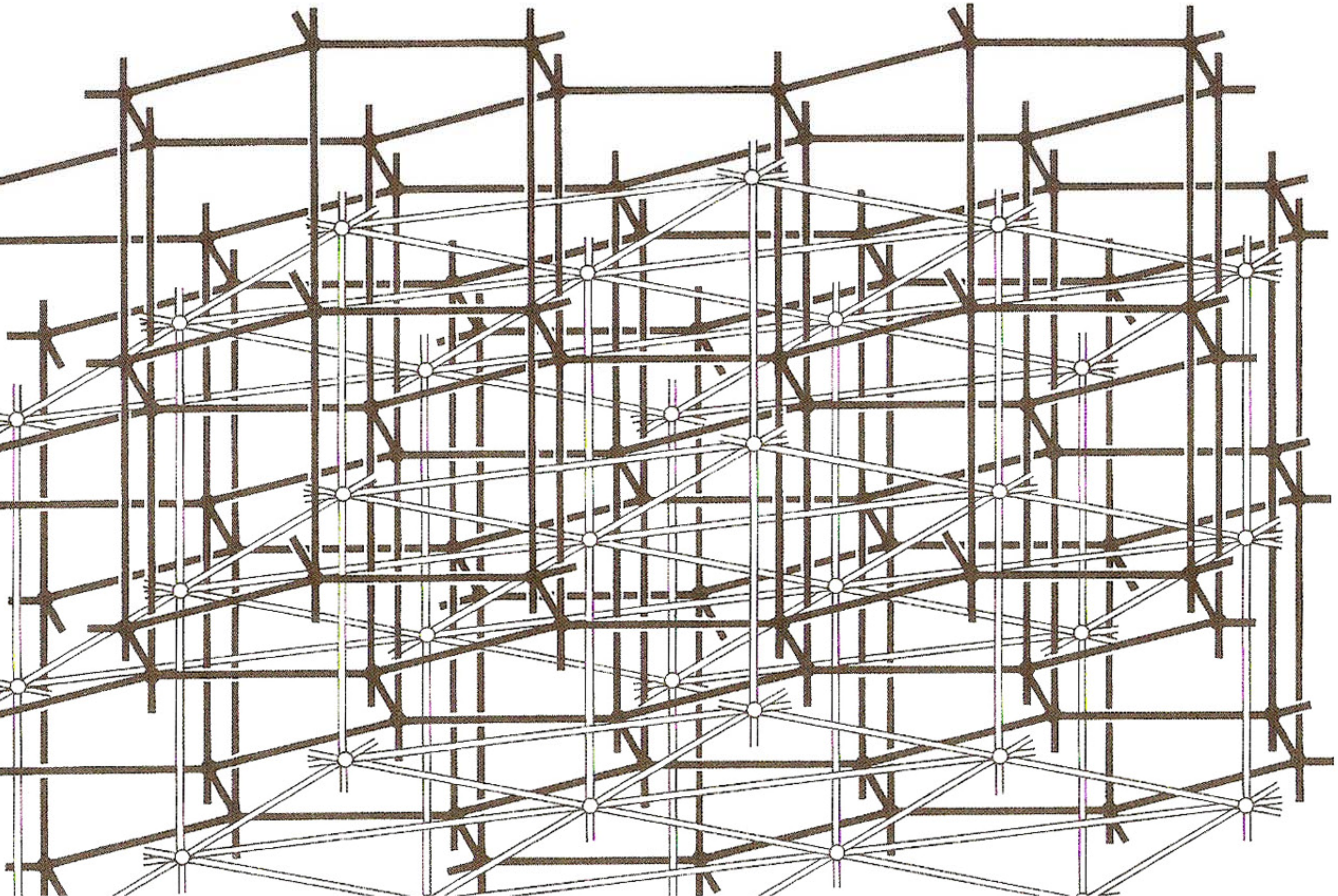


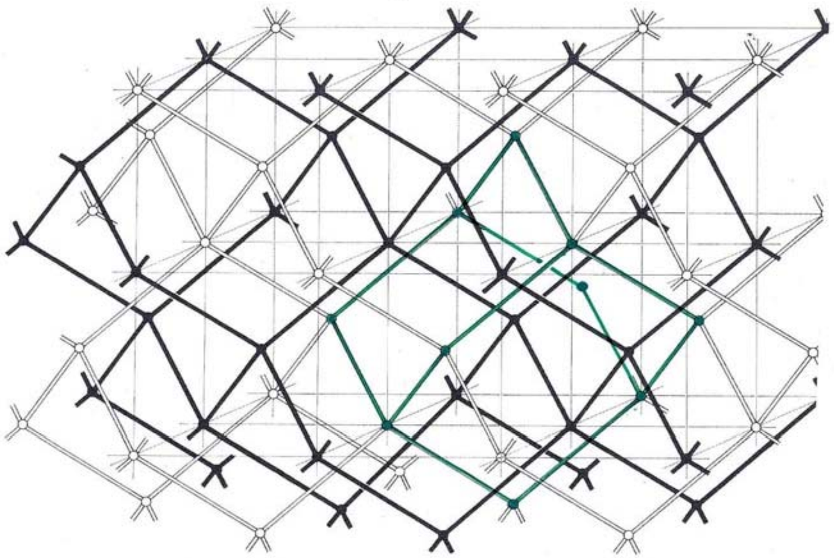


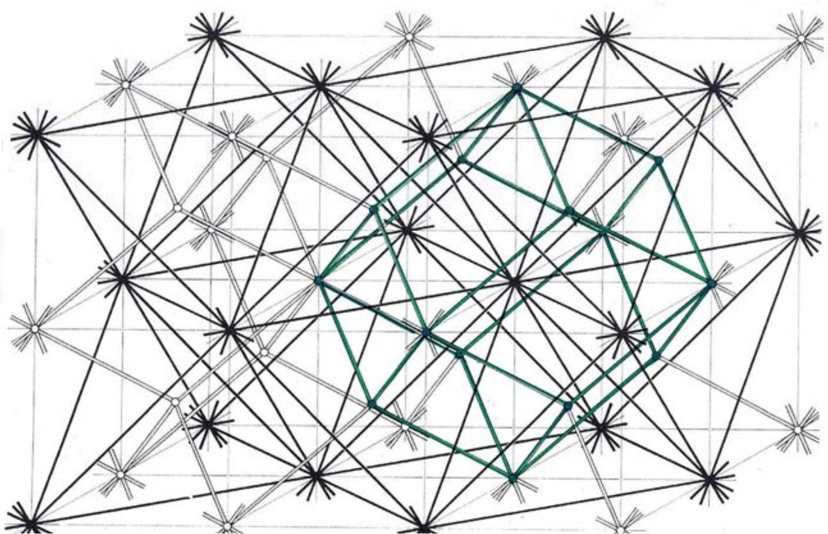

$H_{3^5} = 1.5116a$
$H_{3^4} = 0.8165a$
$b \cong 2.328a$
3^7_D - configuration
and relation to the
'diamond-lattice.
(1.)
$b : na = 1 : \frac{30 + \frac{6 \cdot 4}{2}}{2} a = 21a$
Density Diamond Lattice = $1.3\ b/b^3$
for $(2.328a)^3 \longrightarrow 1.3 \cdot 21a$
(2.) Density of $3^7_D = \frac{1.3 \cdot 21}{2.328^3}\ a/a^3$

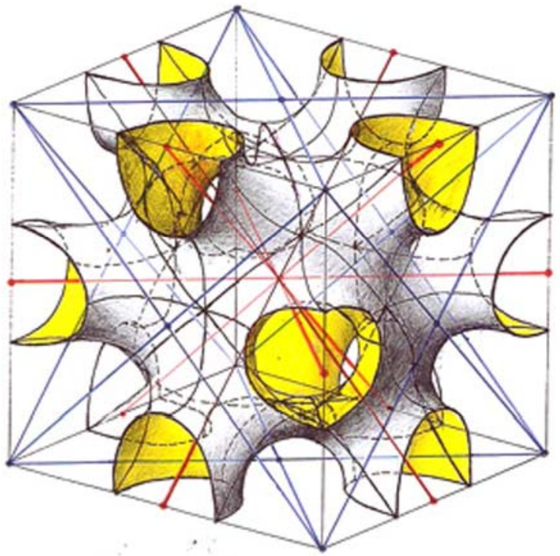
Networks, in general, come in dual reciprocally related pairs.
Every 3D network is genetically associated with a cellular close packing of polyhedral (mostly finite) volumetric solids.
Any pair of dual networks is associated with a unique, topologically and symmetrically determined hyperbolical sponge-surface, subdividing the entire space between the two.

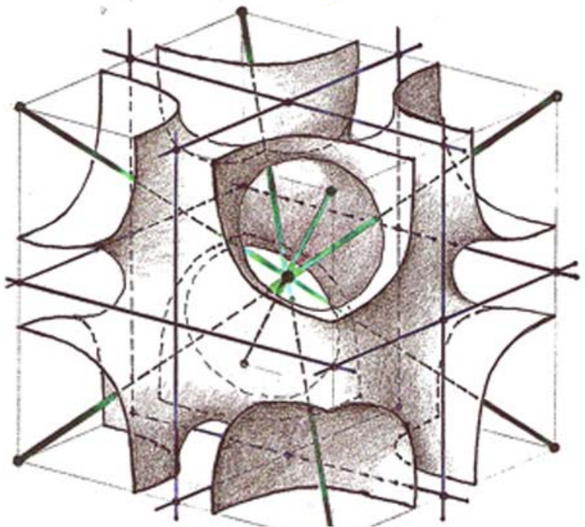


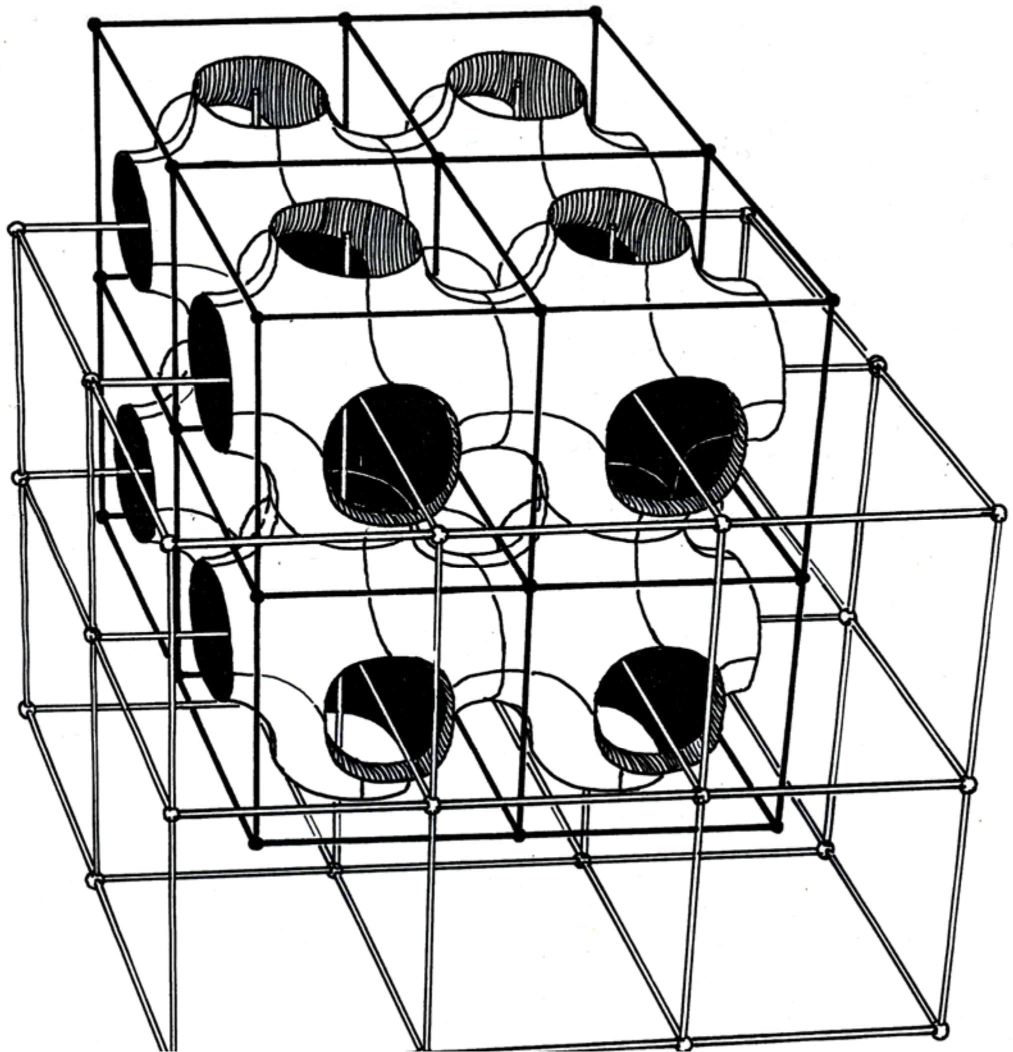




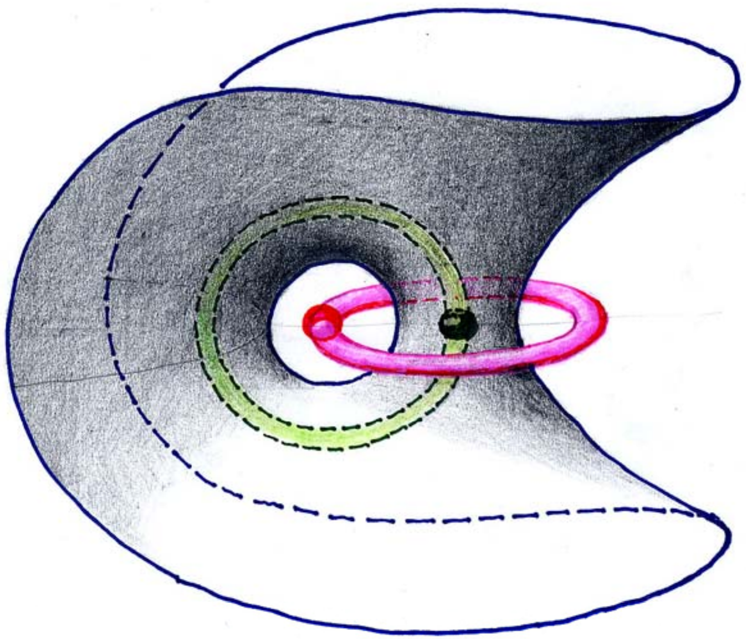


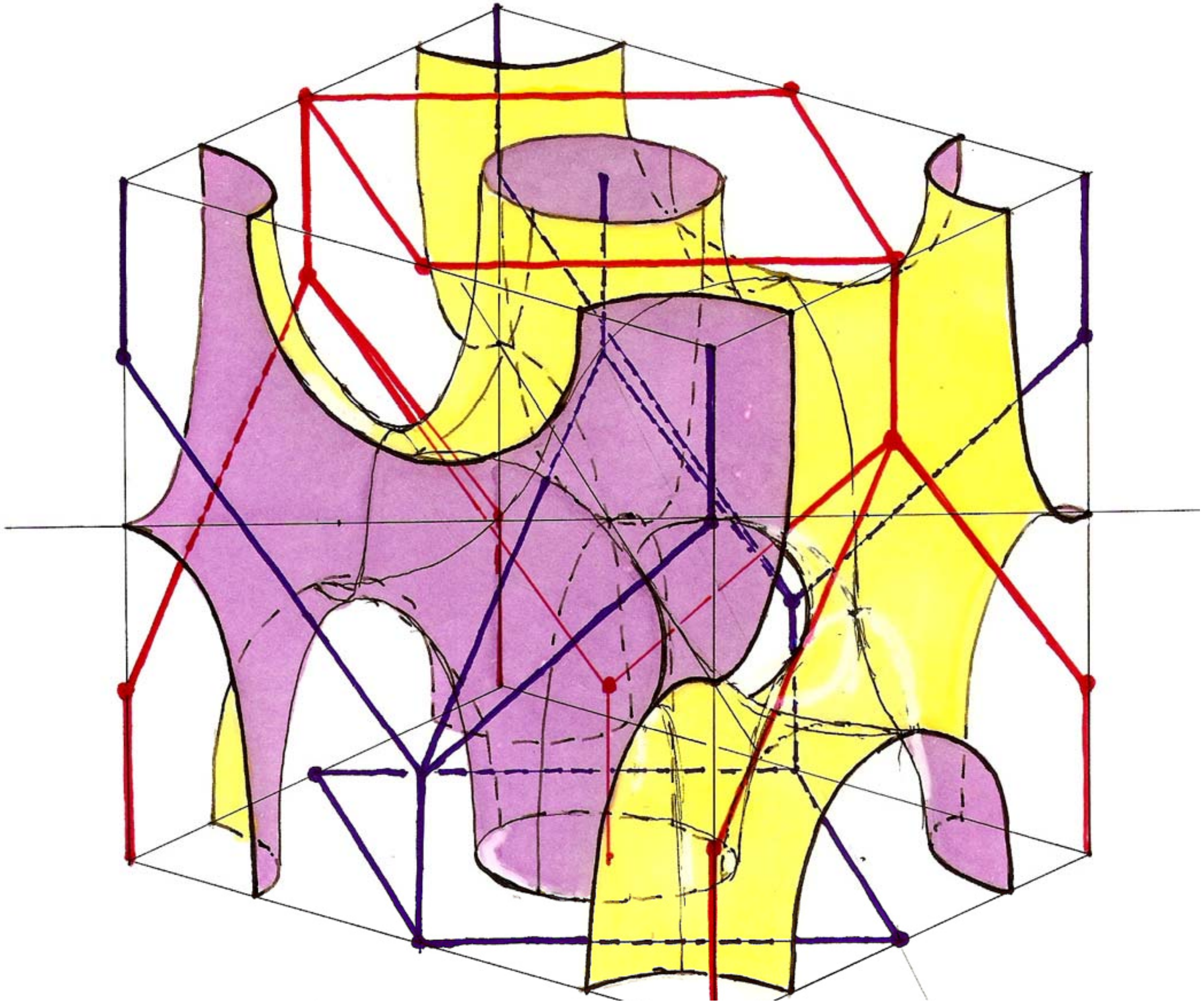


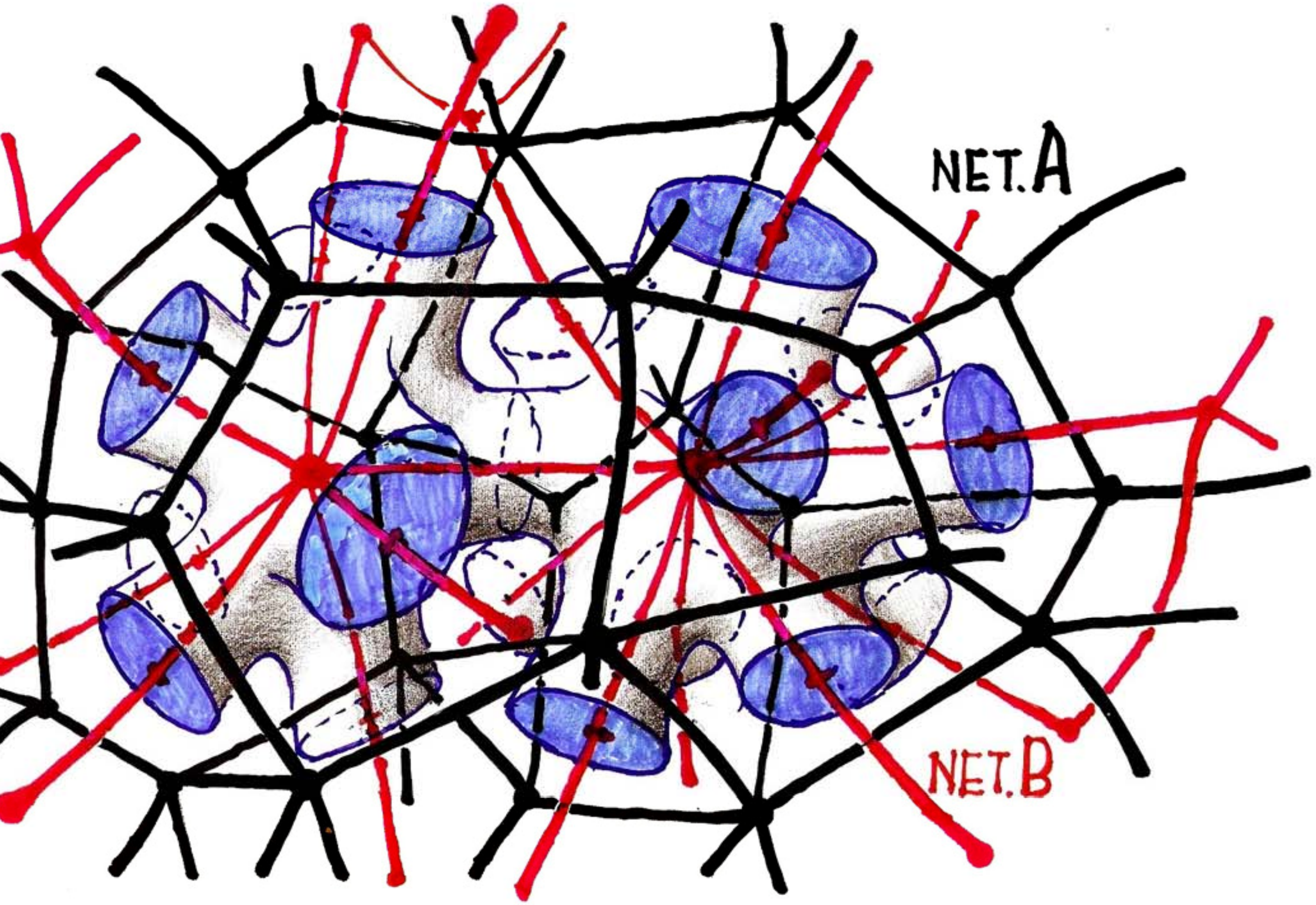




A TRINITY OF TWO DUAL (MOST PRIMITIVE) 3-D NETWORKS AND THE ASSOCIATED (RECIPROCAL) SPONGE SURFACE, SUBDIVIDING THE SPACE BETWEEN THE TWO.

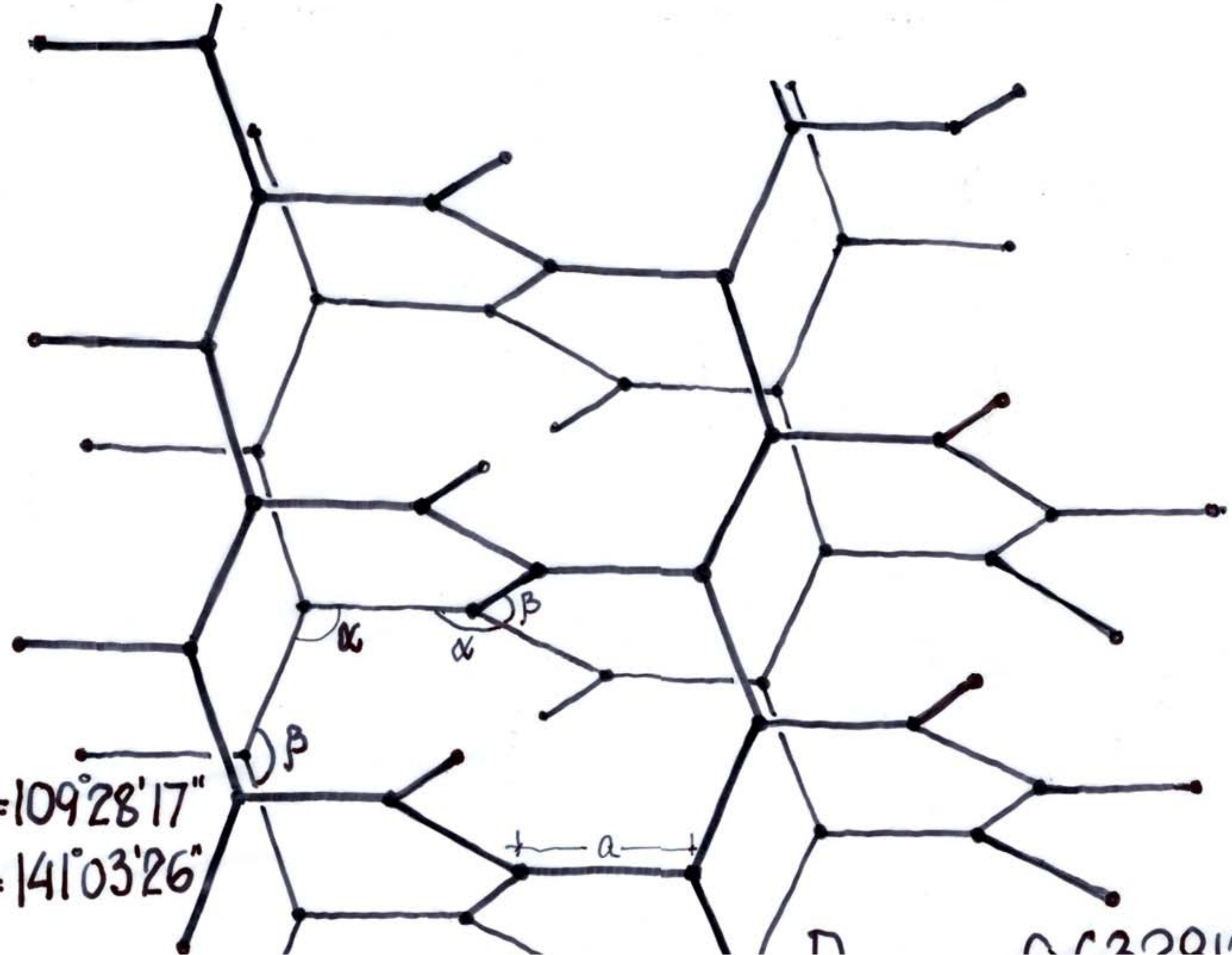






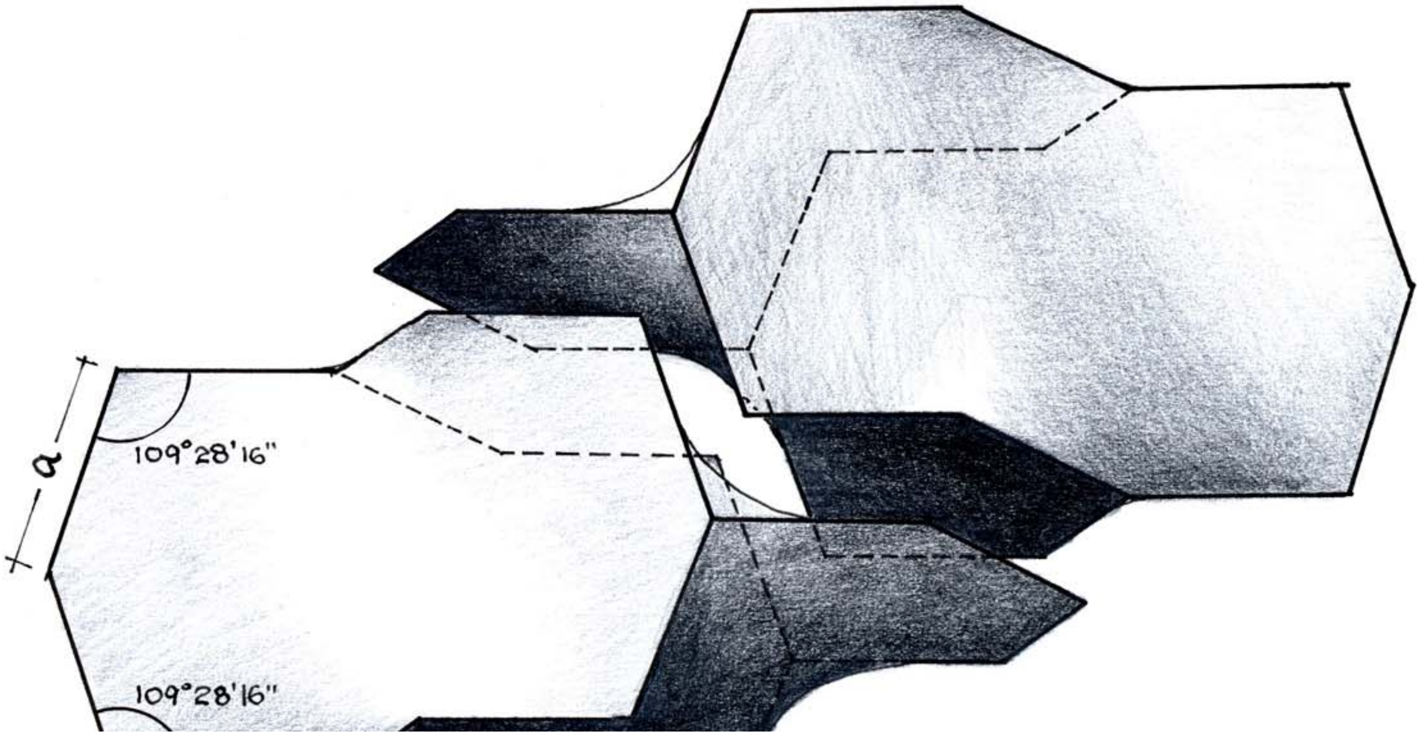
TWO (NON PERIODIC) DUAL NETWORKS, A & B AND THE
PONGE SURFACE SUBDIVIDING SPACE BETWEEN THE TWO

UNIFORM TRIVALENT SPACE LATTICE – 10_3^{10}

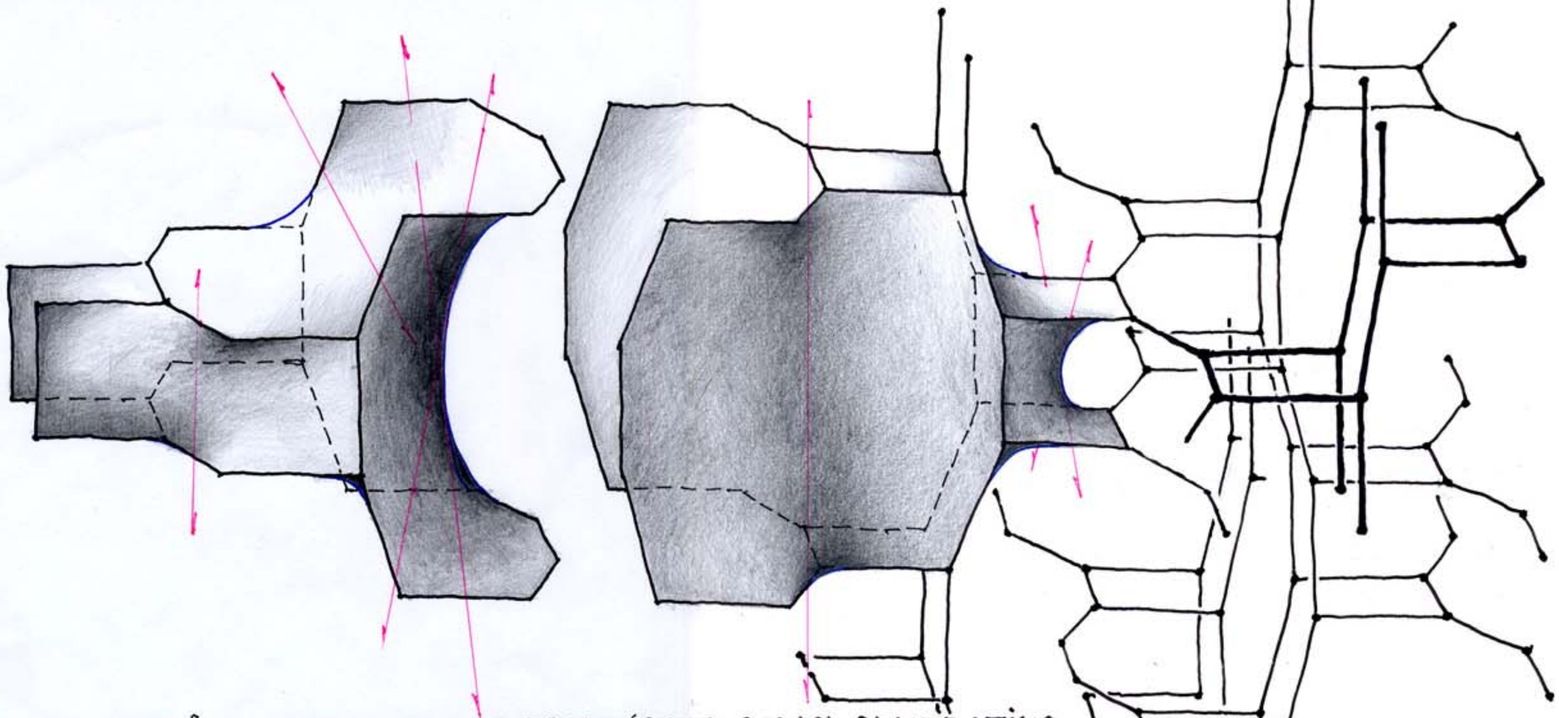


= 109°28'17"
= 141°03'26"

CA-TETRAHEDRON, A SELF CLOSE-PACKING SADDLE-POLYHEDRON,
NERATING THE UNIFORM TRIVALENT SPACE LATTICE-10_3^{10}.



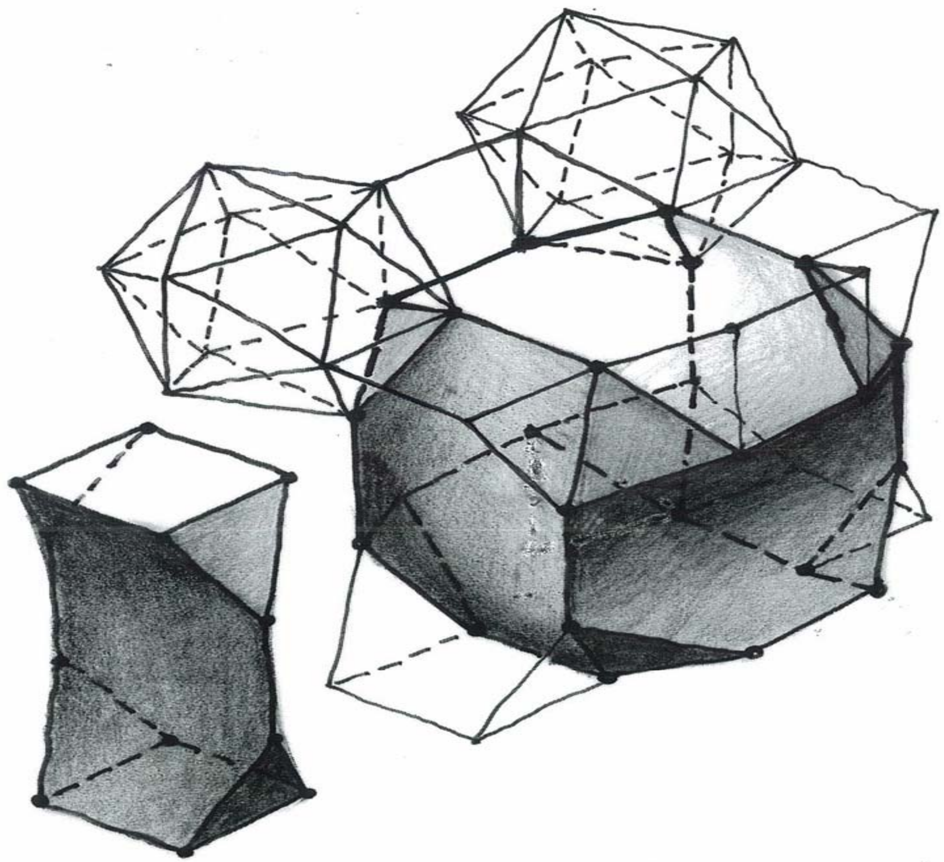
$n.(3-8^{3} \cdot 12^{2} \cdot 16^{2}_{17}) = 0.596104349\ a/a^{3}$



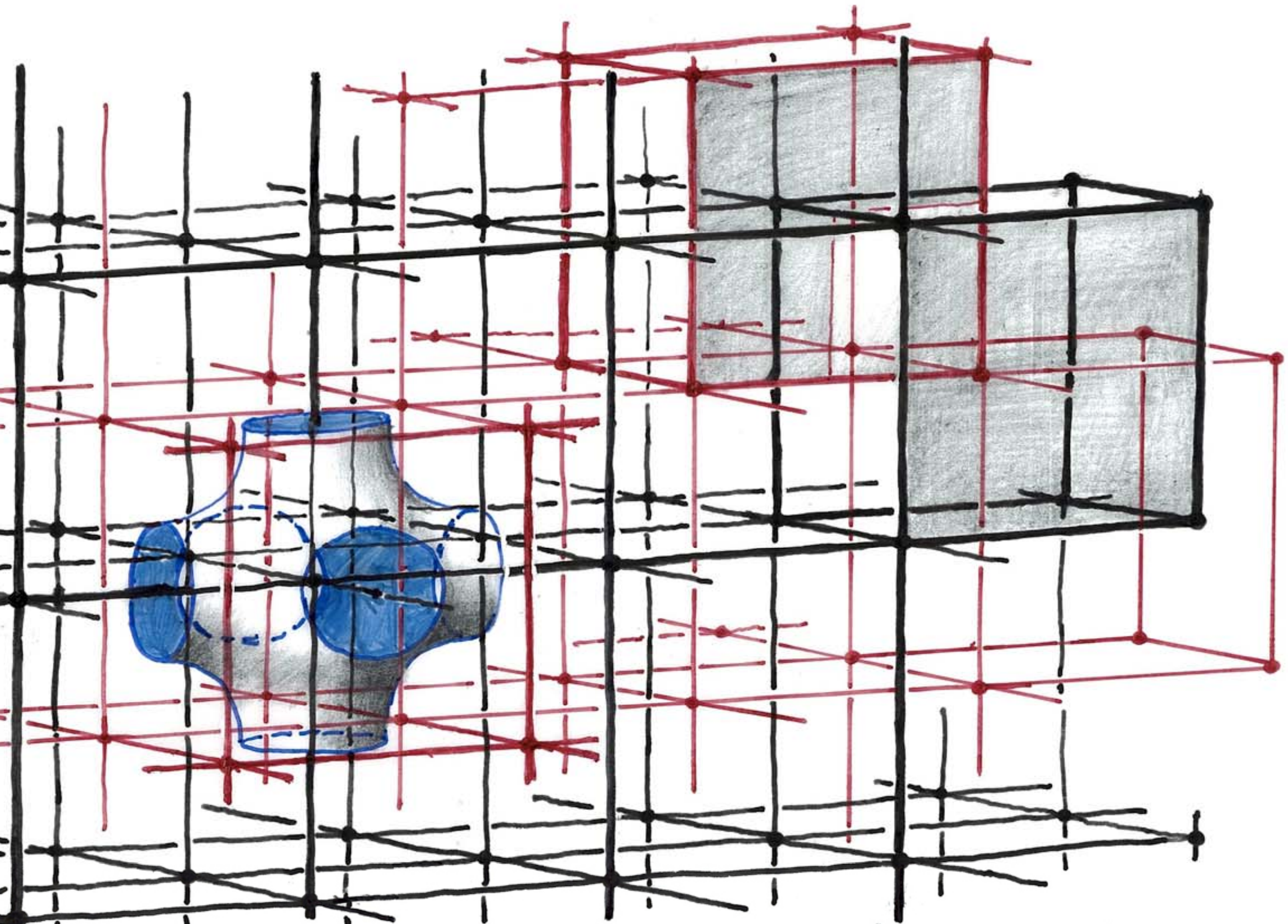
CLOSE PACKİNG SADDLE POLYHEDRON (88 12·8 12 16) GENERATİNG

Cellular, loose or compact close **space-packings** and their polyhedral entities-solids, represent the morphological imagery of the segregated habitat solutions of living multitudinous societies in zoology, botany or the virus domains and the structures of all material crystalline aggregations as well.

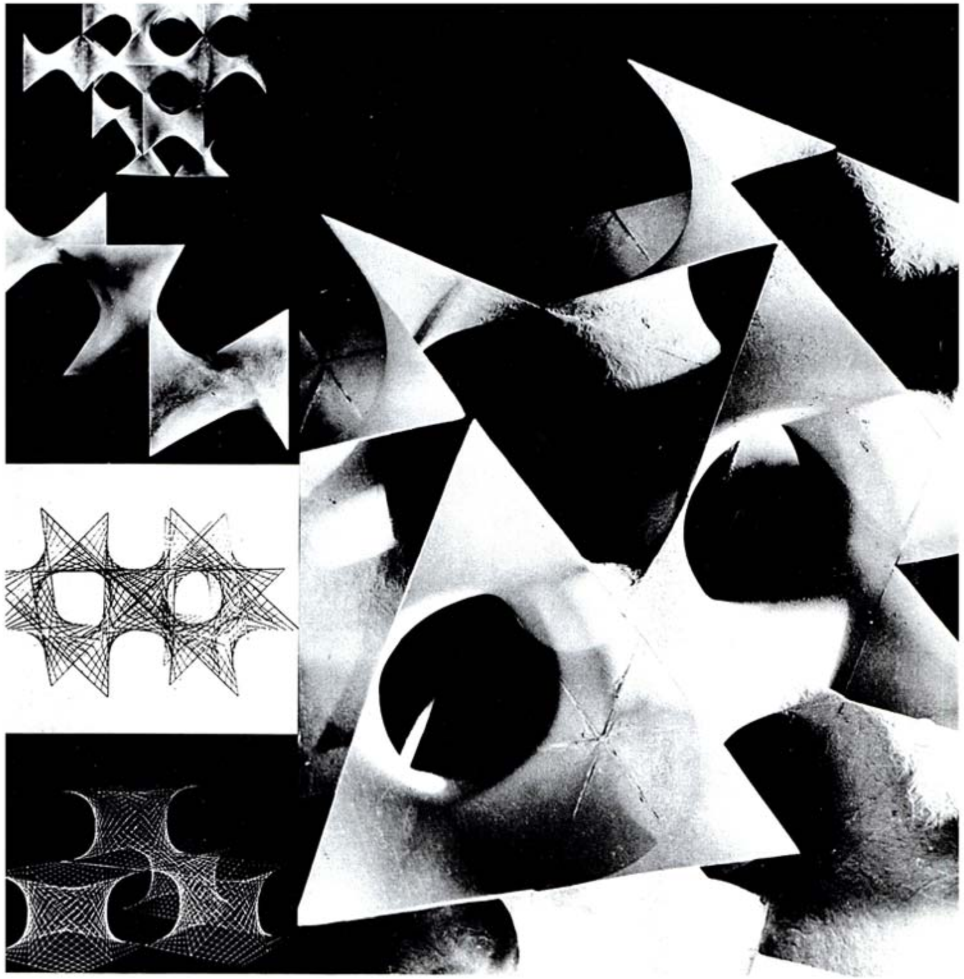
The finite cell units, mostly shaped like “saddle polyhedra”, (having hyperbolical curved faces, with one or two, and no more than two faces, meeting at every edge) conform with the Euler’s theorem and formula of $V-E+F=2K$ (where V,E,F&K stand for vertices, Edges, Faces and the Euler characteristic K, respectively). For finite polyhedra $K=2$.



THREE SOLIDS, CLOSE PACKING OF WHICH
GENERATE THE UNIFORM HEXAVALENT 6

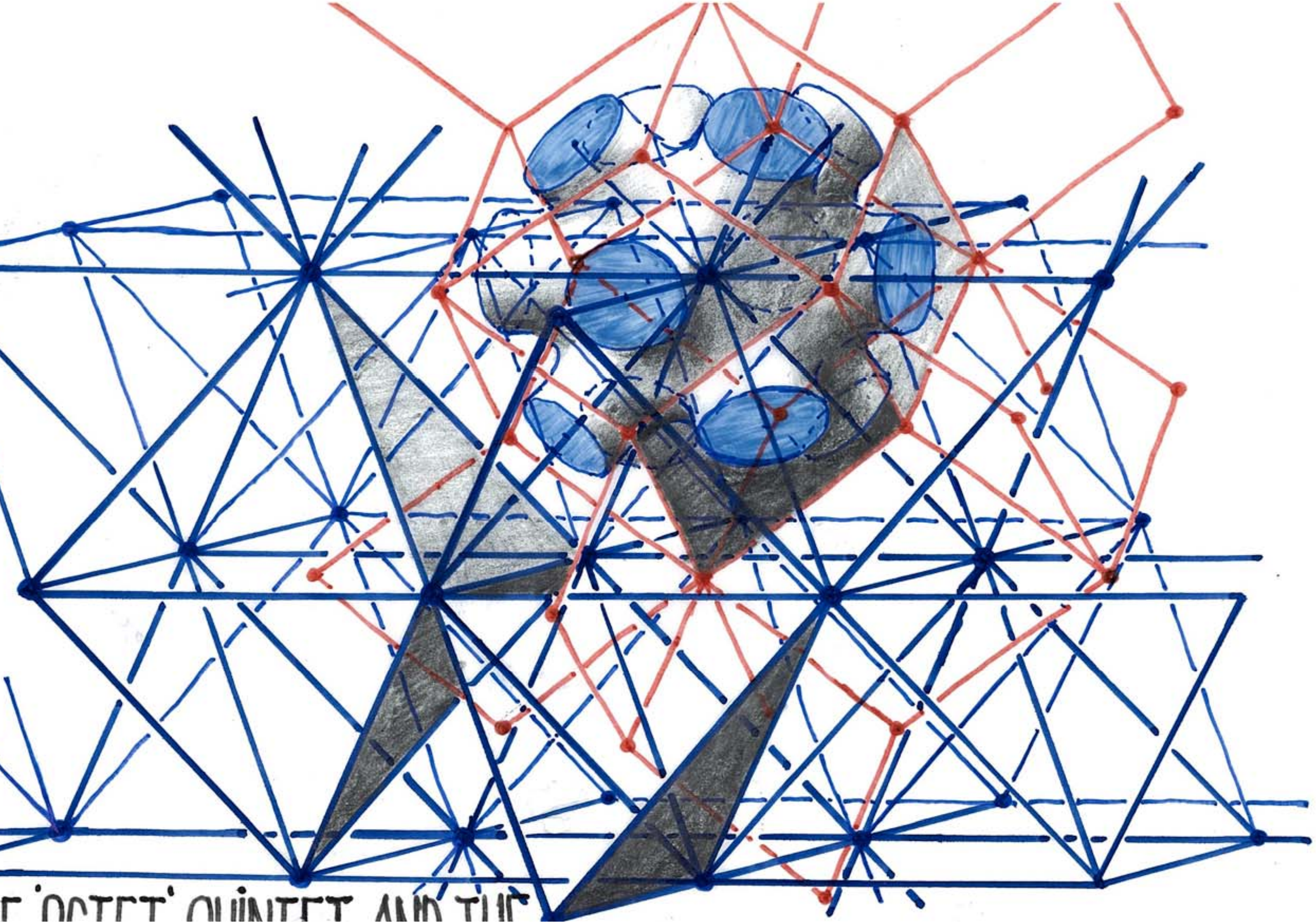


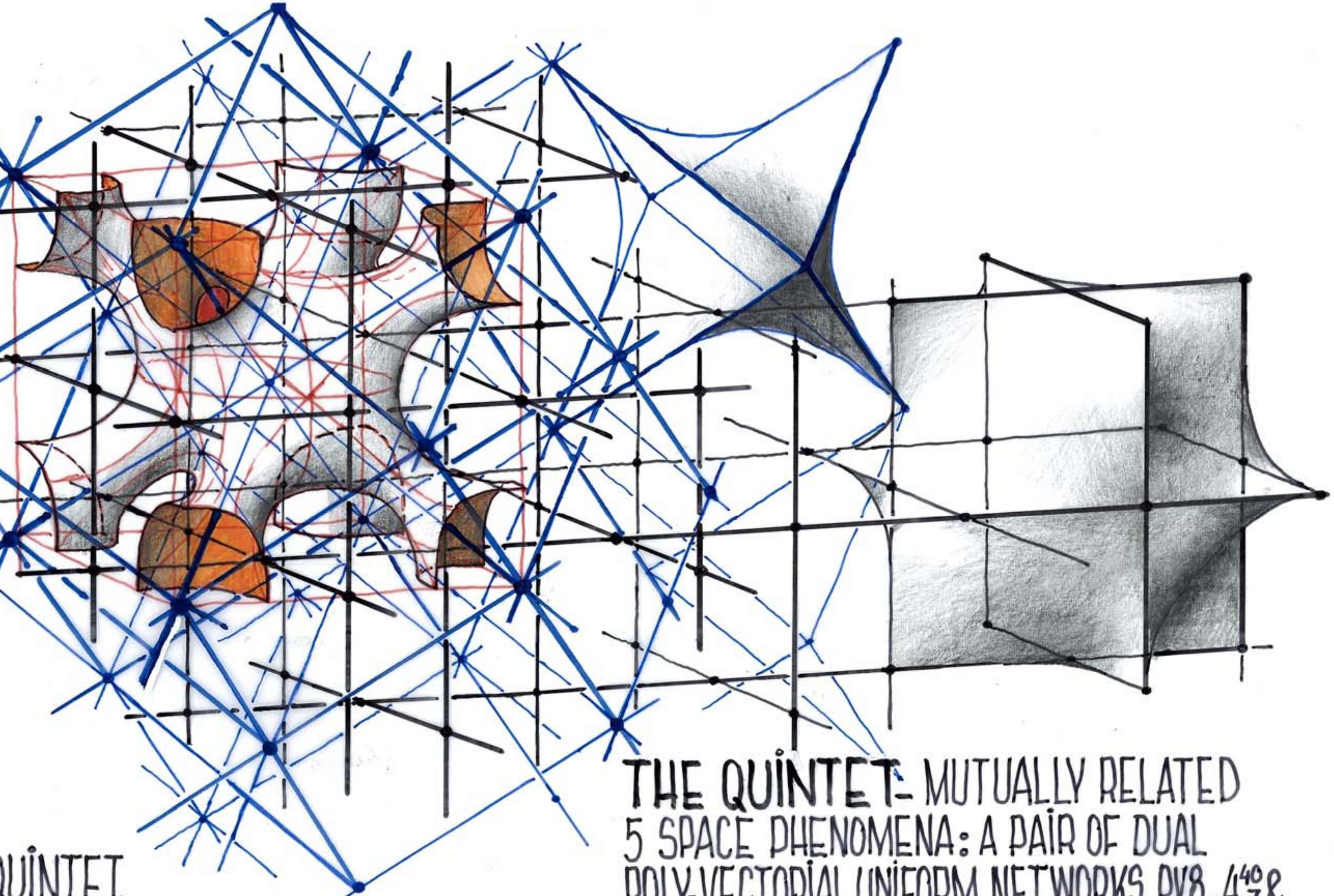
E CUBIC QUINTET, WITH THE DUAL PAIR OF CUBIC HEXAVALENT



Periodic minimal and polyhyparic surfaces of genus 3 , subdividing space between two cubic lattices into two identical subspaces.

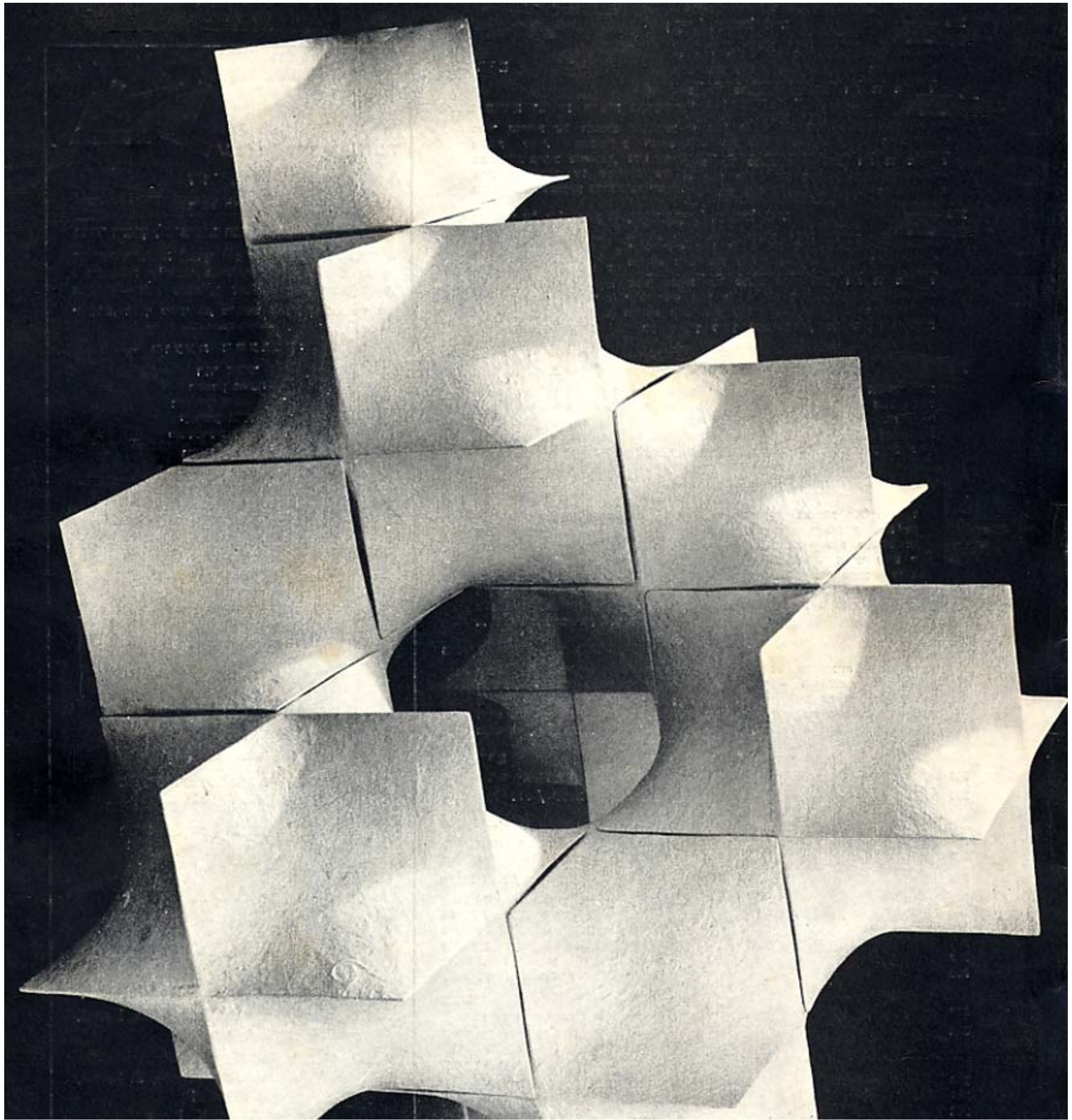
The described five features of 3D-Space, namely the **dual networks pair,** the **two associated close-packing modes (**with their respective polyhedral solids and the associated **hyperbolical sponge surface,** all together represent a **‘quintuplet assembly’** which encompasses the essence of the 3D space phenomenology.

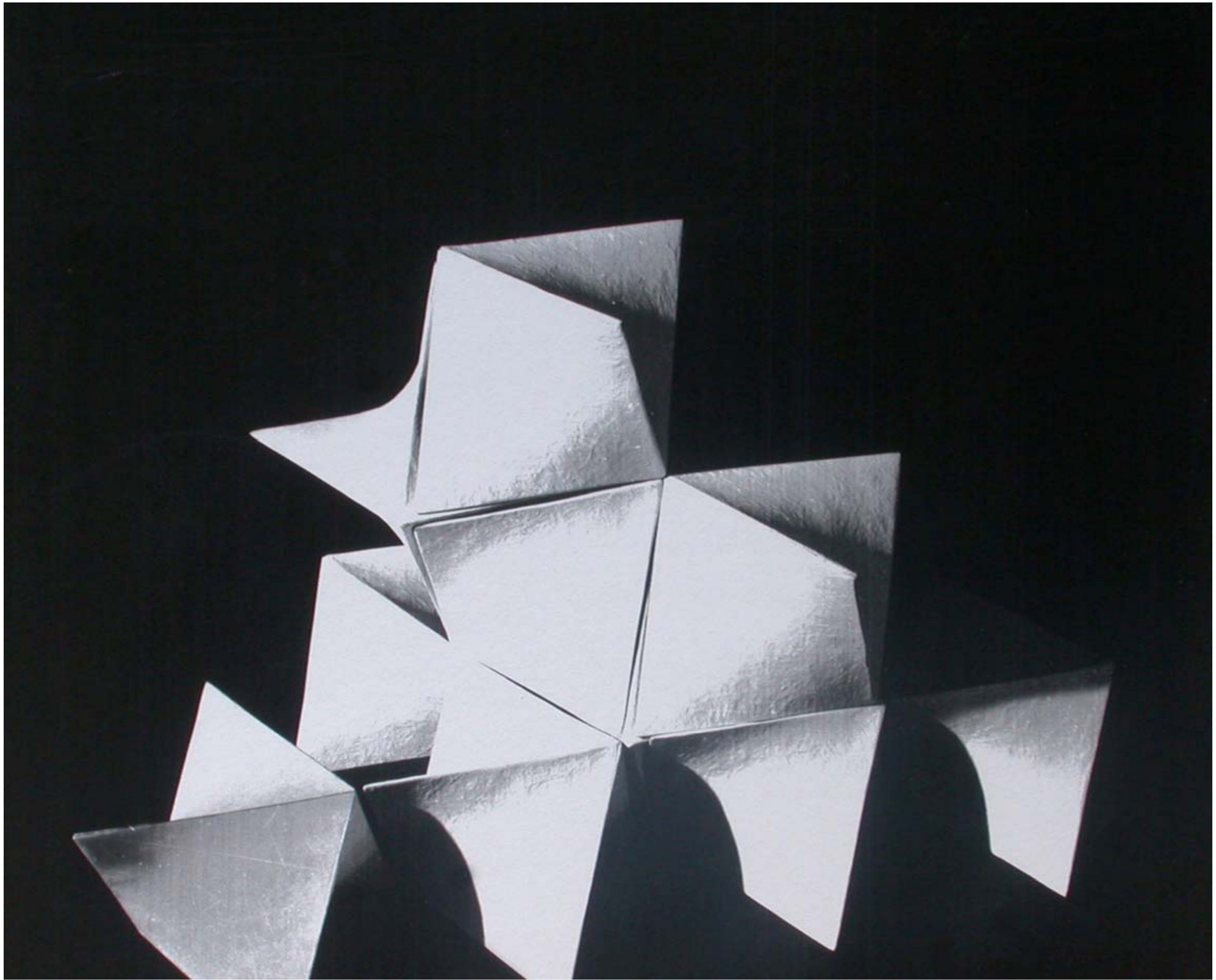

E 'OCTET' QUINTET AND THE



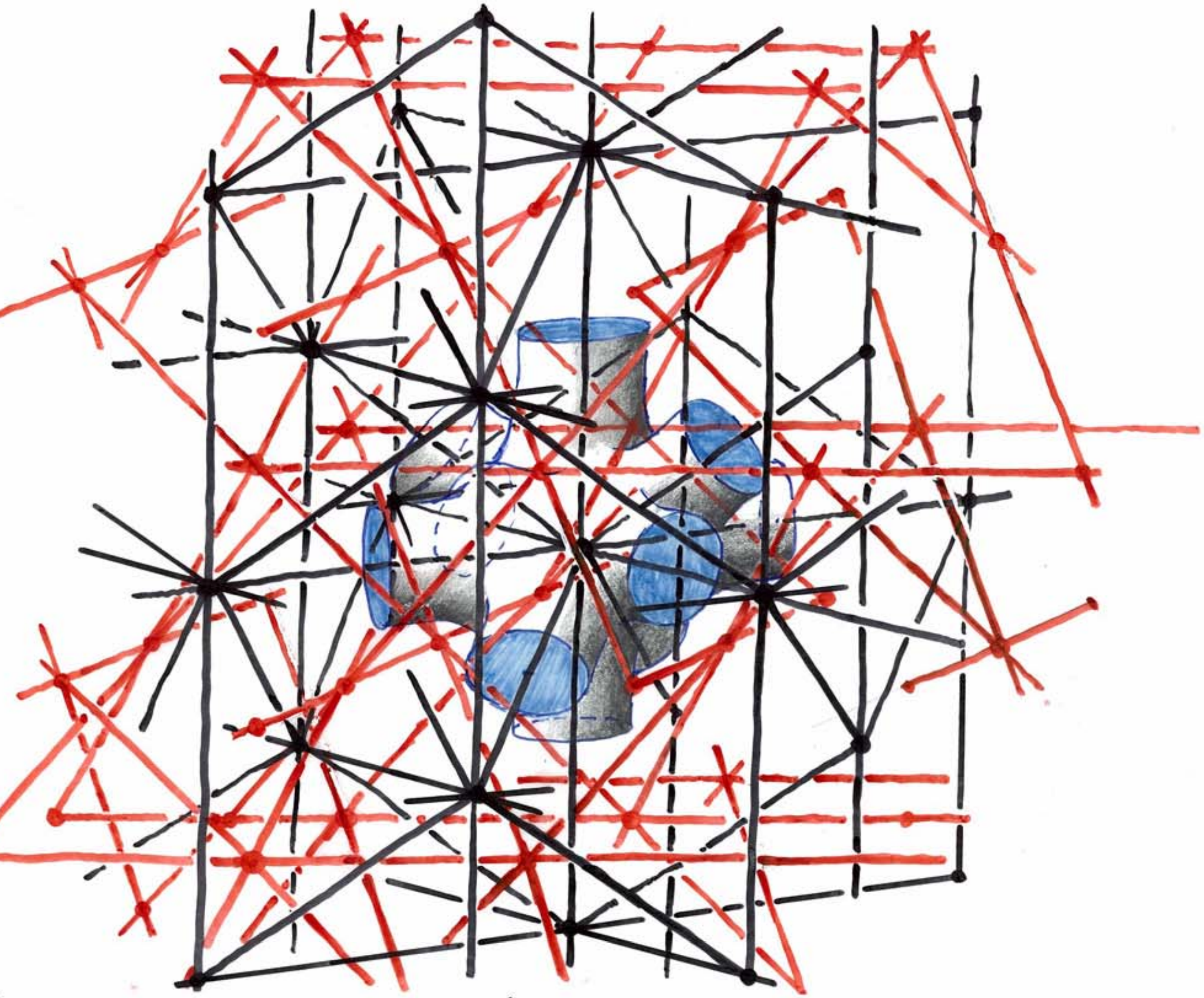
QUINTET,

THE QUINTET- MUTUALLY RELATED
5 SPACE PHENOMENA: A PAIR OF DUAL
POLY-VECTORIAL UNIFORM NETWORKS, PV.8-4$^{40}_{7}$&





Surface-partitions, plane, spherical but mostly hyperbolical, sponge-like space sub-divisions, are probably the most abundant forms in nature, on every possible scale of the physical-biological reality. Partitions define our personal, family or communal and national territorial-spatial expansion boundaries and the limits of our control, thus defining the boundaries between the **interior** and the **exterior** as predominant features of our environment.

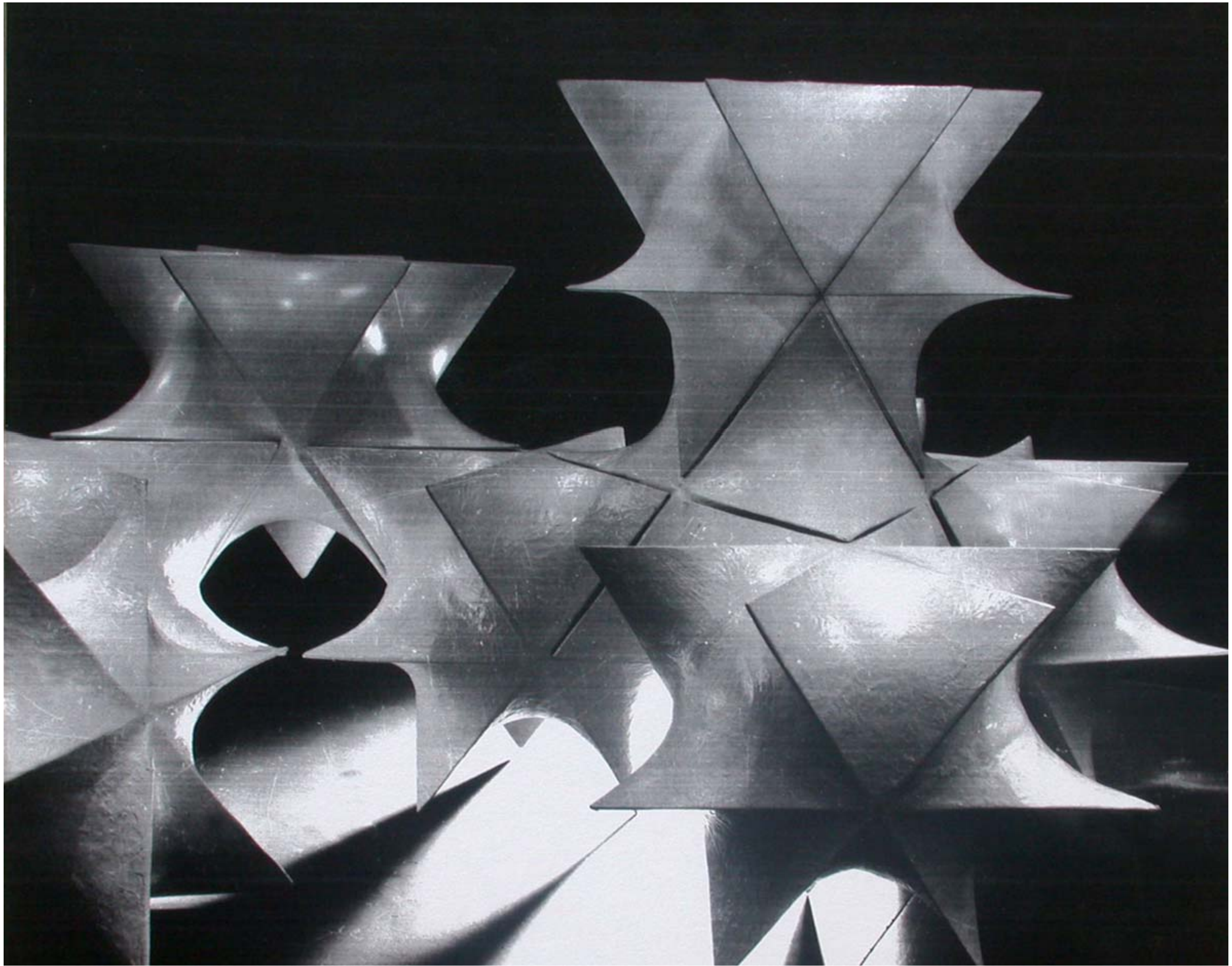


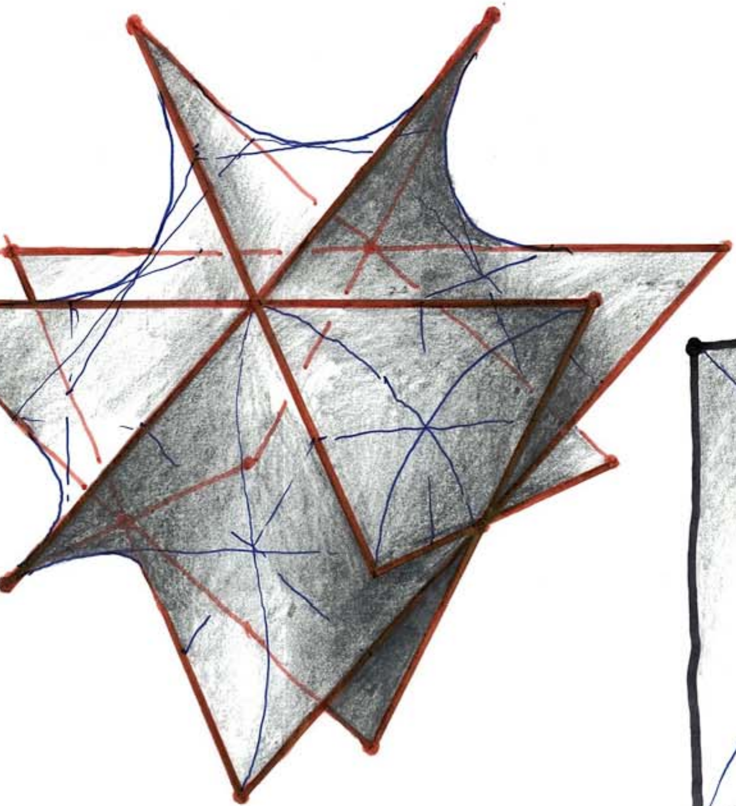
$(4^2.6)^6_{19}$

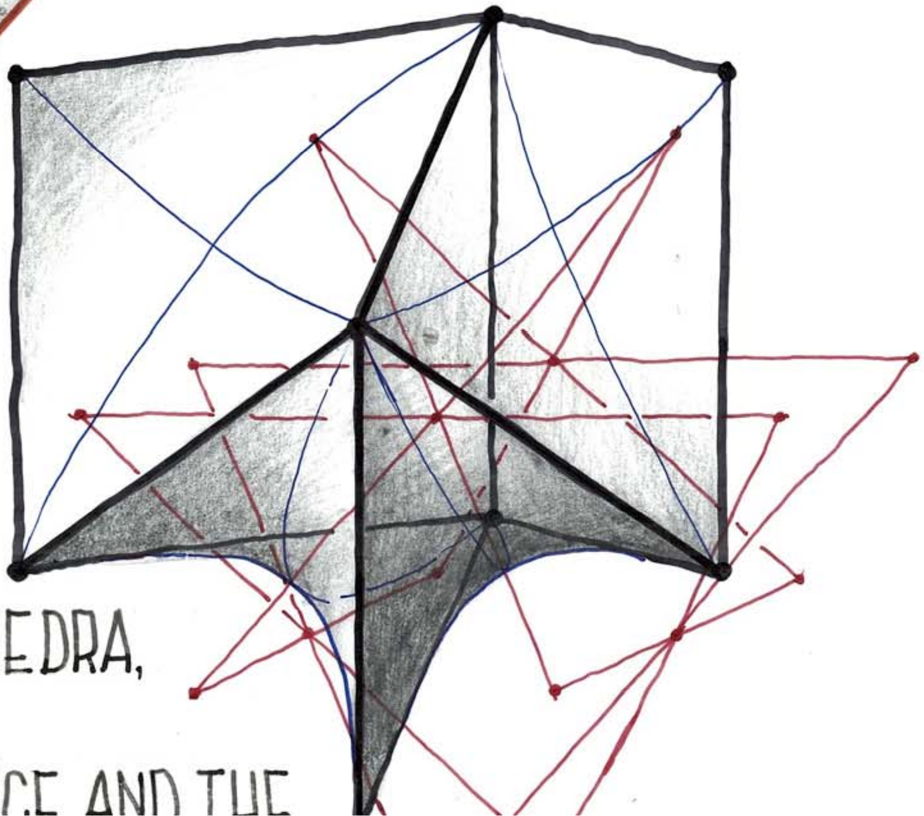
$g_{T.U.} = 19$
$\Sigma\alpha = 10\pi$
Val. = 18

$V_{T.U.} = 9$
$E_{T.U.} = 81$
$F_{T.U.} = 36$

IFORM POLY VECTORIAL HEXAVALENT PV6-$4^3 6^9_{10}$ SPACE LATTICE



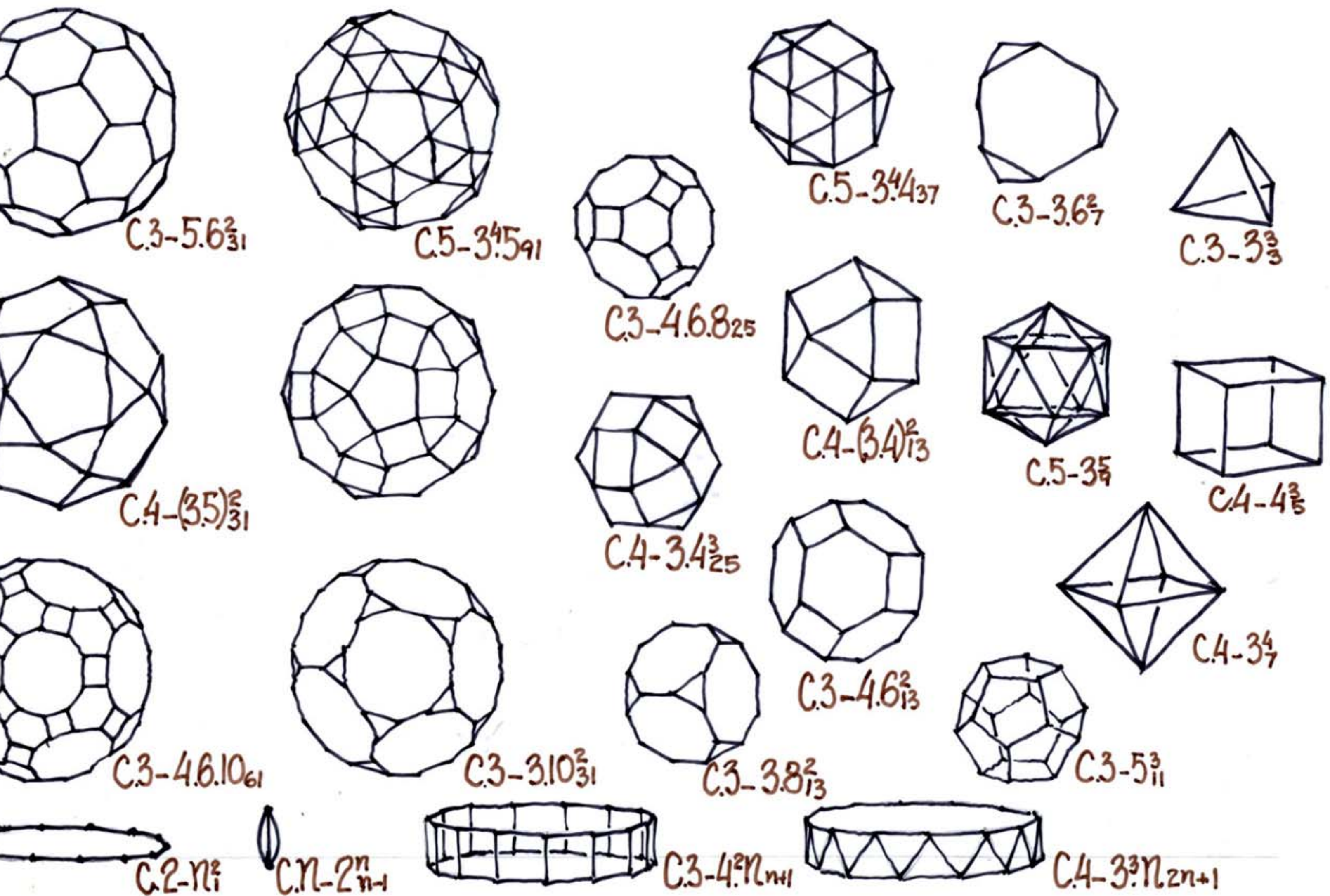




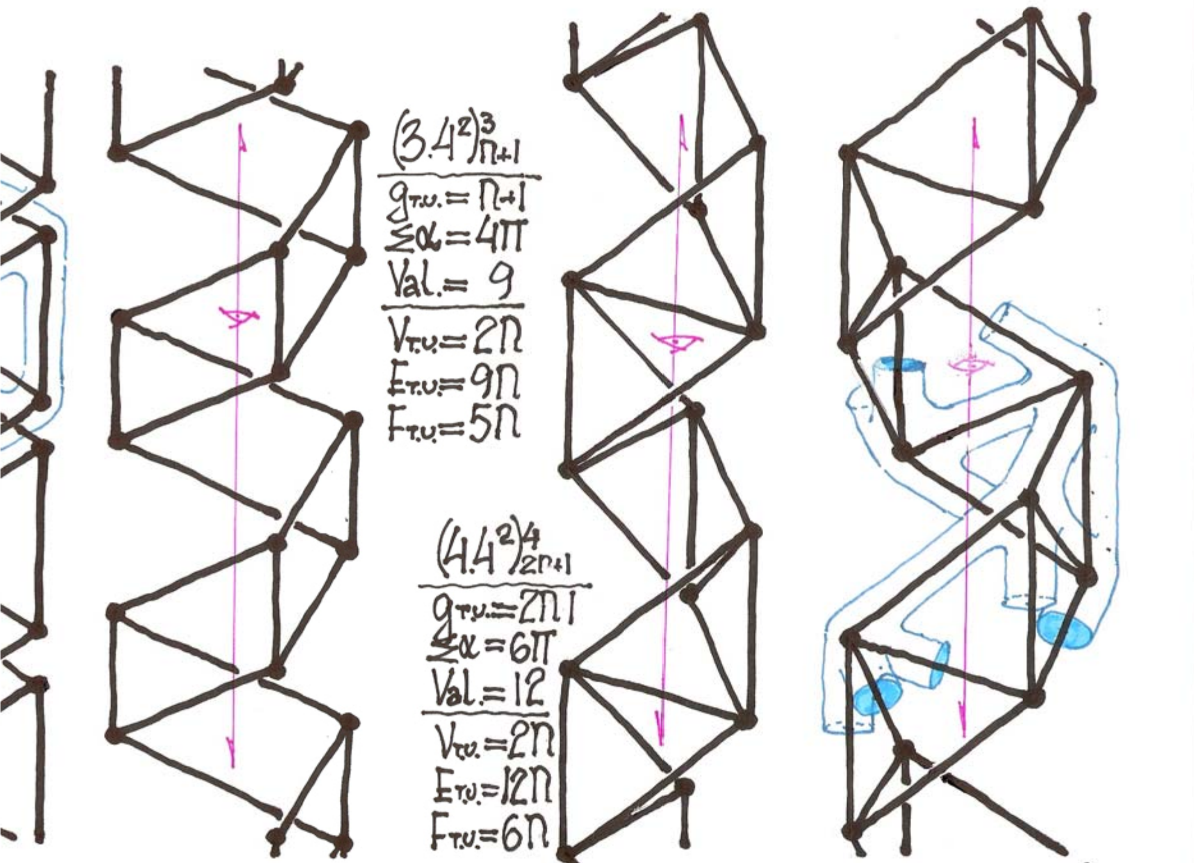
...SE PACKING SADDLE POLYHEDRA,
...AHEDRON GENERATING THE
...FORM DV6-$4^3 6^9_{10}$ SPACE LATTICE AND THE

every dual networks pair the associated sponge surface subdividing between the two and the two associated close-packing modes describe an inter-relating **quintuplet,** in which **every four components can be accurately defined and derived from the fifth.**

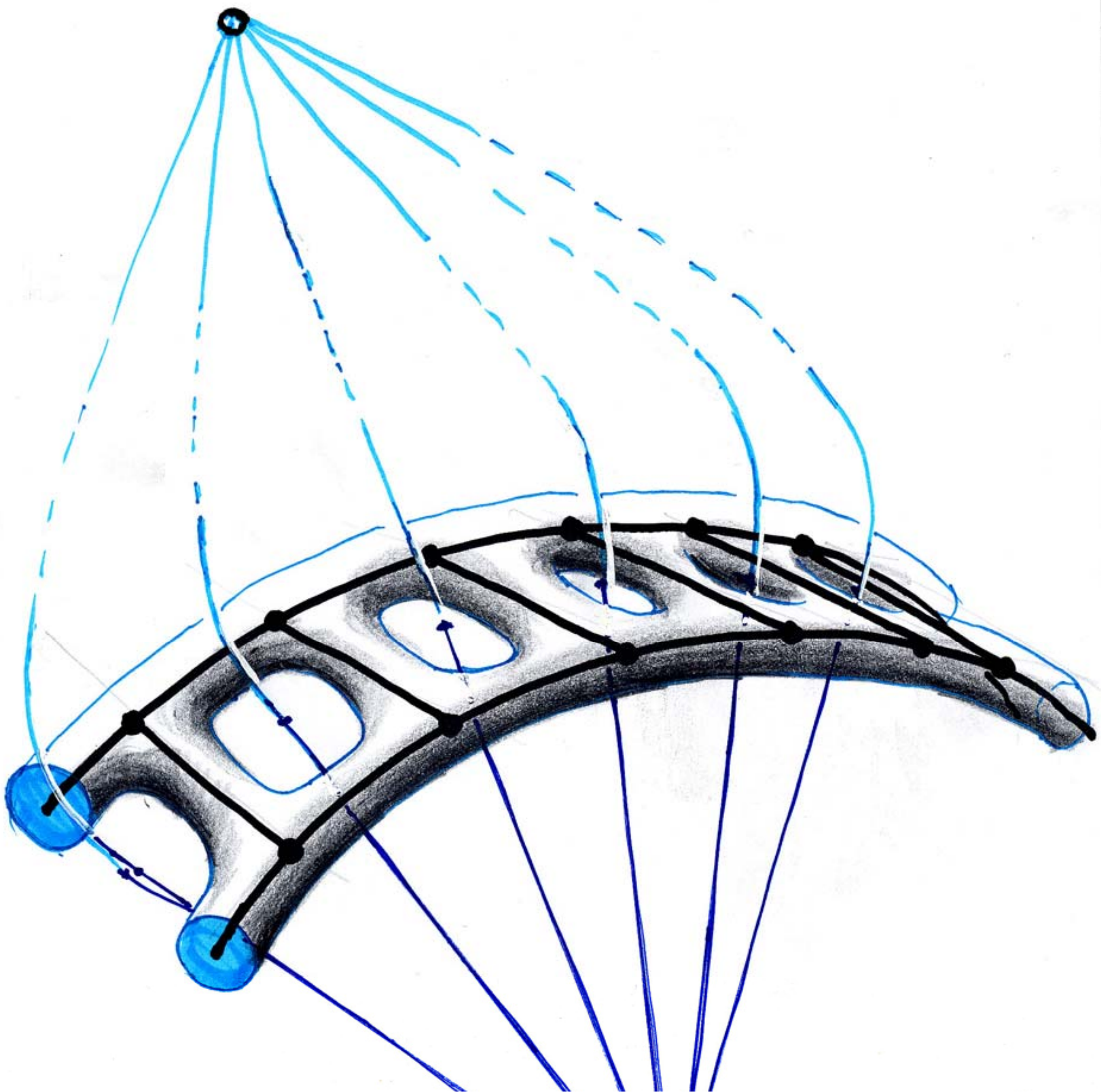
The number of topologically different space networks, sponge surface partitions and cellular space-packings amounts to infinity, even when topologically-symmetrically constrained, as periodic features.

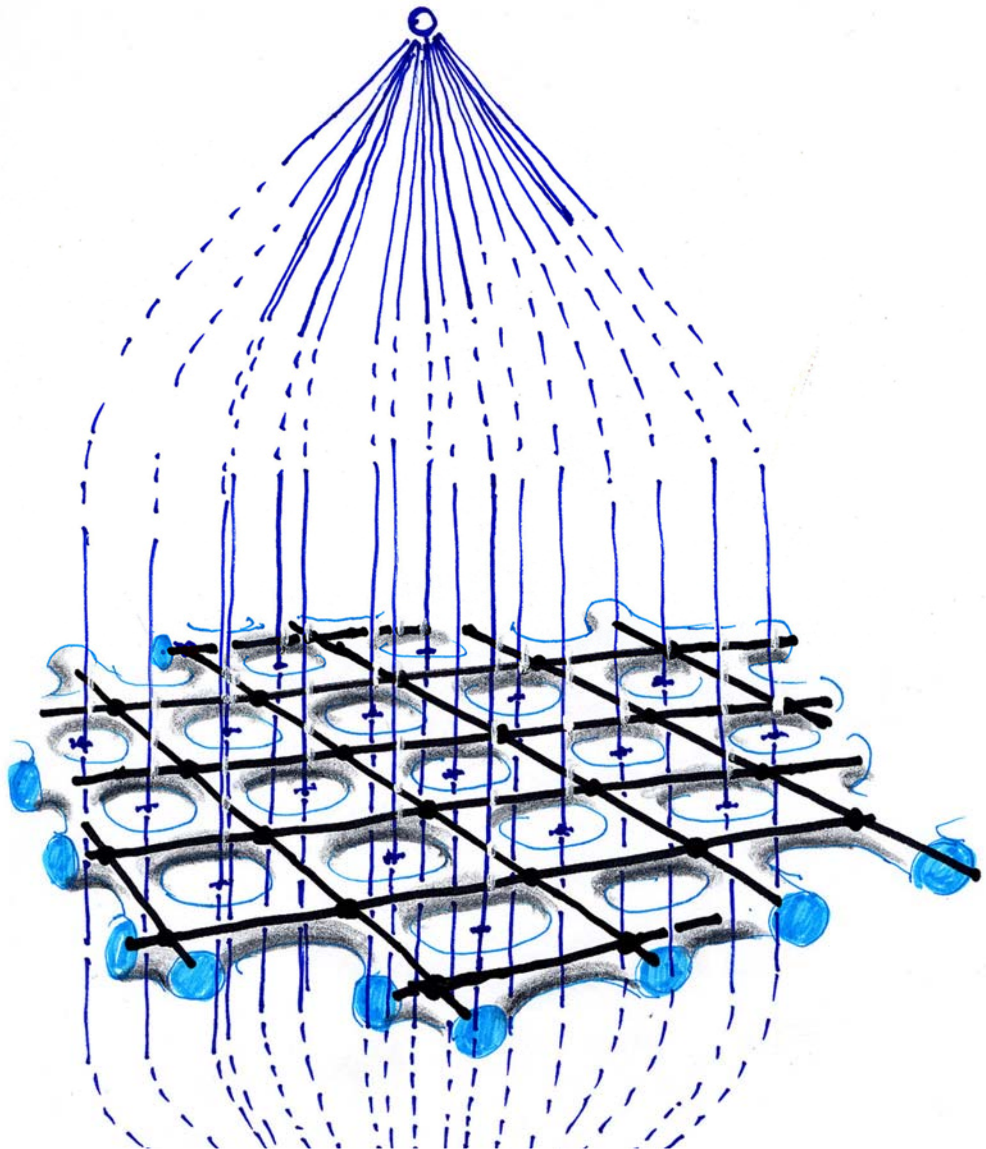


C.3-5.6^2_{31}
C.5-$3^4.5_{91}$
C.5-$3^4.4_{37}$
C.3-3.6^2_7
C.3-3^3_3
C.3-$4.6.8_{25}$
C.4-$(3.5)^2_{31}$
C.4-$(3.4)^2_{13}$
C.5-3^5_{19}
C.4-4^3_5
C.4-3.4^3_{25}
C.4-3^4_7
C.3-4.6^2_{13}
C.3-$4.6.10_{61}$
C.3-3.10^2_{31}
C.3-3.8^2_{13}
C.3-5^3_{11}
C.2-n^2_1
C.n-2^n_{n-1}
C.3-$4^2.n_{n+1}$
C.4-$3^3.n_{2n+1}$



M AXIAL TRIVALENT-($A3-4^2_{n+1}$) AND TETRAVALENT ($A4-3^3_{2n+1}$) HELICOIDAL
TTICES AND RELATED SPONGE SURFACES AND UNIFORM SPONGE POLYHEDRA.





$n=2$ $\left(4.^{2}5\right)^{5}_{19}$

$g = 19 = 9n+1$

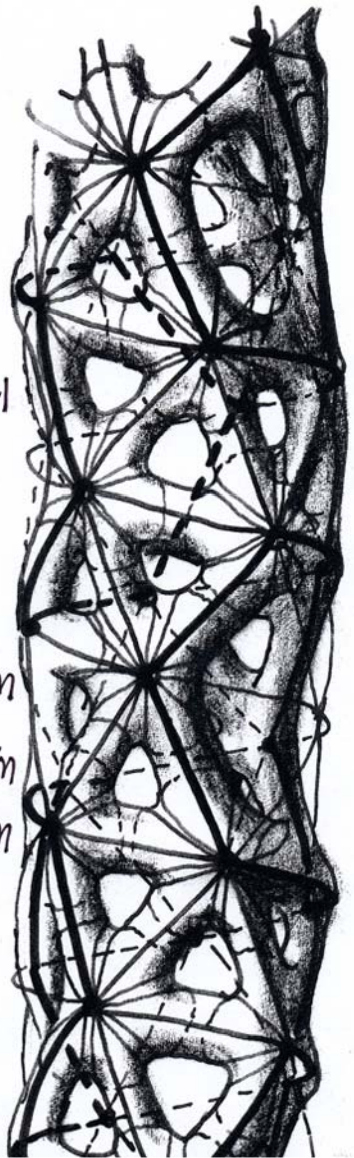
$\Sigma\alpha_{av.} = 8\pi$

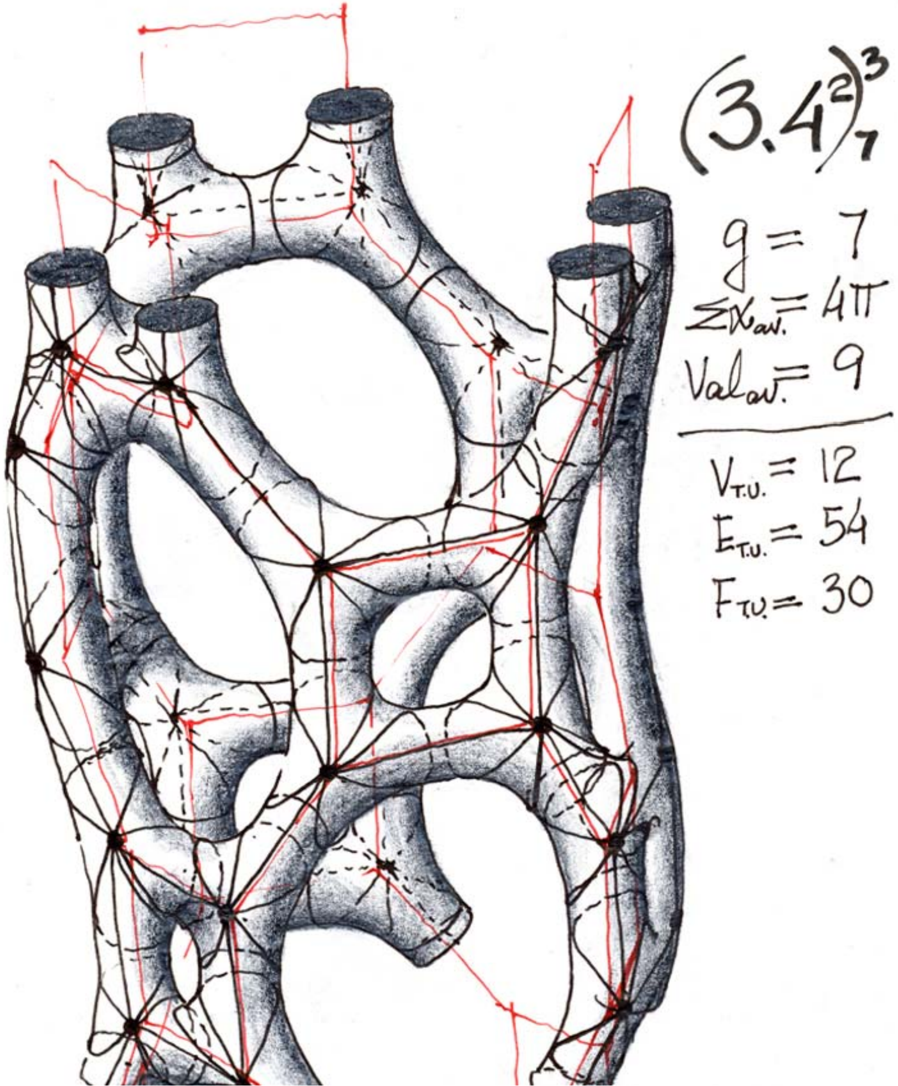
$Val_{av.} = 15$

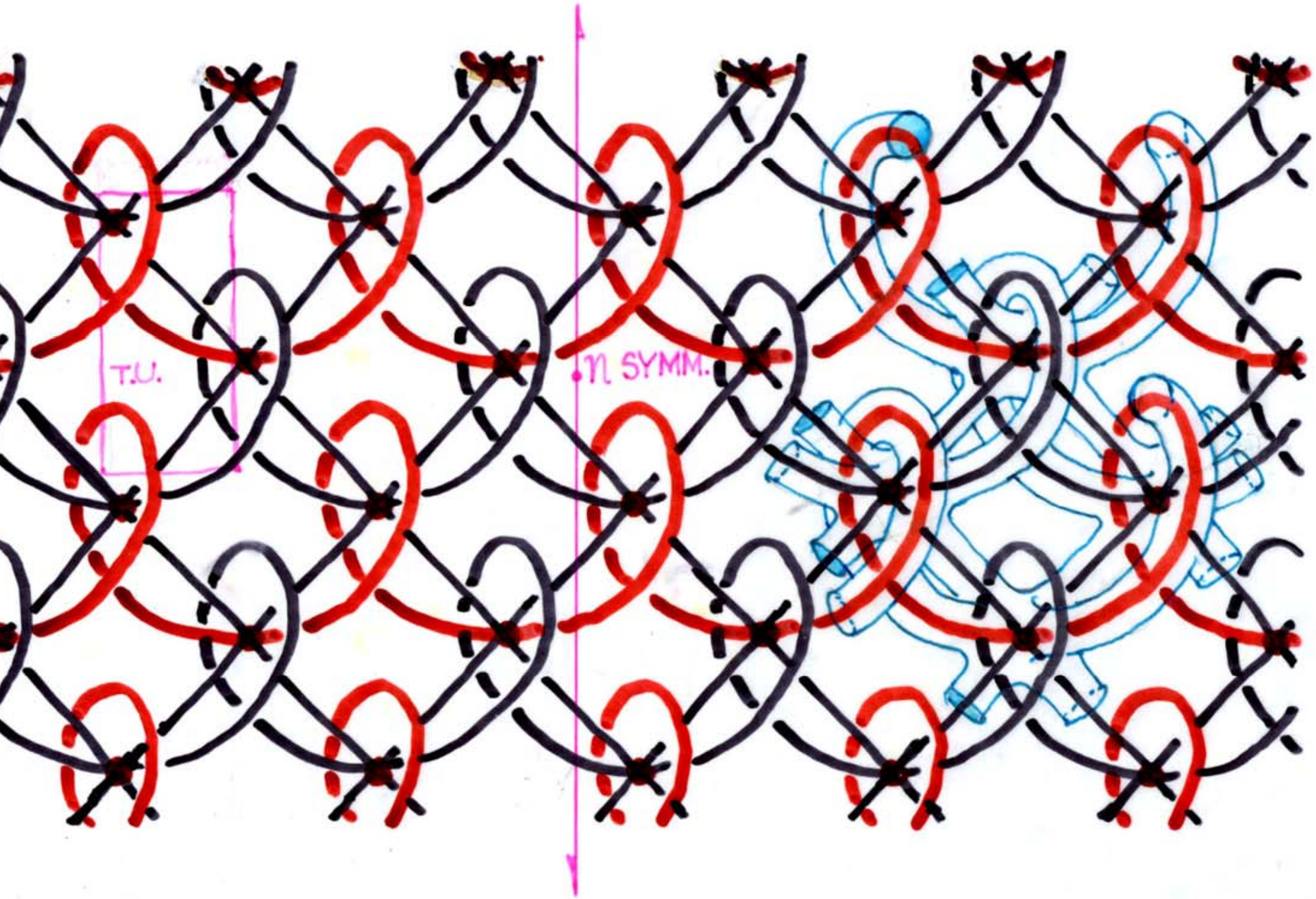
$V_{T.U.} = 6 \times 2 = 6n$

$E_{T.U.} = 45 \times 2 = 45n$

$F_{T.U.} = 21 \times 2 = 21n$




$(3.4^2)^3_7$
$g = 7$
$\Sigma K_{av.} = 4\pi$
$Val_{av.} = 9$
$V_{T.U.} = 12$
$E_{T.U.} = 54$
$F_{T.U.} = 30$



$(4^2 6)_{2n+1}$

$g_{T.U.} = 2n+1$

$\Sigma\alpha = 10\pi$

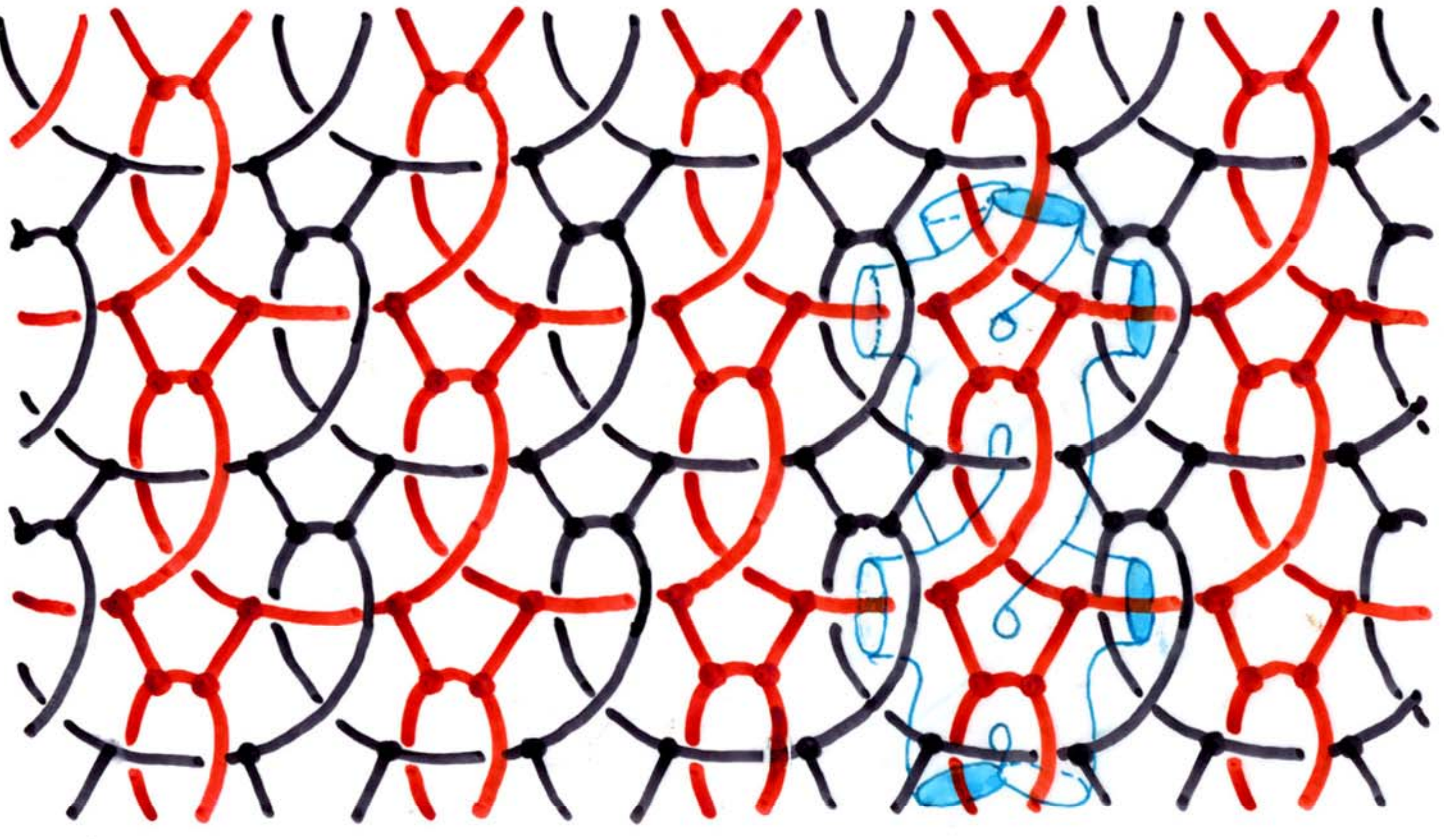
Val. = 18

$V_{T.U.} = n$

$E_{T.U.} = 9n$

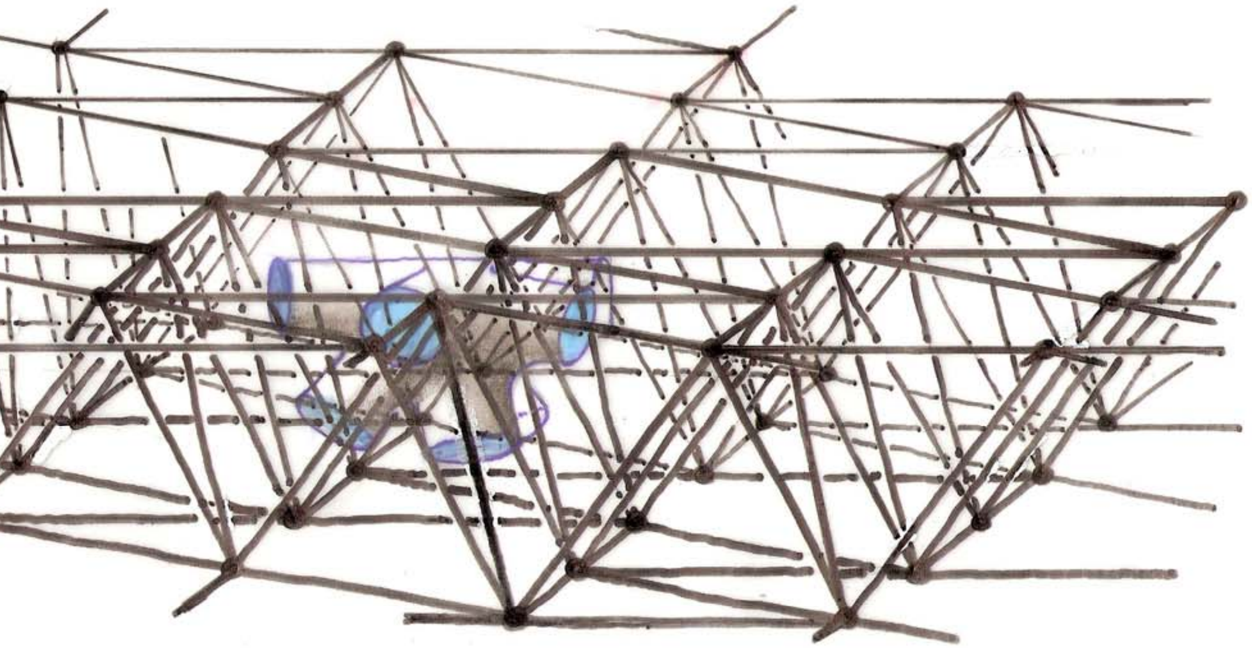
$F_{T.U.} = 4n$

ERIODIC (EQUIVERTEX) AXIAL HEXAVALENT A6-$4^4 3^6_{2n+1}$ SPACE



TWO IDENTICAL MUTUALLY ENTANGLED 3-D NETWORKS

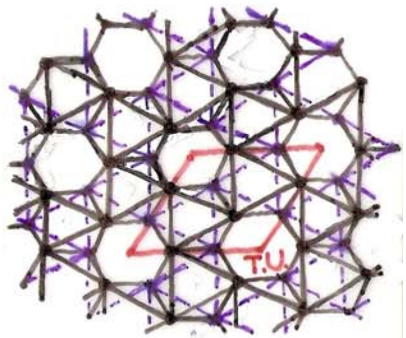
.11.10



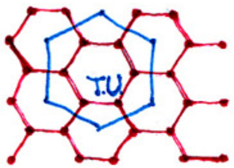
$(4^2 7)^7_{16}$

$g_{T.U.} = 16$
$\Sigma \alpha = 12\pi$
$Val. = 21$

$V_{T.U.} = 6$
$E_{T.U.} = 63$
$F_{T.U.} = 27$



...NIFORM DOUBLE-LAYER HEPTAVALENT DLZ-$3^4 4^3 6^3_{16}$ SPACE LATTICE



$(4.4^2)^4_{13}$

$g_{T.U.} = 13$

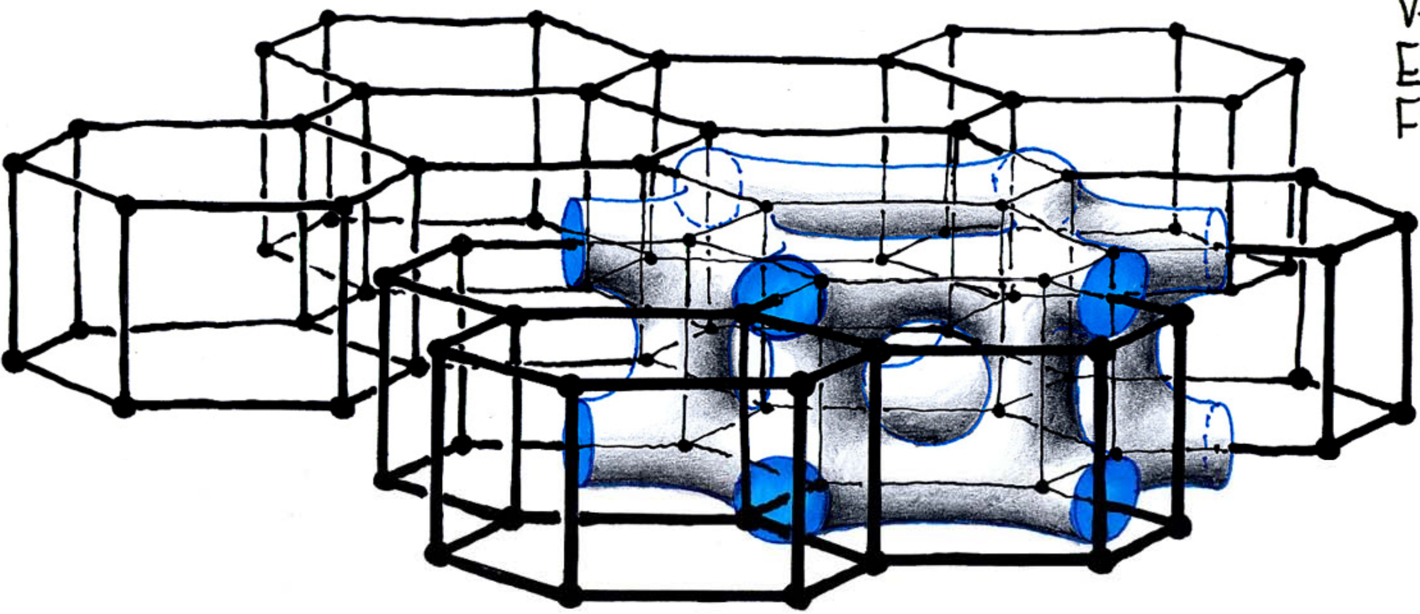
$\Sigma\alpha = 6\pi$

$Val. = 12$

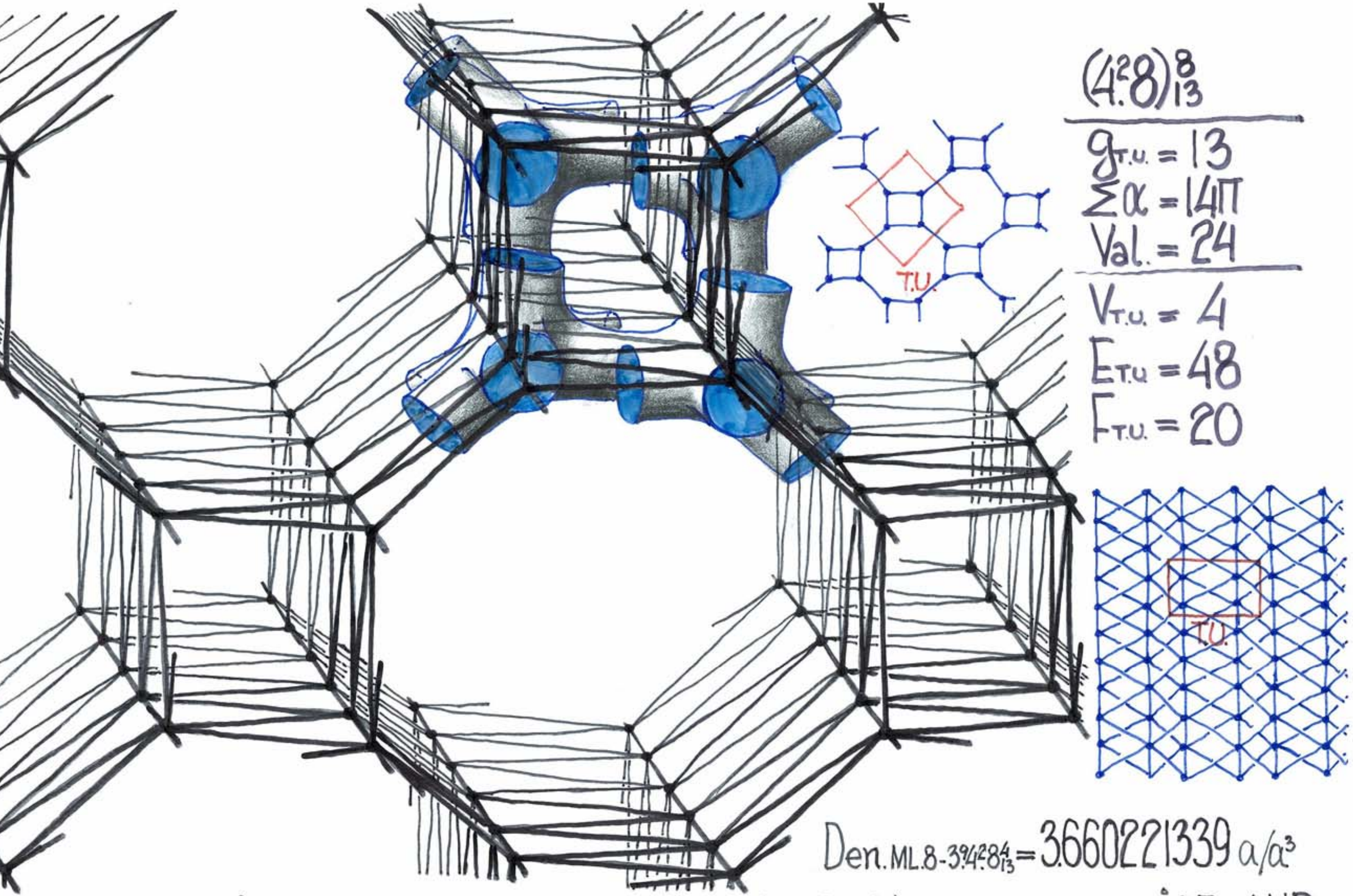
$V_{T.U.} = 12$

$E_{T.U.} = 72$

$F_{T.U.} = 36$



IFORM DOUBLE LAYER TETRAVALENT P4 4363 SPACE LATTICE



$(4^2.8)^{8}_{13}$

$g_{T.U.} = 13$

$\Sigma\alpha = 14\pi$

$Val. = 24$

$V_{T.U.} = 4$

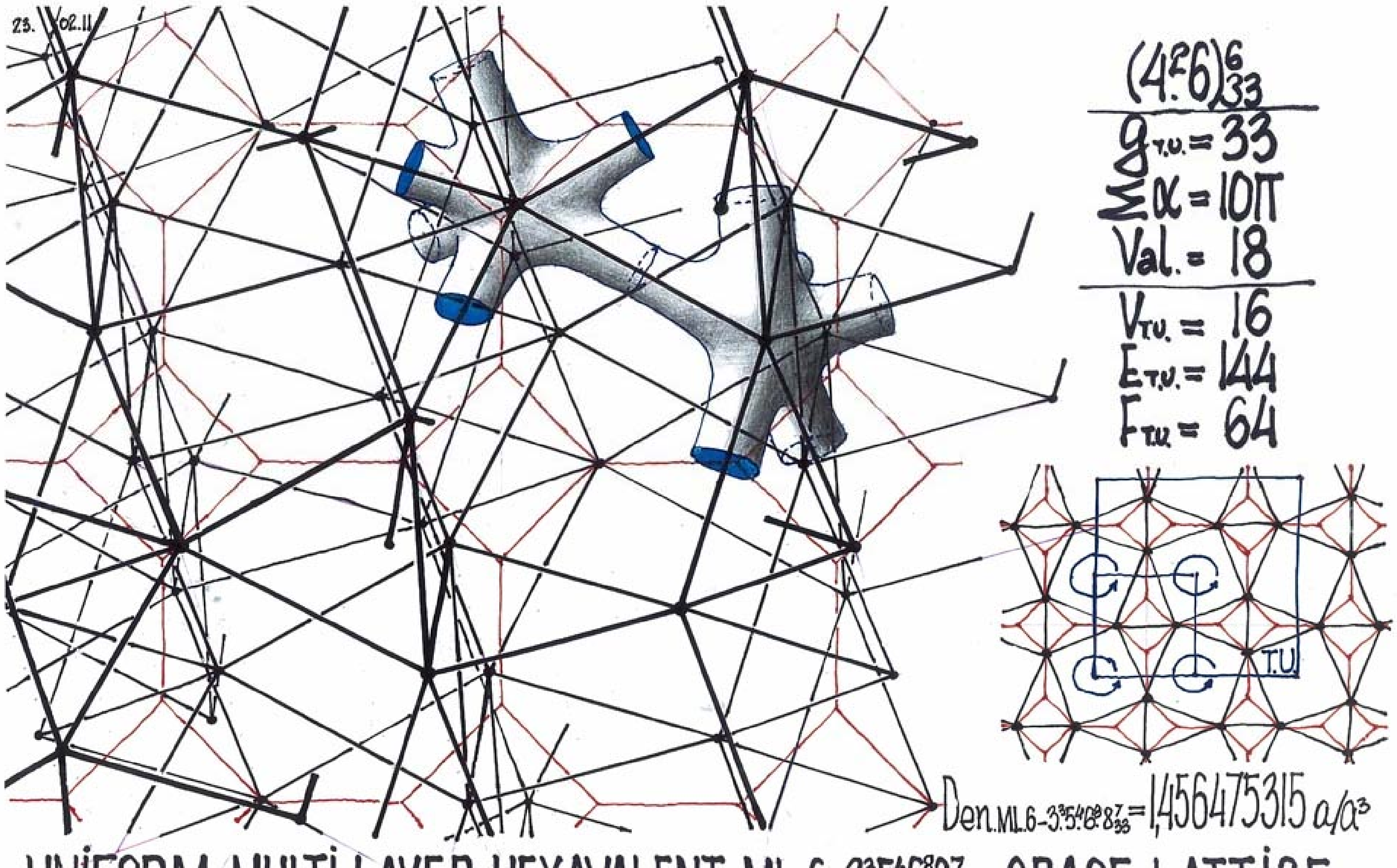
$E_{T.U} = 48$

$F_{T.U.} = 20$

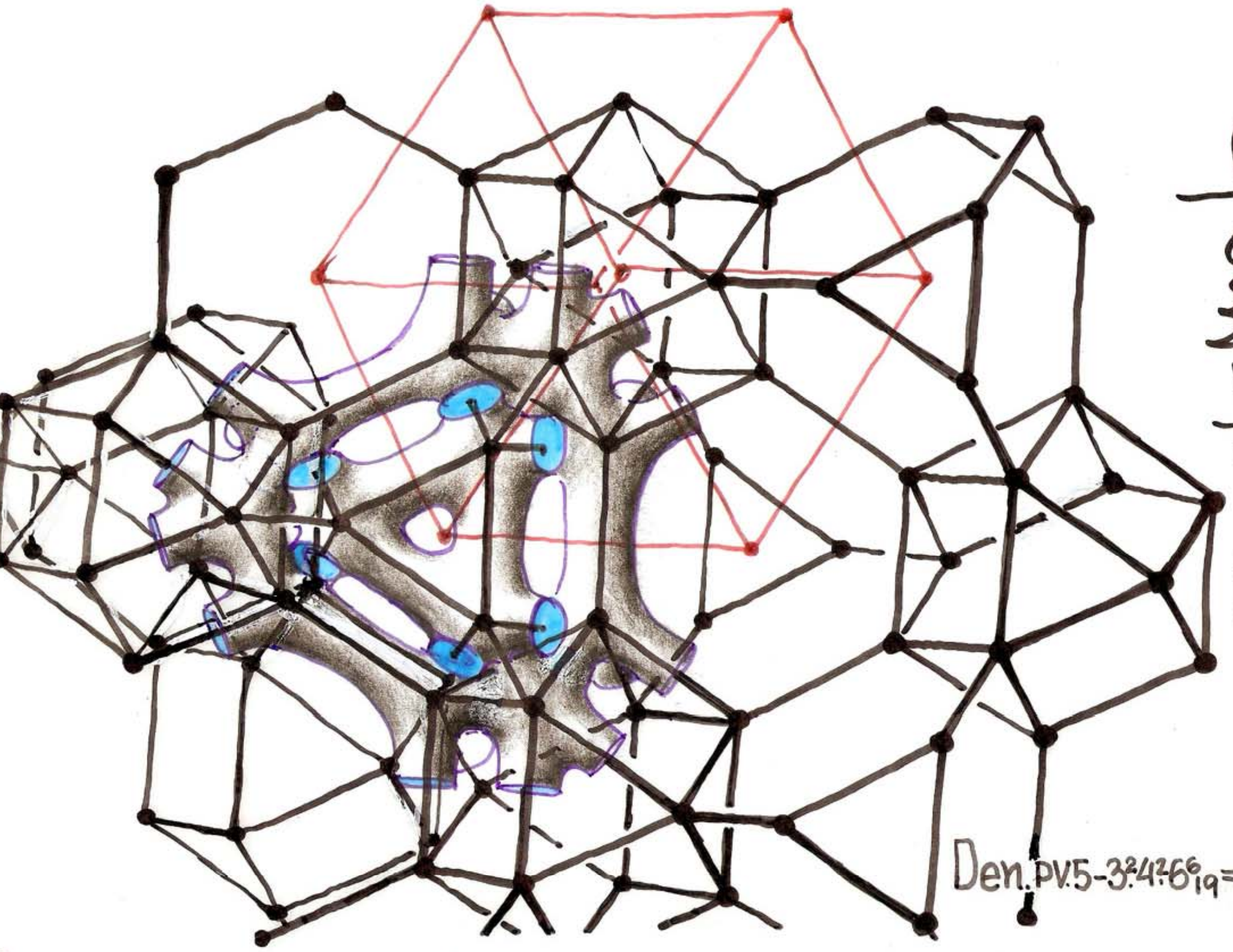
$Den._{ML.8-3^9.4^2.8^4_{13}} = 3.660221339\ a/a^3$

...IFORM MULTİ-LAYER OCTAVALENT ML.8-3⁹4²8⁴ SPACE LATTİCE AND

If the ‘quintuplet is periodic-symmetrical in nature, all the five associated components share in the same symmetry regime (adhering to same symmetry group).



UNIFORM MULTI-LAYER HEXAVALENT $ML.6-3^3.5^4.6^8.8^7_{33}$ SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON



$$(4^2.5)^5_{19}$$

$$g_{T.U.} = 19$$

$$\sum \alpha = 8\pi$$

$$Val. = 15$$

$$V_{T.U.} = 12$$

$$E_{T.U.} = 90$$

$$F_{T.U.} = 42$$

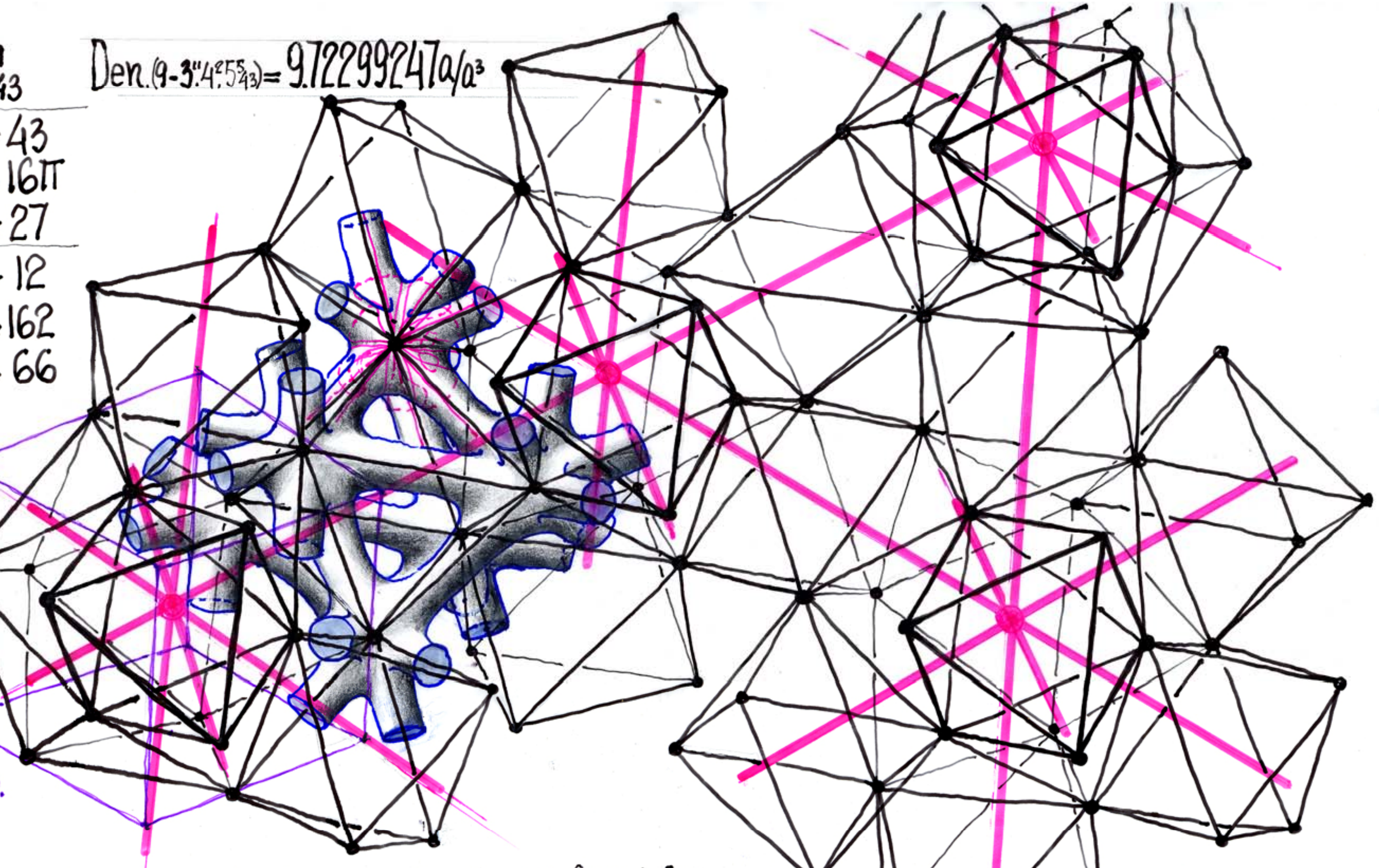
Den. PV.5 - $3^2.4^2.6^6_{19}$ = 2.121320341 a/a

İFORM POLYVECTORİAL PENTAVALENT PV5 $3^2 4^2 6^6$ SPACE LATTİCE

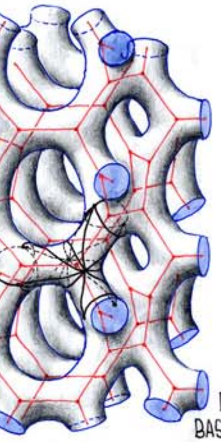
Den.$(9\text{-}3^{11}4^{2}5^{5}_{43}) = 9.72299247\,a/a^{3}$

43
16π
27

12
162
66



RM 9-VALENT $9\text{-}3^{11}4^{2}5^{5}_{43}$ SPACE-LATTICE (WITH LOCAL



$(3^3)^3_4$

$g = 4$

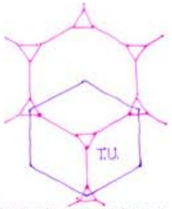
$\Sigma\alpha = 3\pi$

$Val. = 9$

$V_{T.U.} = 12$

$E_{T.U.} = 54$

$F_{T.U.} = 36$



UNIFORM SPONGE POLYHEDRON

PERIODIC SPONGE SURFACE ON THE BASIS OF A UNIFORM TRIVALENT LATTICE

$3^3.4^4_7$

$g = 7$

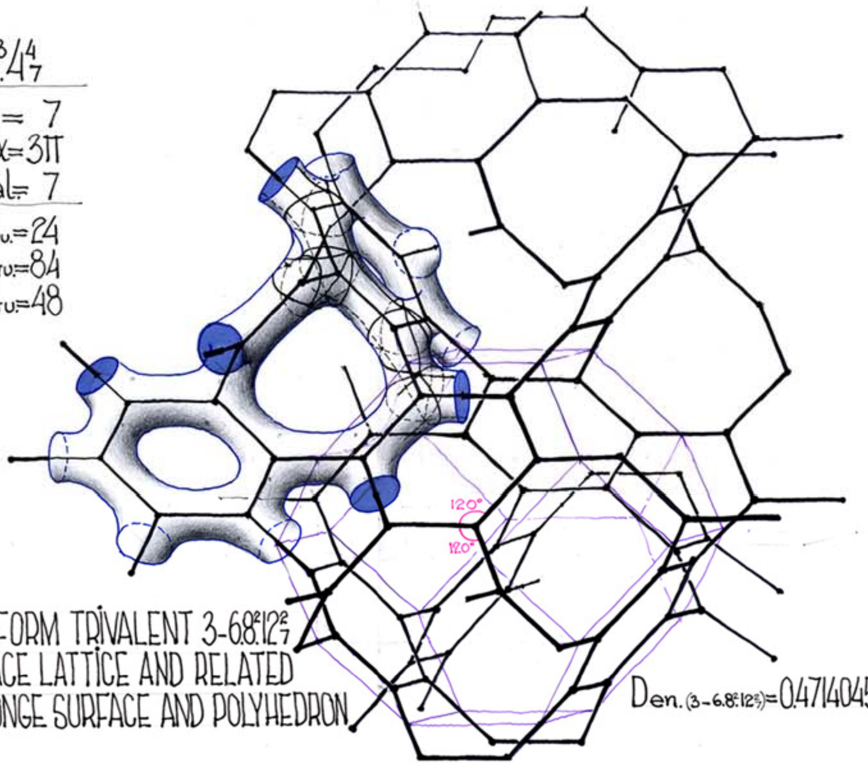
$\Sigma\alpha = 3\pi$

$Val. = 7$

$V_{T.U.} = 24$

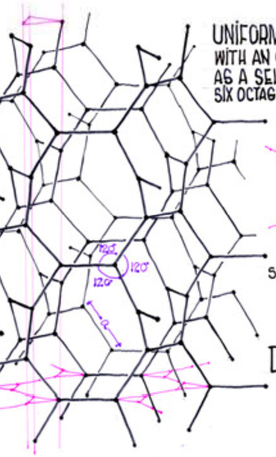
$E_{T.U.} = 84$

$F_{T.U.} = 48$

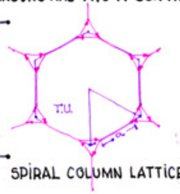


UNIFORM TRIVALENT $3\text{-}6.8^2.12^2_7$ SPACE LATTICE AND RELATED SPONGE SURFACE AND POLYHEDRON

$Den._{(3-6.8^2.12^2_7)} = 0.47140452\ a/a^3$

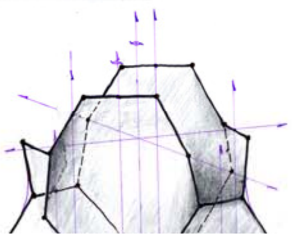


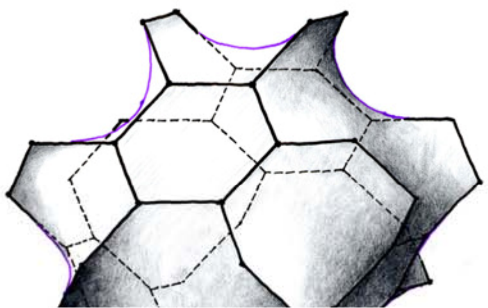
UNIFORM TRIVALENT SPACE LATTICE WITH AN OCTAHEDRAL SADDLE POLYHEDRON AS A SELF CLOSE PACKING SOLID, WITH SIX OCTAGONS AND TWO 14-GON FACES.

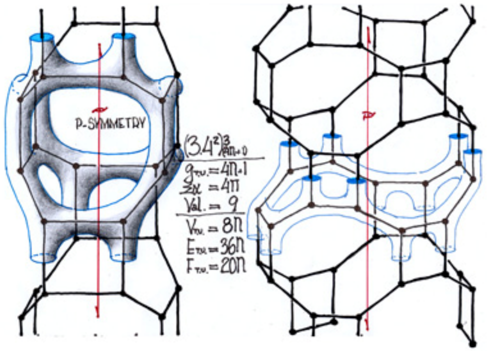


SPIRAL COLUMN LATTICE

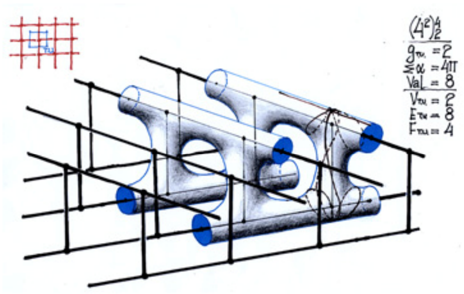
$Den._{8^6.14^2} = 0.254558441\ a/a^3$



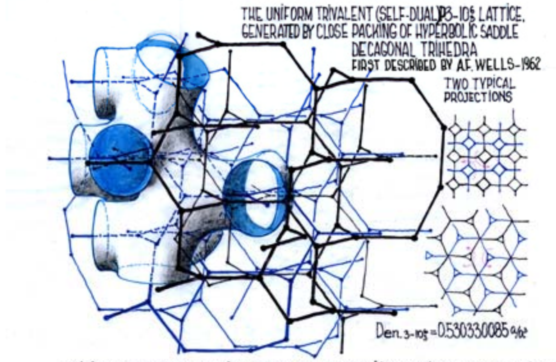




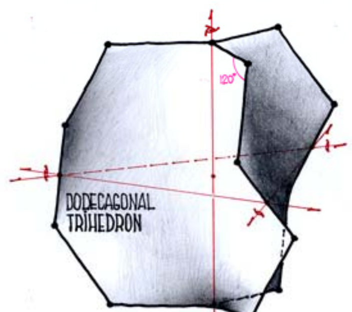
A UNIFORM AXIAL TRIVALENT A3-4.8²4n SPACE LATTICE AND RELATED SPONGE SURFACES AND UNIFORM SPONGE POLYHEDRA.



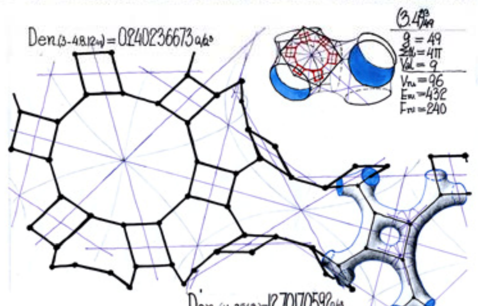
B UNIFORM, DOUBLE-LAYER, TRIVALENT SPACE-LATTICE AND THE RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.



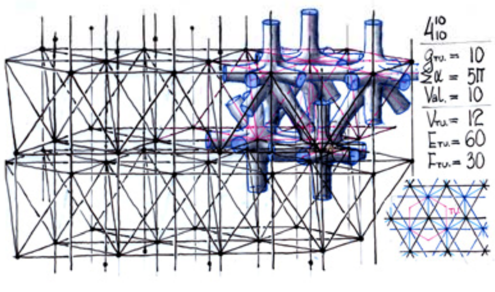
C TRINITY- THE MUTUALLY RECIPROCAL TWO DUAL (IDENTICAL) UNIFORM SPACE LATTICES AND THE HYPERBOLIC SPONGE SURFACE, SUBDIVIDING THE SPACE BETWEEN THE TWO.



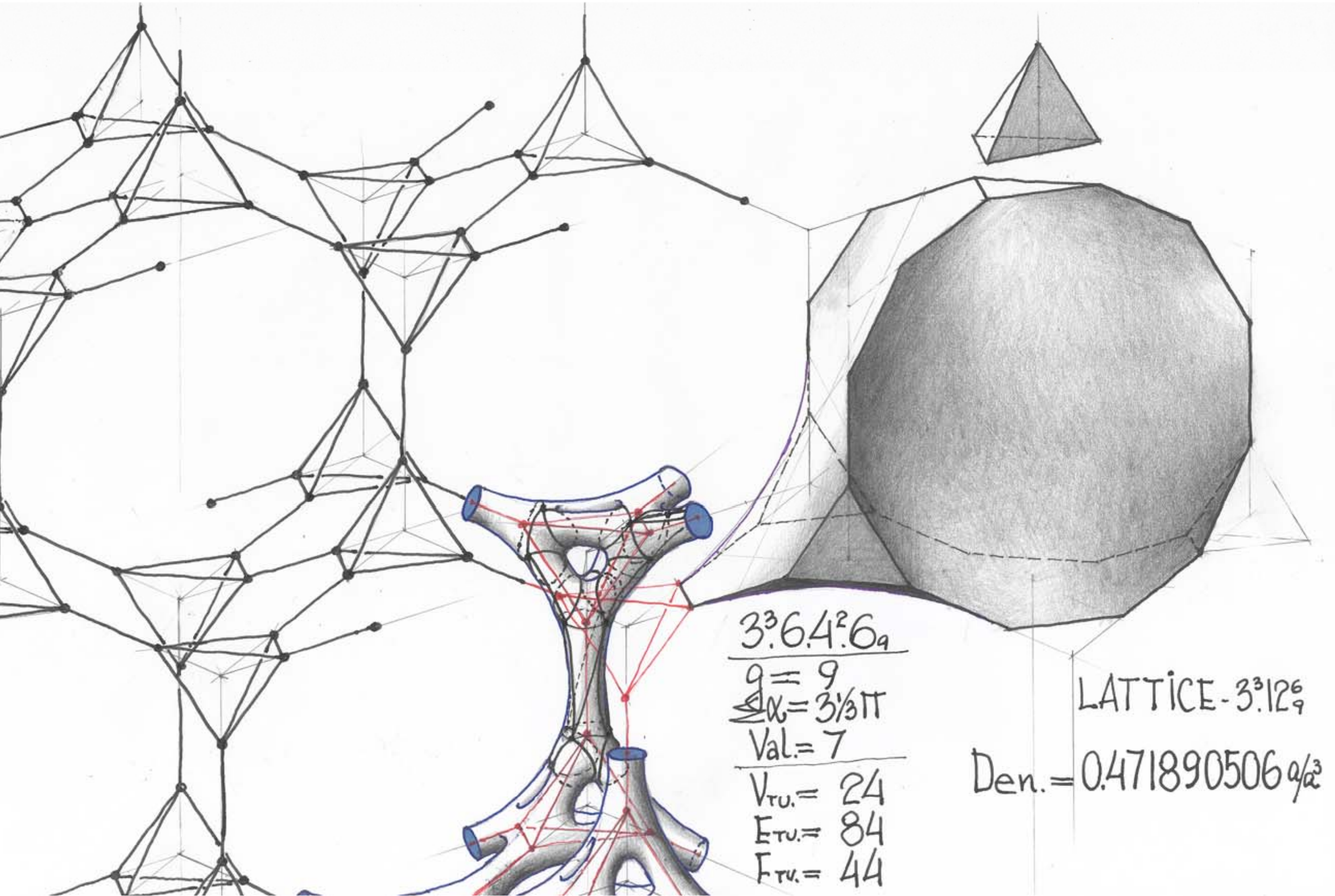
D SELF CLOSE PACKING SADDLE POLYHERON GENERATING UNIFORM TRIVALENT POLYVECTORIAL P3-10³ SPACE-LATTICE.



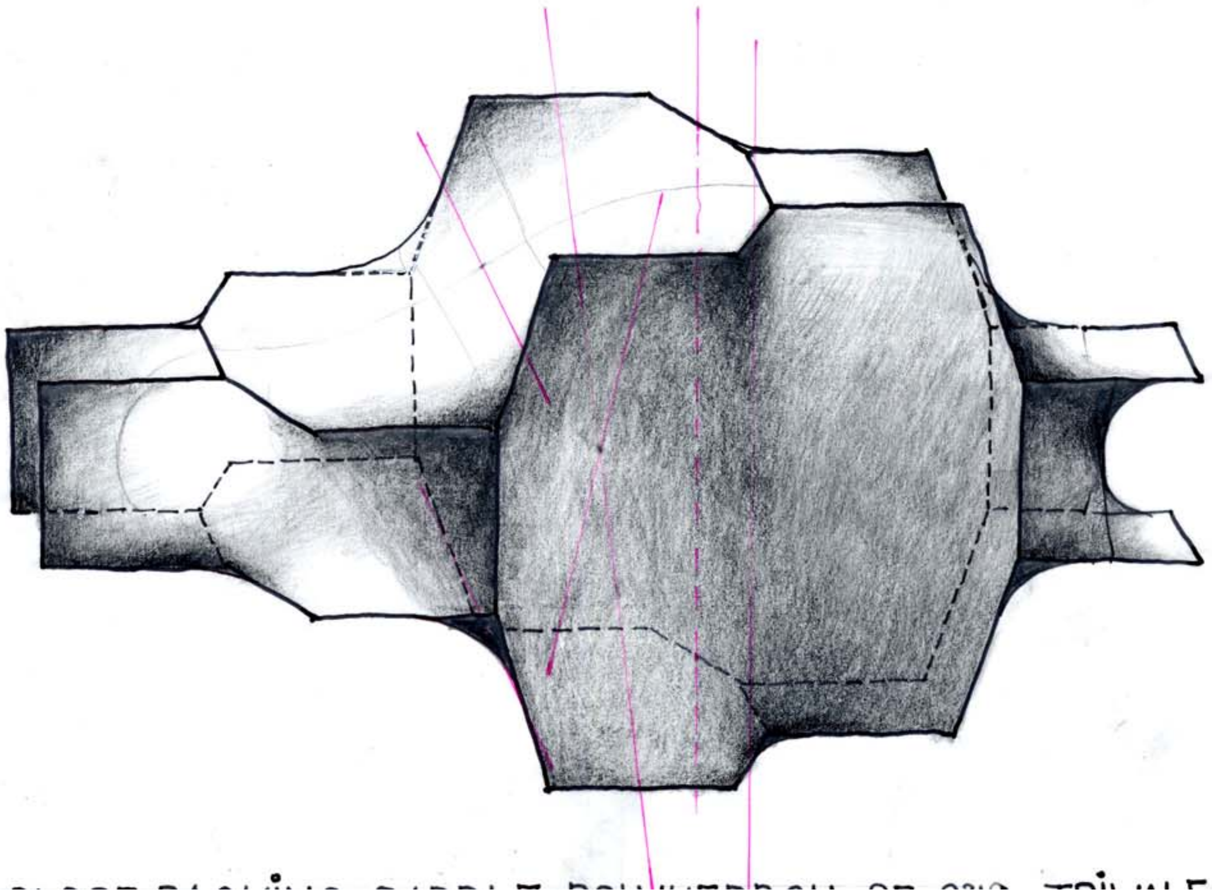
E UNIFORM TRIVALENT P3-4.8.12 SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON

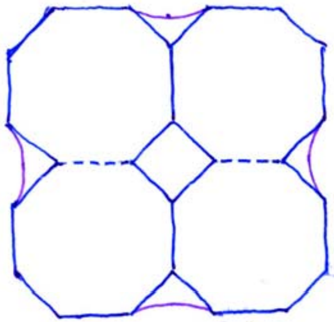


F UNIFORM 11-VALENT P11-3 4 SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON

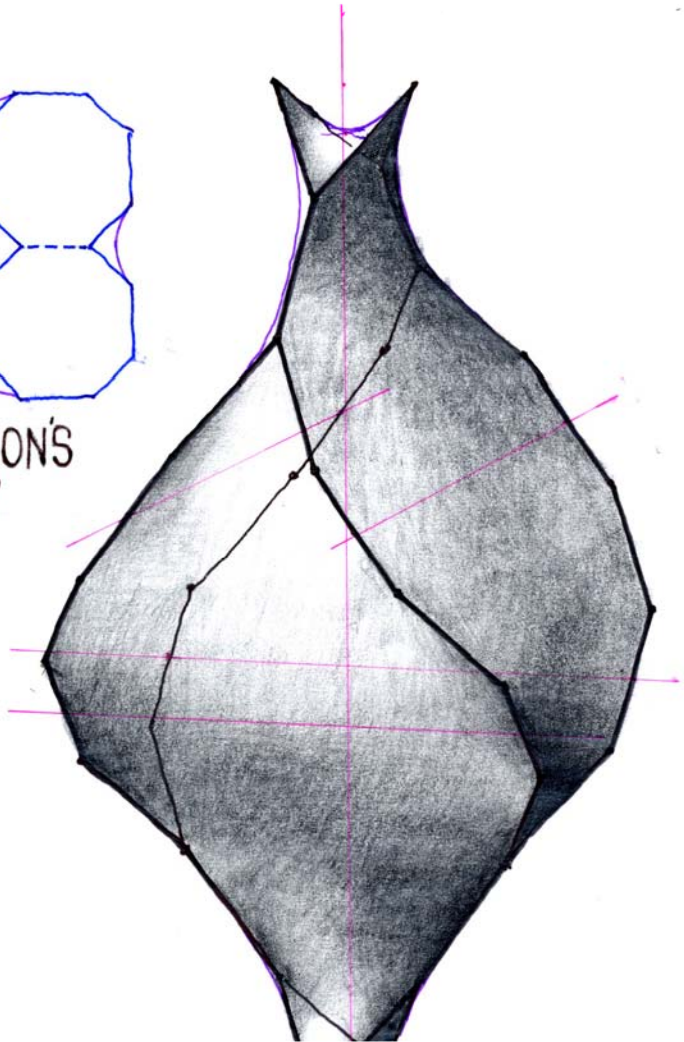


3³.6.4².6₉
g = 9
∢α = 3⅓π
Val. = 7
V TU. = 24
E TU. = 84
F TU. = 44
LATTICE - 3³.12⁶₉
Den. = 0.471890506 a/a³

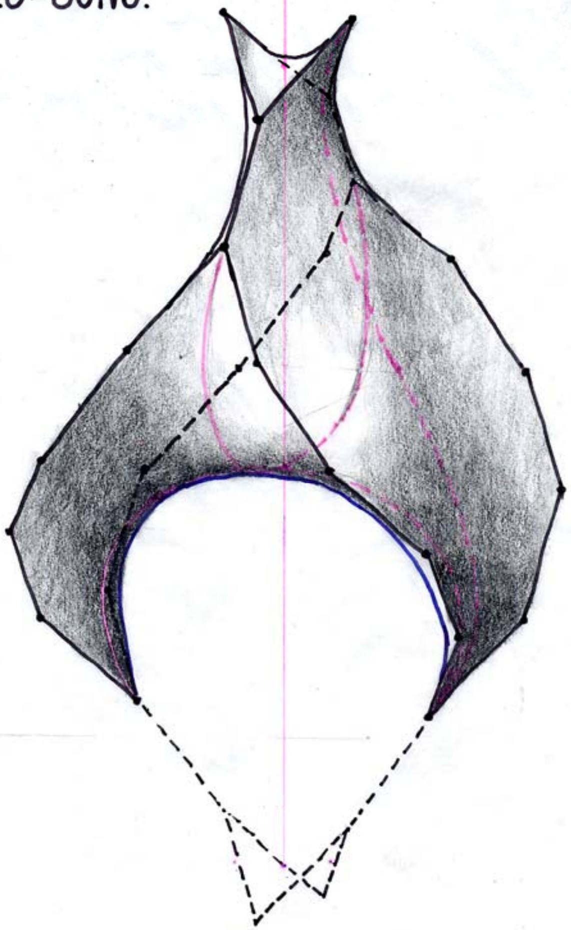




POLYHEDRON'S
OVERVIEW



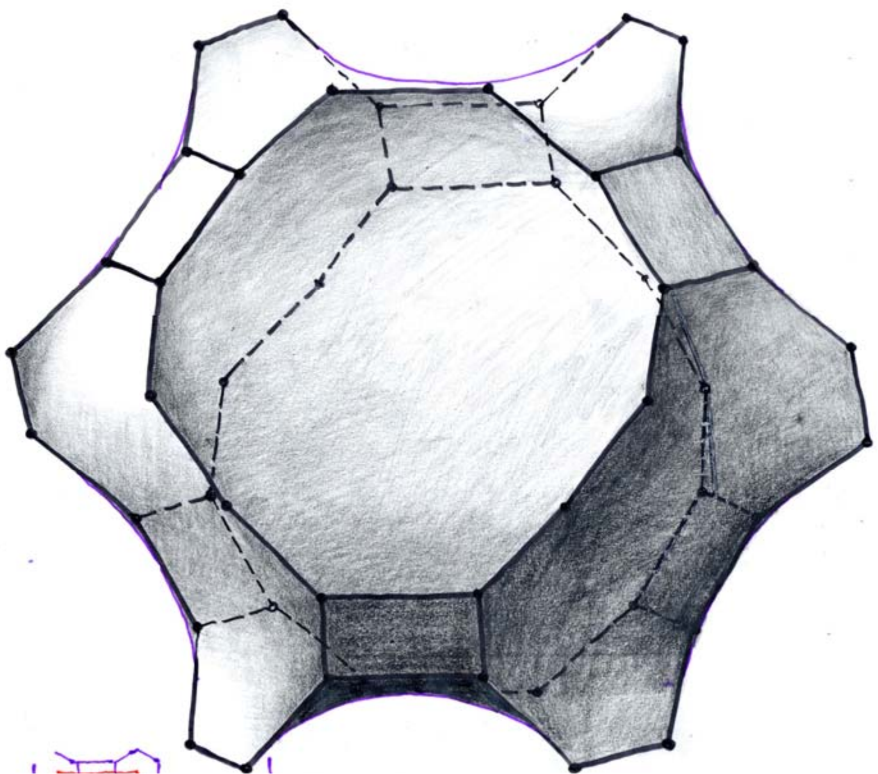
TETRAHEDRAL SADDLE POLYHEDRON
WITH 4; 14 & 20-GONS.

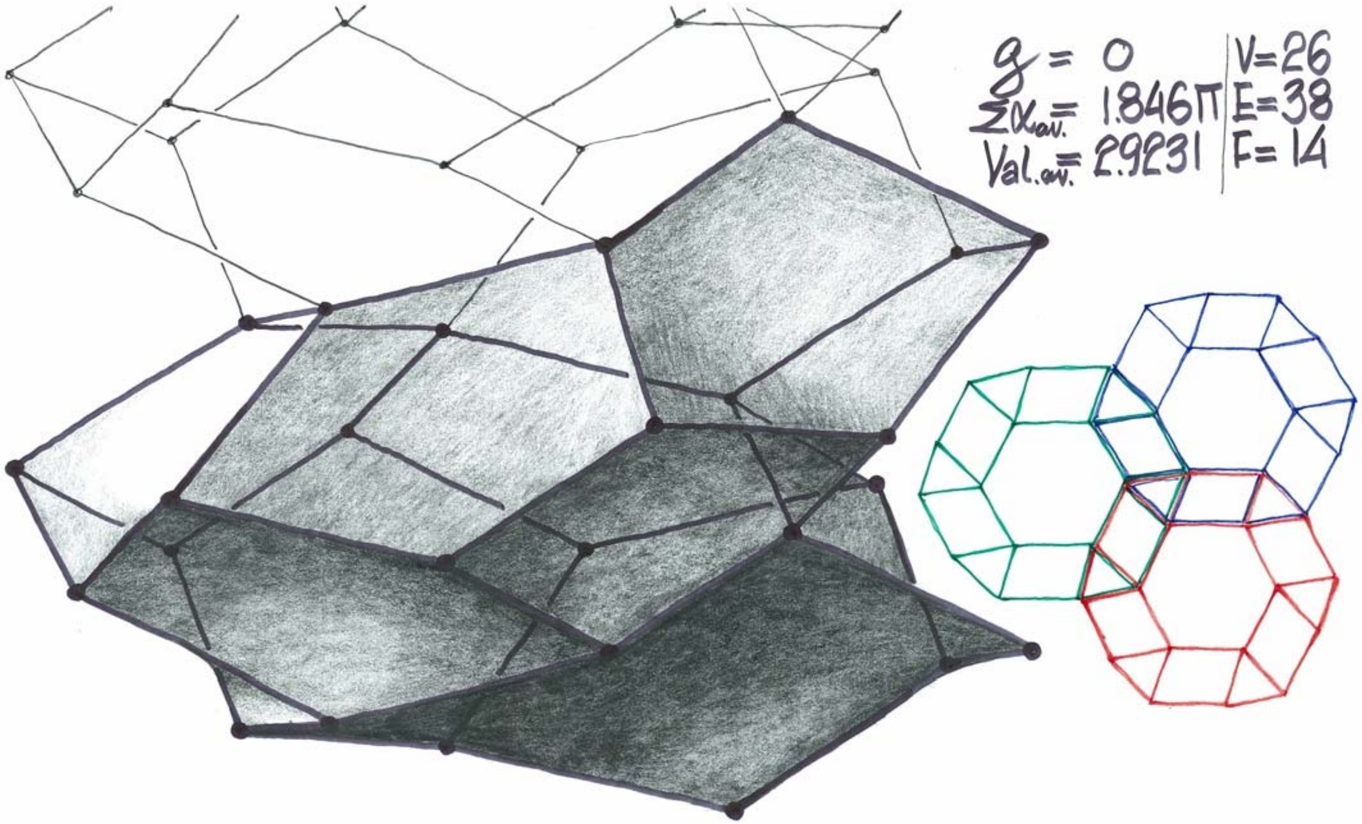


A SELF CLOSE PACKING SADDLE POLYHEDRON

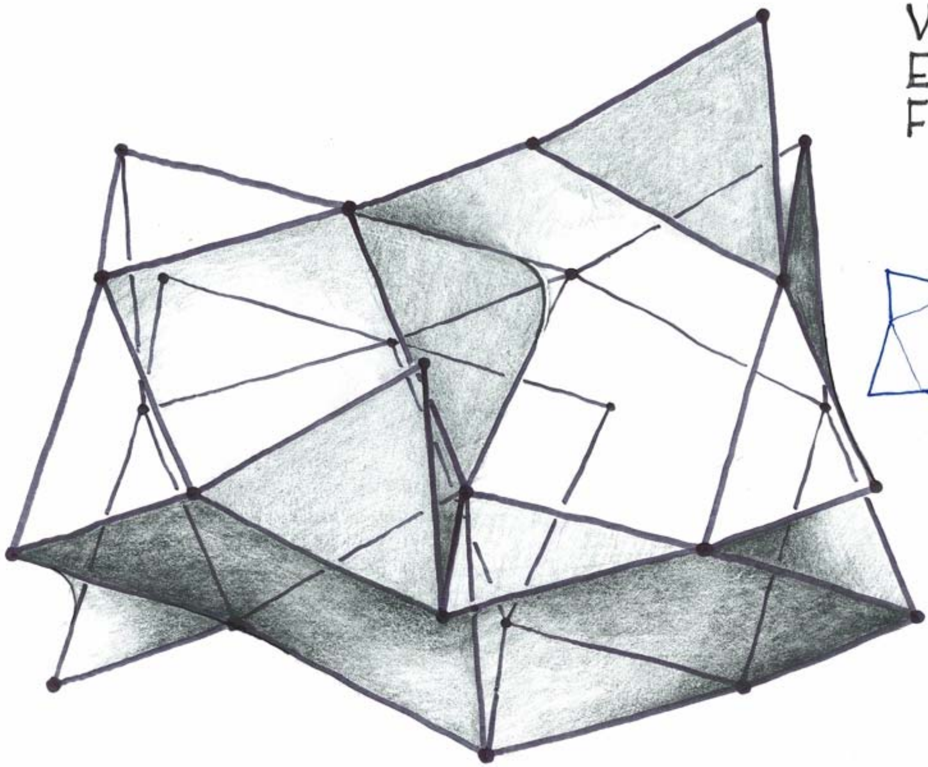
SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE

- THE 4.12^{4}



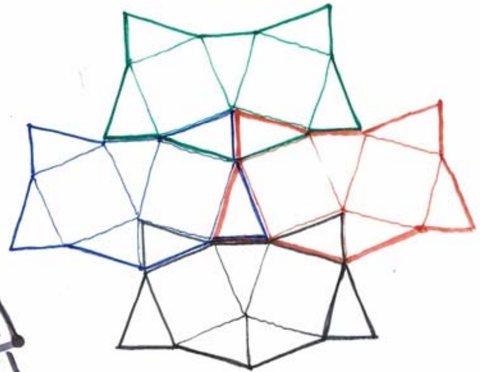


SELF-PACK SADDLE POLYHEDRON GENERATING THE UNIFORM MULTI-LAYER TETRAVALENT ML.4-$5^{48}.8^{30}.9^{16}_{7}$ SPACE LATTICE.

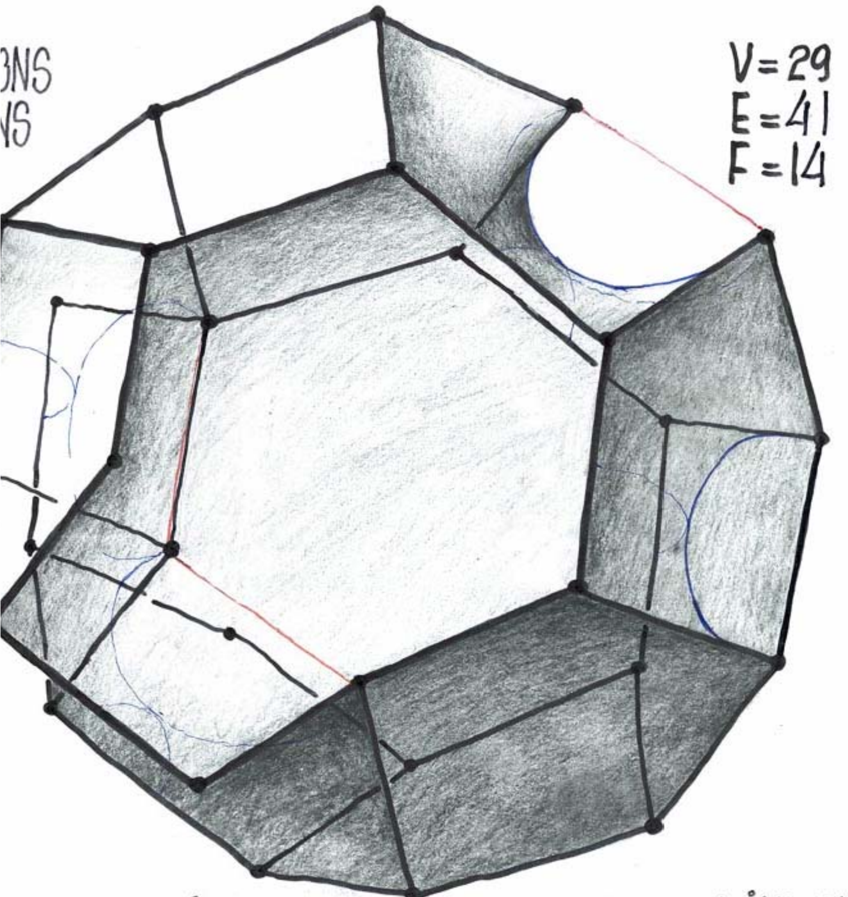


V = 26
E = 48
F = 24

g = 0
$\Sigma\alpha_{av.} = 1.8462\pi$
$Val._{av.} = 3.7308$



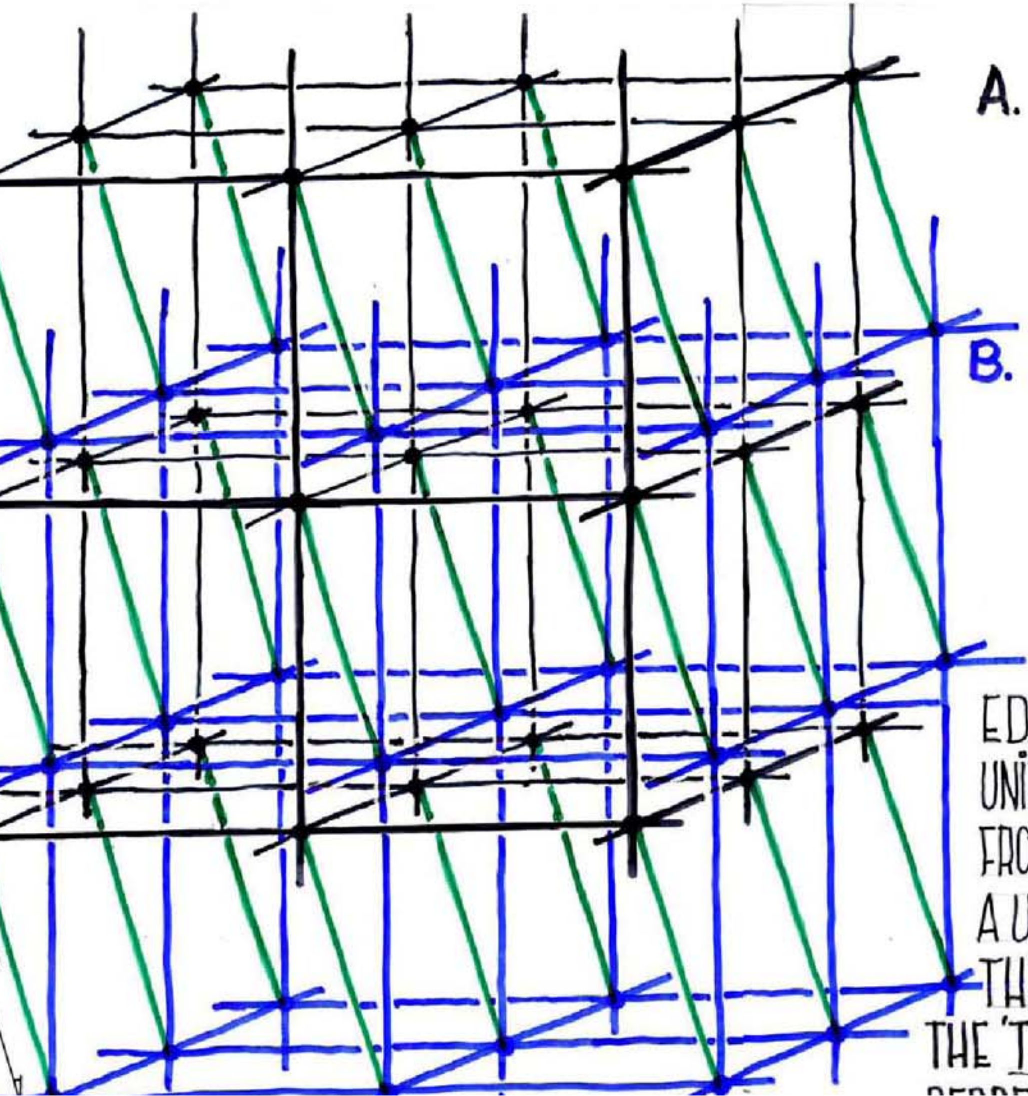
SELF-PACK SADDLE POLYHEDRON GENERATING THE UNIFORM MULTI-LAYER HEXAVALENT ML.6-$3^3.5^4.6^8.8^7_{33}$ SPACE LATTICE.



CLOSE PACKING SKEW POLYHEDRON GENERATING THE MULTI-LAYER TETRAVALENT ML4-$5^{48}.8^{30}.9^{16}_{7}$ SPACE LATTICE.

Every dual networks pair, the associated sponge surface subdividing between the two and the two associated close-packing modes describe an inter-relating ‘quintuplet in which **every four components can be accurately defined and derived from the fifth.**

The number of topologically different space networks, partition-surfaces and cellular space-packings is infinite (for each category), even when periodic in nature, due to topological similarities' or symmetry constraints.



ENTANGLED NETWORKS

EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT (CUBIC) LATTICE FROM A-TO B-POSITION, RESULTING IN A UNIFORM SEPTAVALENT LATTICE, THE DENSITY OF WHICH IS 7.00 a/a^3. THE '<u>TRANSLATION LATTICE</u>' IS A 3-D

In conclusion

In his monumental publication: **“Structural Inorganic Chemistry”** (1962), in a chapter discussing the **‘Geometric Basis of Crystal Chemistry’,** referring to 3D networks, A.F.Wells makes a startling factual observation: **“The theory of these nets does not appear to be known,** and in fact no attempt to derive them systematically seems to have been made”..(pp. 101). Even his efforts (‘Three Dimensional Nets and Polyhedra’-1977) did not help to resolve the issue in a meaningful way.

A comprehensive theory in any research domain may emerge only after the domain's phenomenology is accounted for and comprehended.

The presented **‘quintuplet of the associated 3D space phenomena’** are central to our perception and understanding of 3D space in general and that of our habitat environment in particular. 3D networks and the associated hyperbolical partition surfaces and the twin close-packing modes represent the most important morphological features of our architectural design imagery and primary, visually embraced notions and features of our 3D space phenomenology.