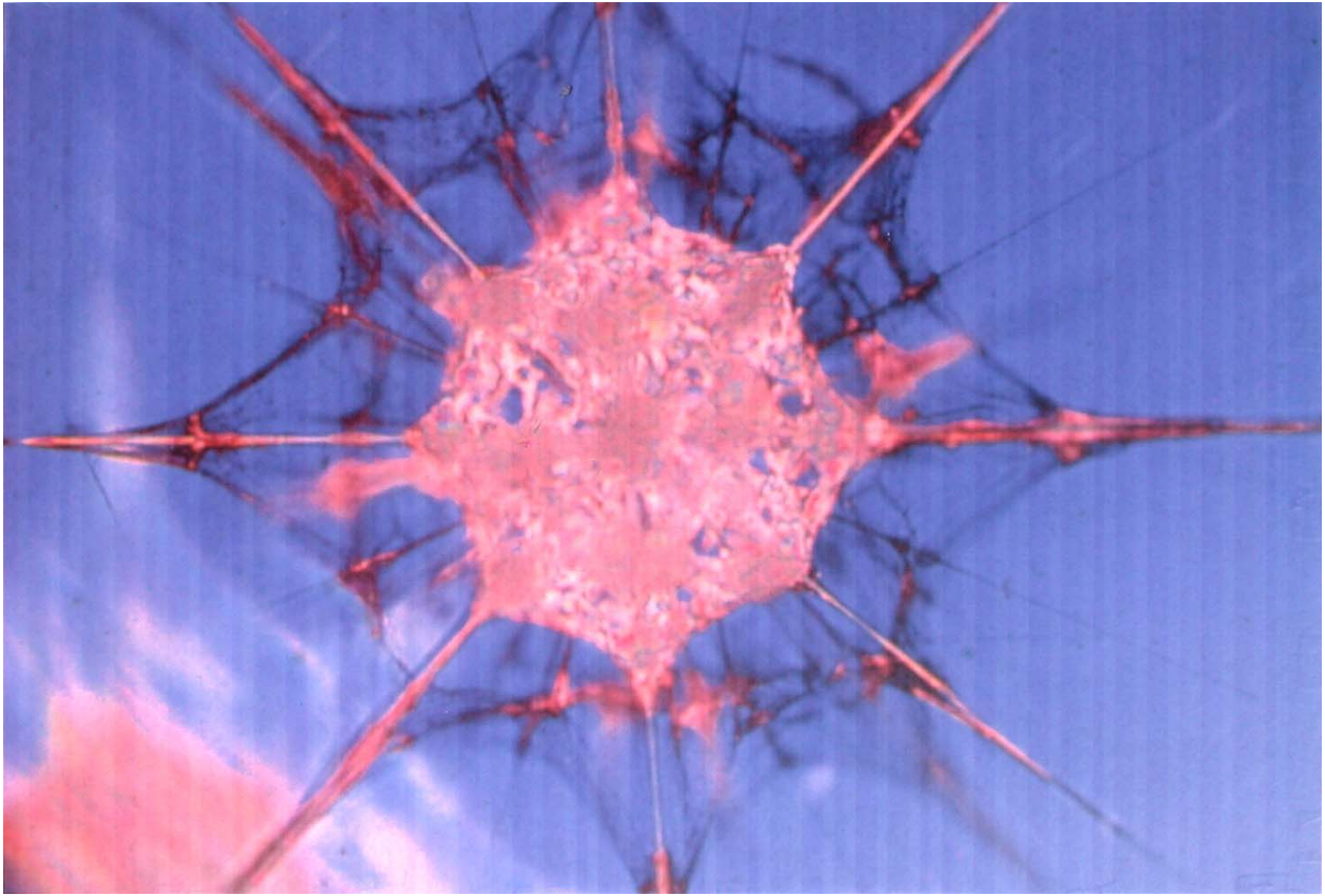
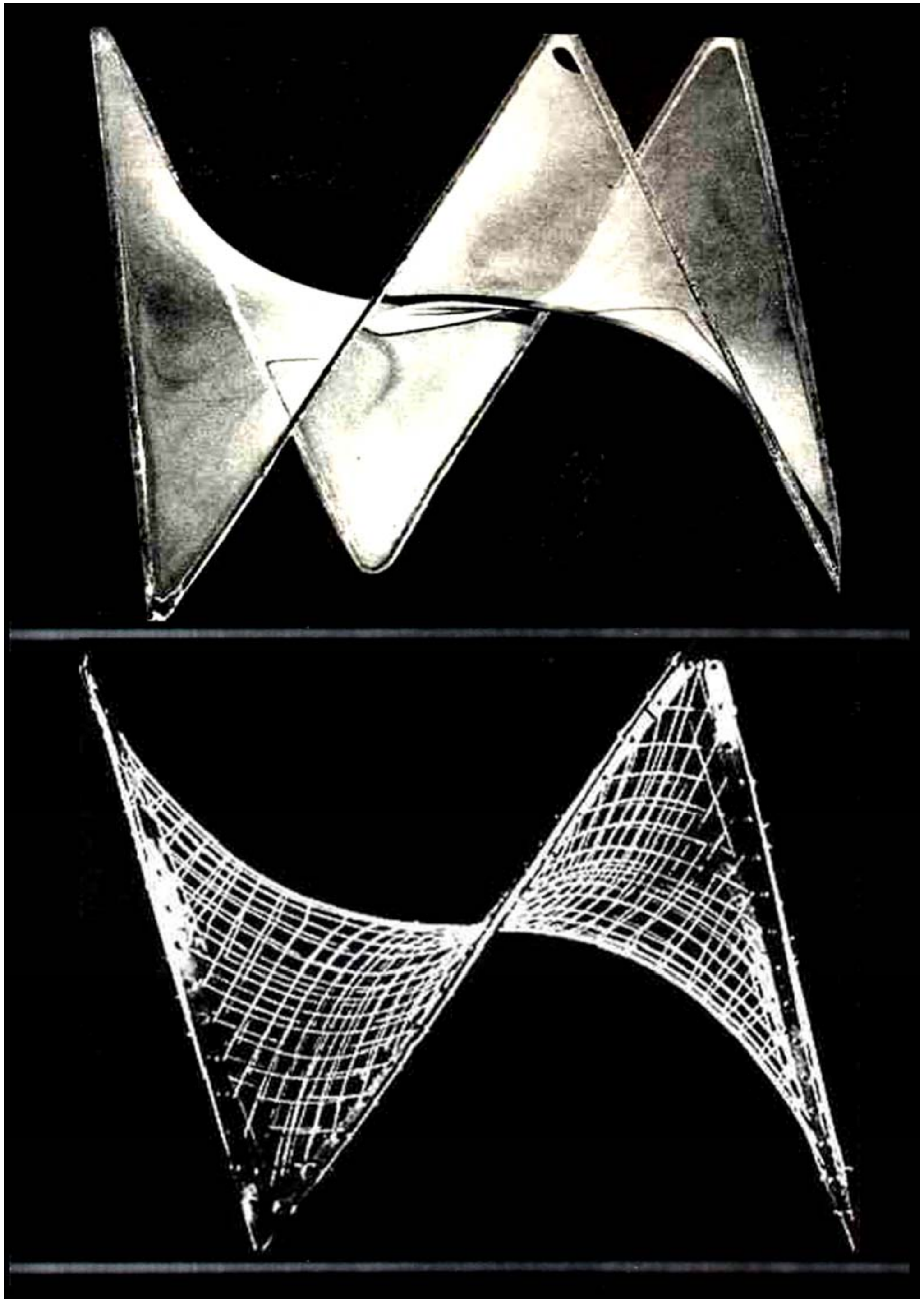
THE PERIODIC TABLE OF THE POLYHEDRA AND THE LATTICE UNIVERSE

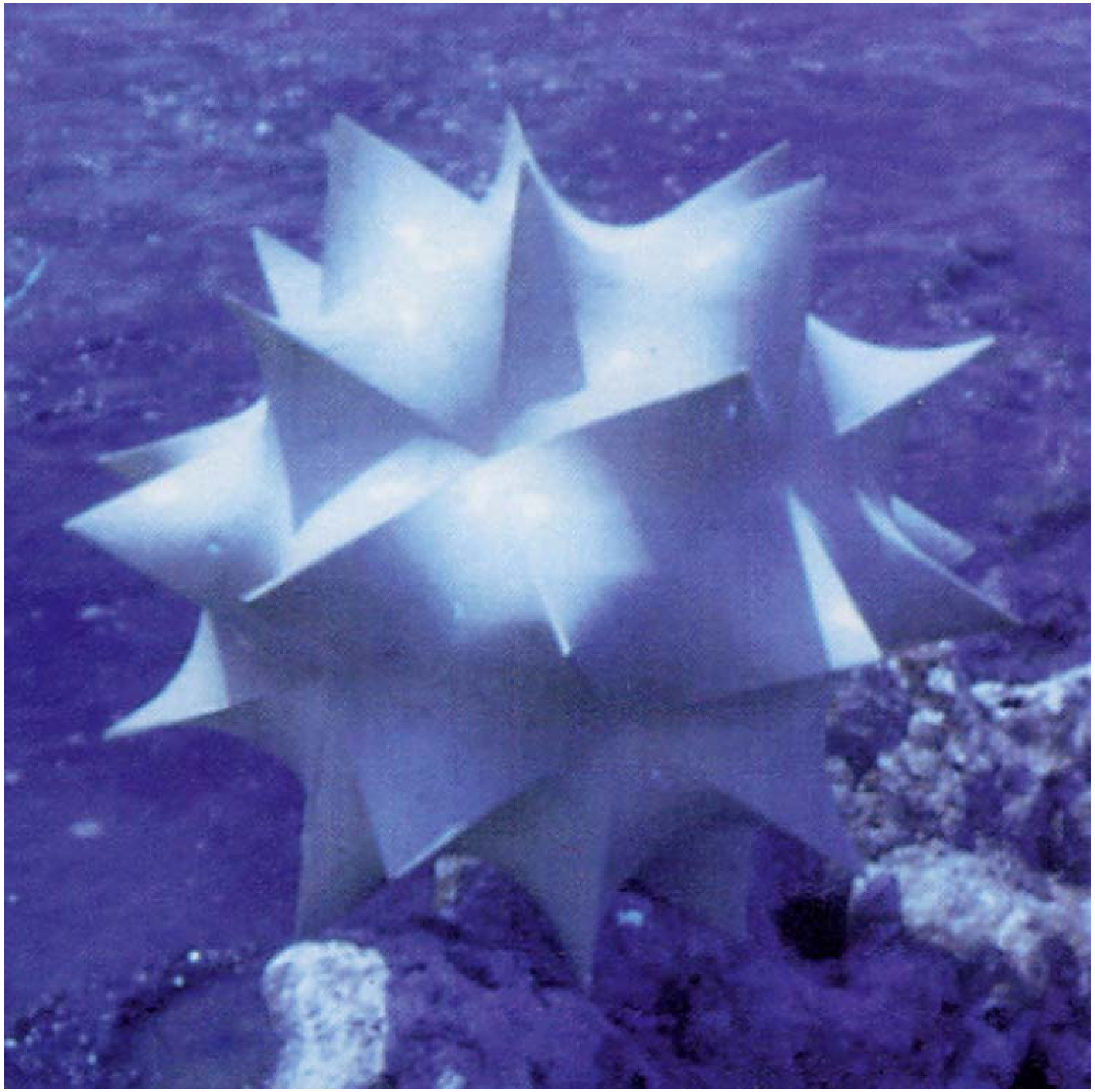
Prof. Michael Burt **Technion, I.I.T. – Israel**

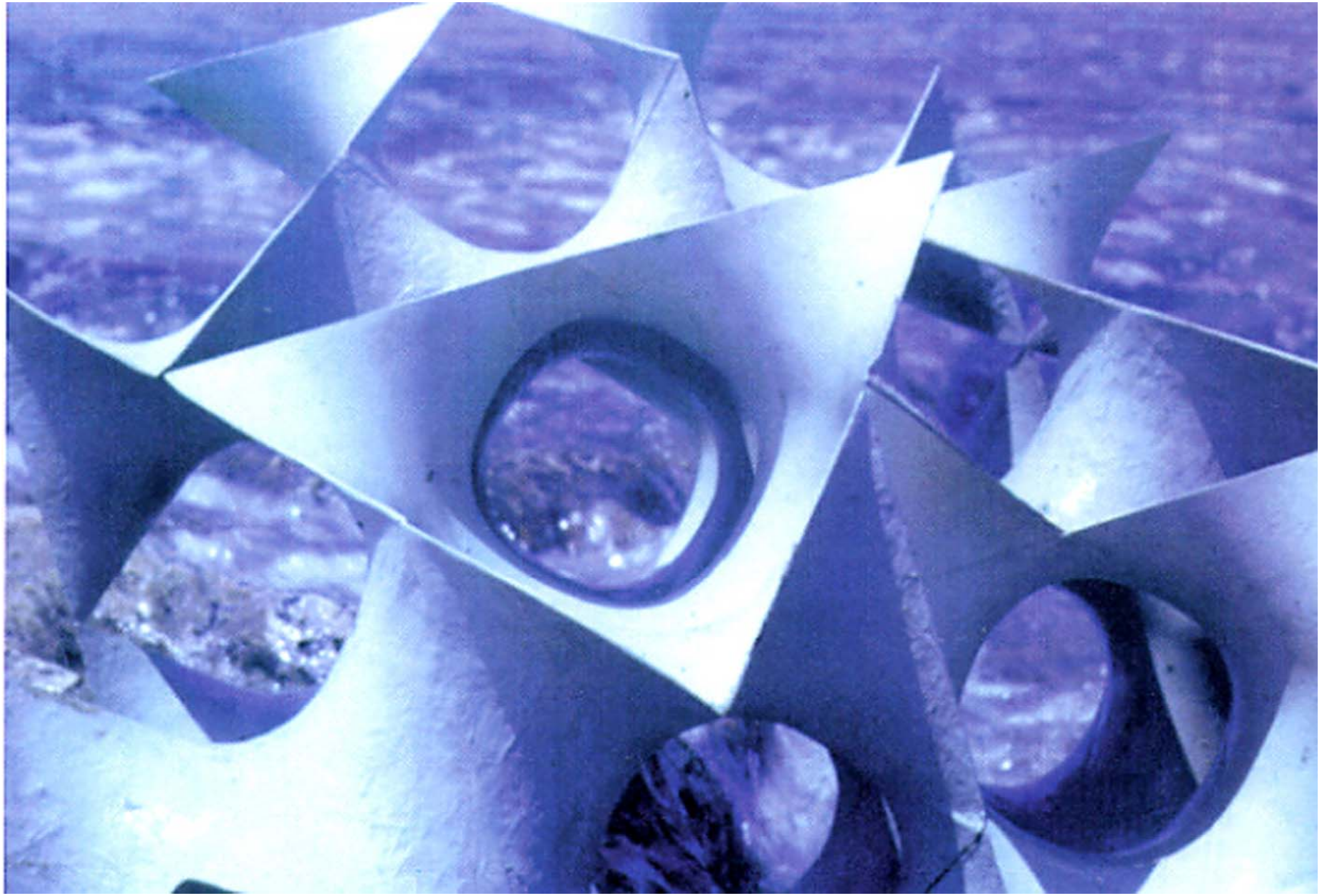
B

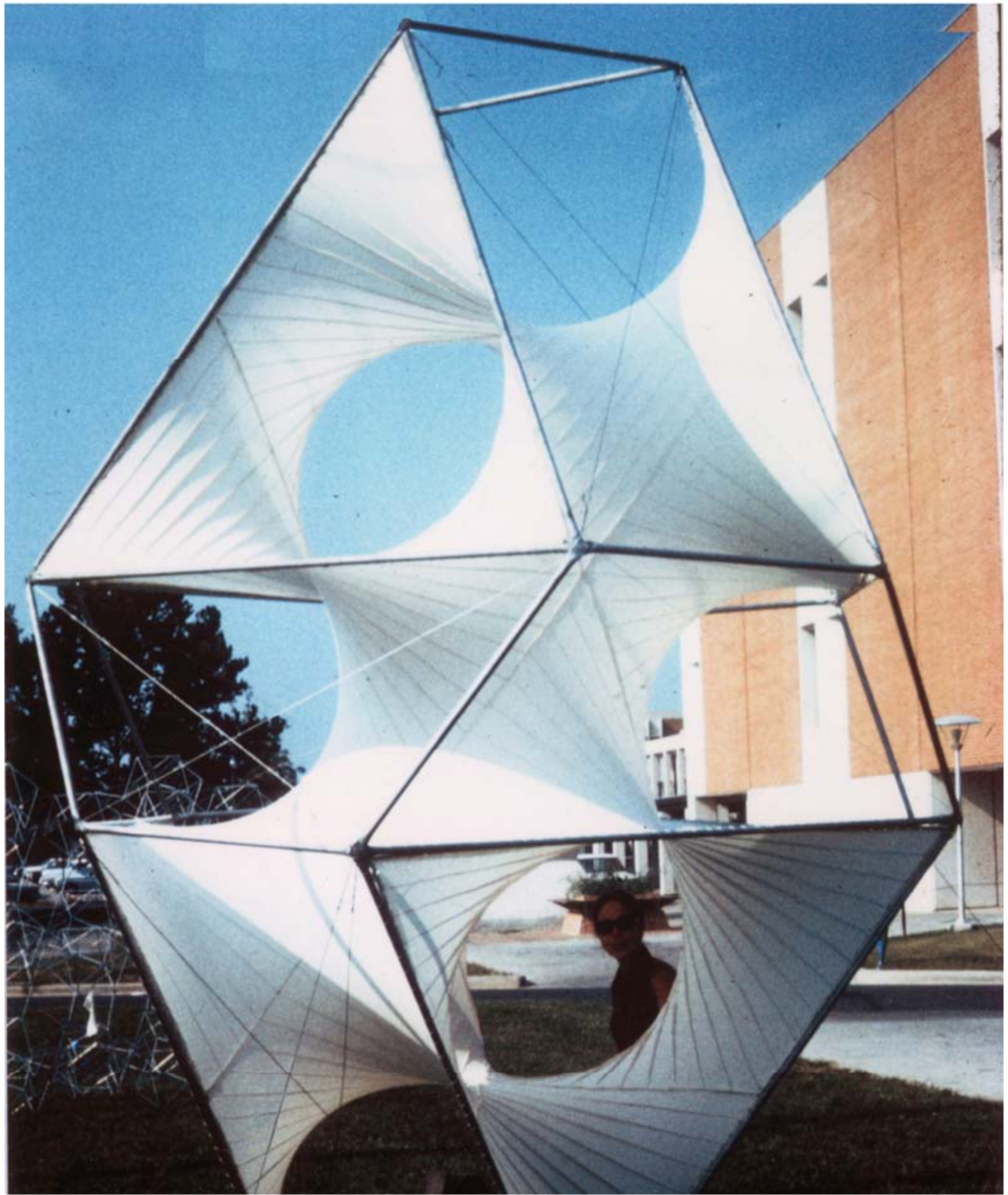
“Our study of natural form”, the essence of morphology, “is part of that wider science of form which deals with the forms assumed by nature under all aspects and conditions, and in a still wider sense, with **forms which are theoretically imaginable”…..(**On Growth and Form – D'Arcy Thompson), "Theoretically" to imply that we are dealing with causal- rational forms.

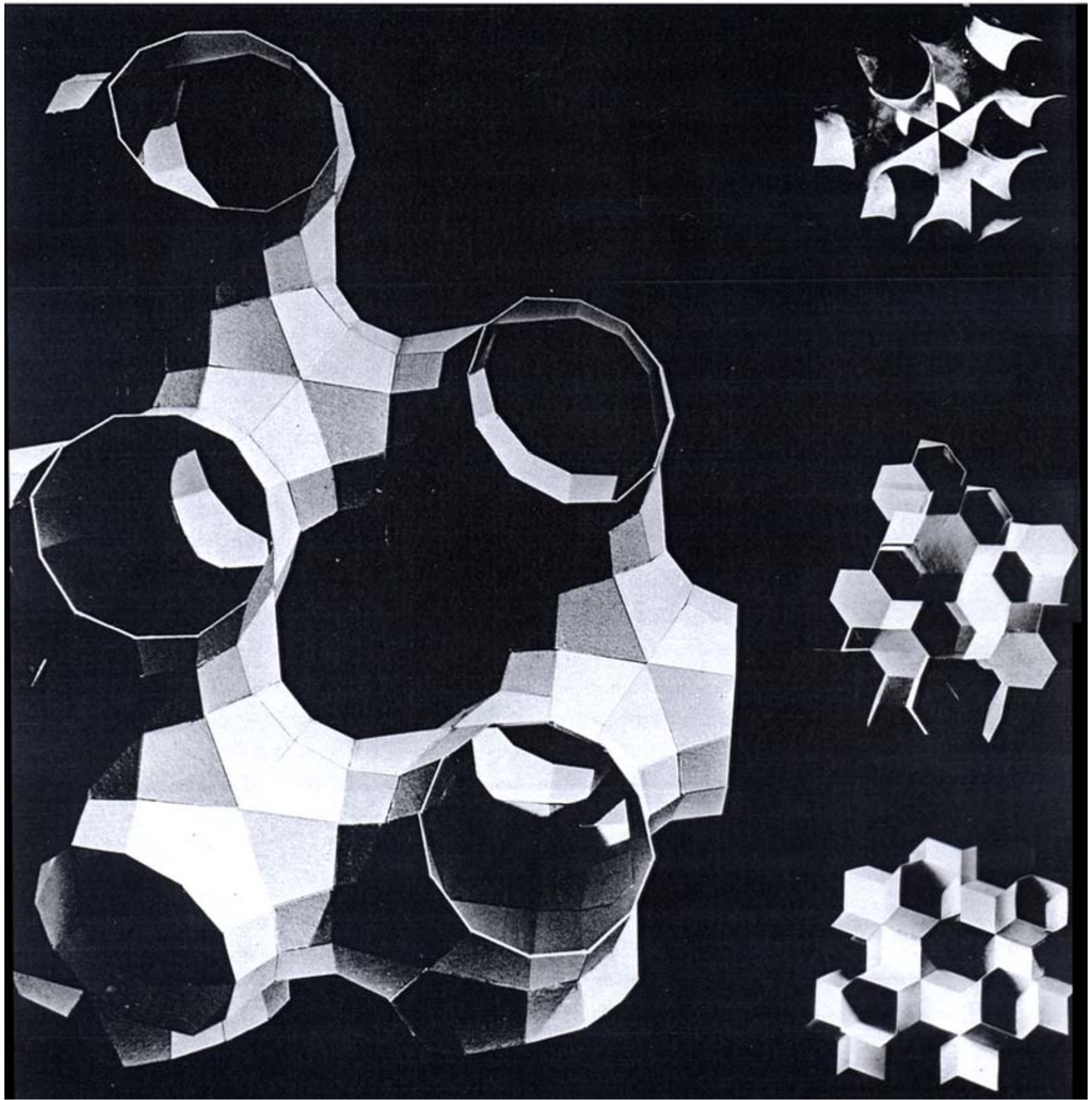


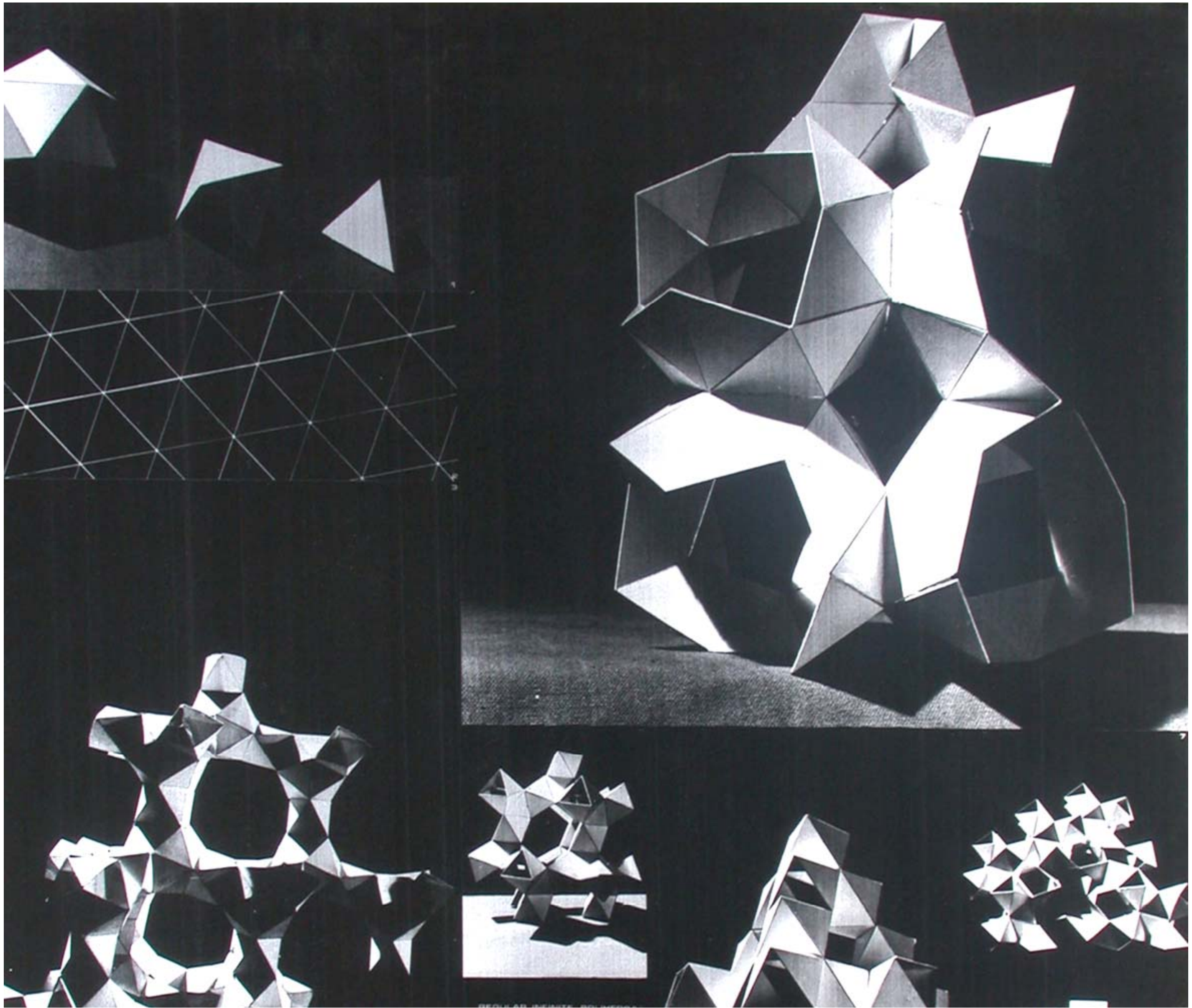


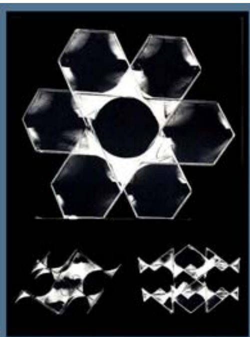






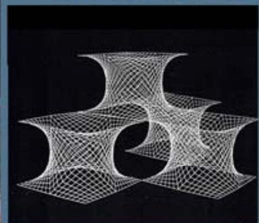


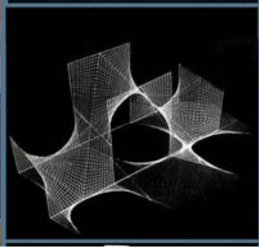


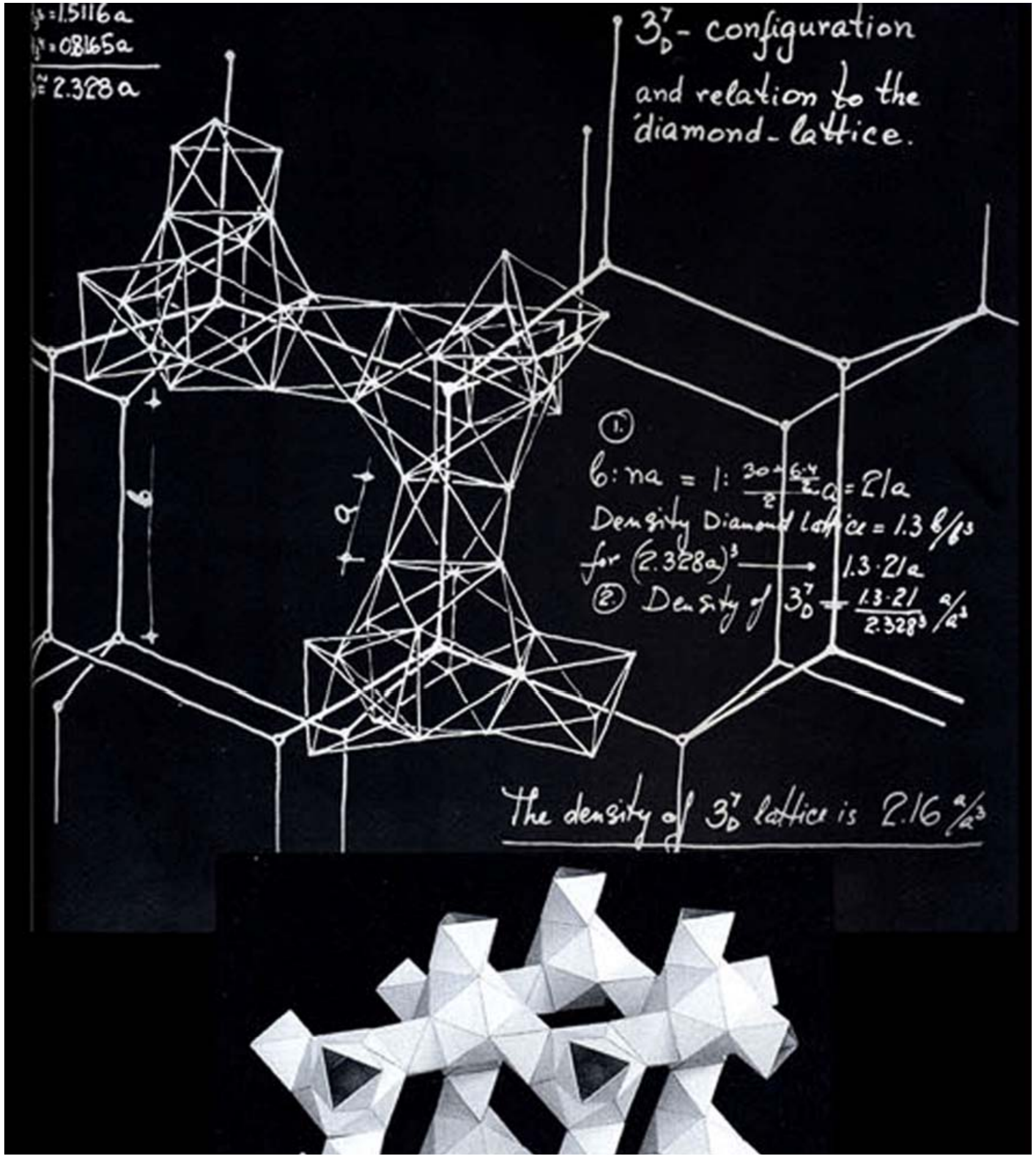


HEXAHYP-7 SHELL STRUCTURE
1969 - HAIFA ISRAEL
AQUAVILLE - PARIS BIENNALE
1969 - PARIS M.BURT

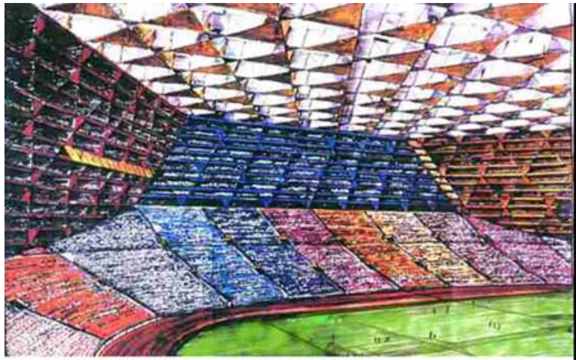
PREVIOUS RESEARCH EFFORTS ON THE THEME OF
HYPERBOLIC SURFACES AND INFINITE POLYHEDRA
AND APPLICATIONS TO LIGHT-WEIGHT STRUCTURES

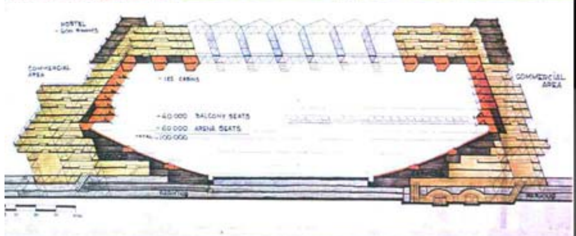


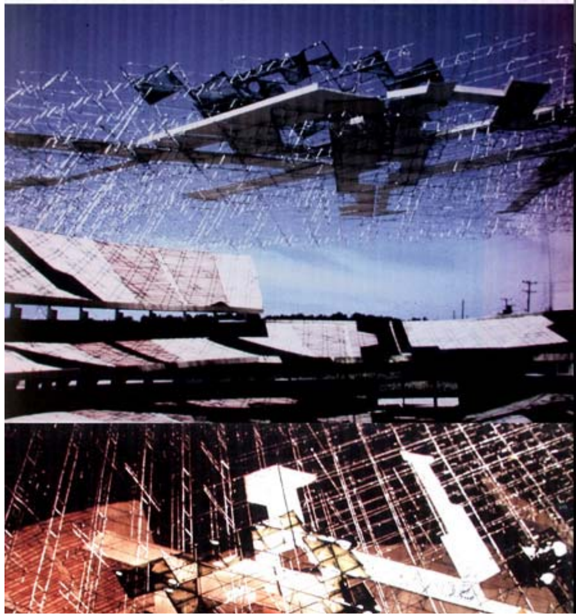


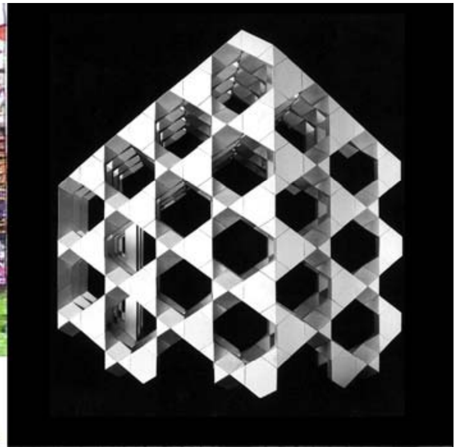


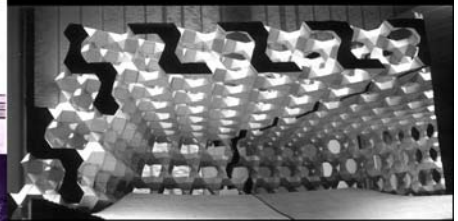
=1.5116a
=0.8165a
= 2.328a
3^7_D - configuration
and relation to the
'diamond-lattice.
1.
Density Diamond lattice = 1.3
for $(2.328a)^3$ → 1.3·21a
2. Density of 3^7_D
The density of 3^7_D lattice is 2.16

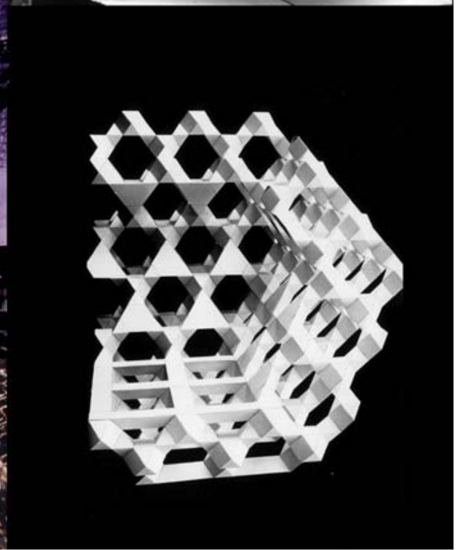



HOSTEL
COMMERCIAL AREA
• 165 CABINS
COMMERCIAL AREA
• 40 000 BALCONY SEATS
• 60 000 ARENA SEATS
TOTAL 100 000

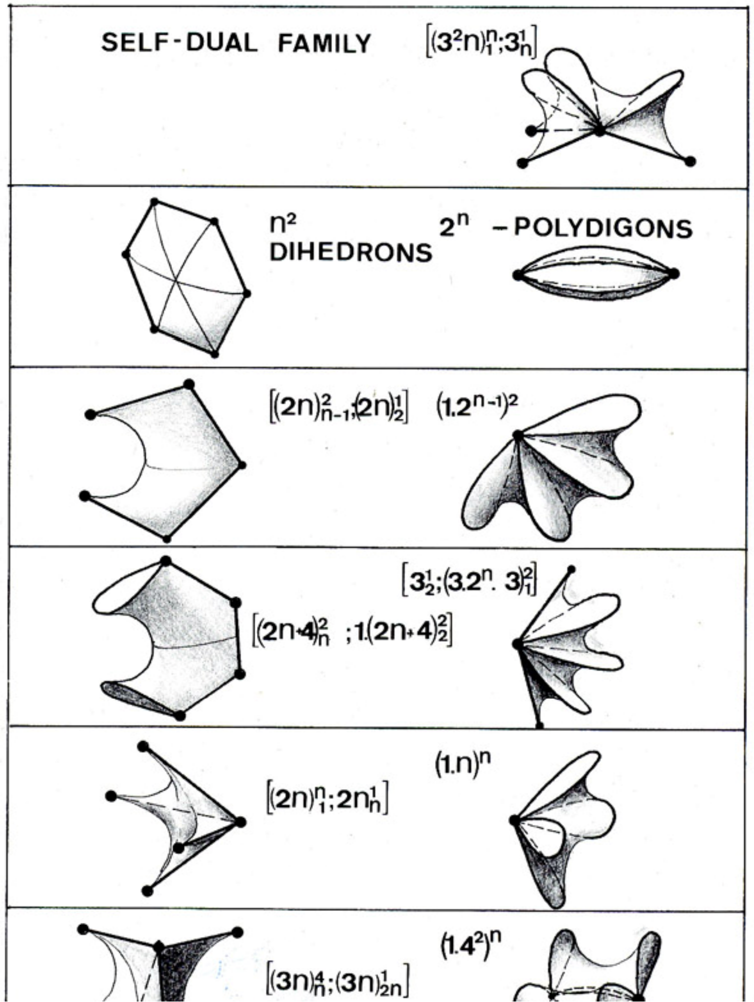




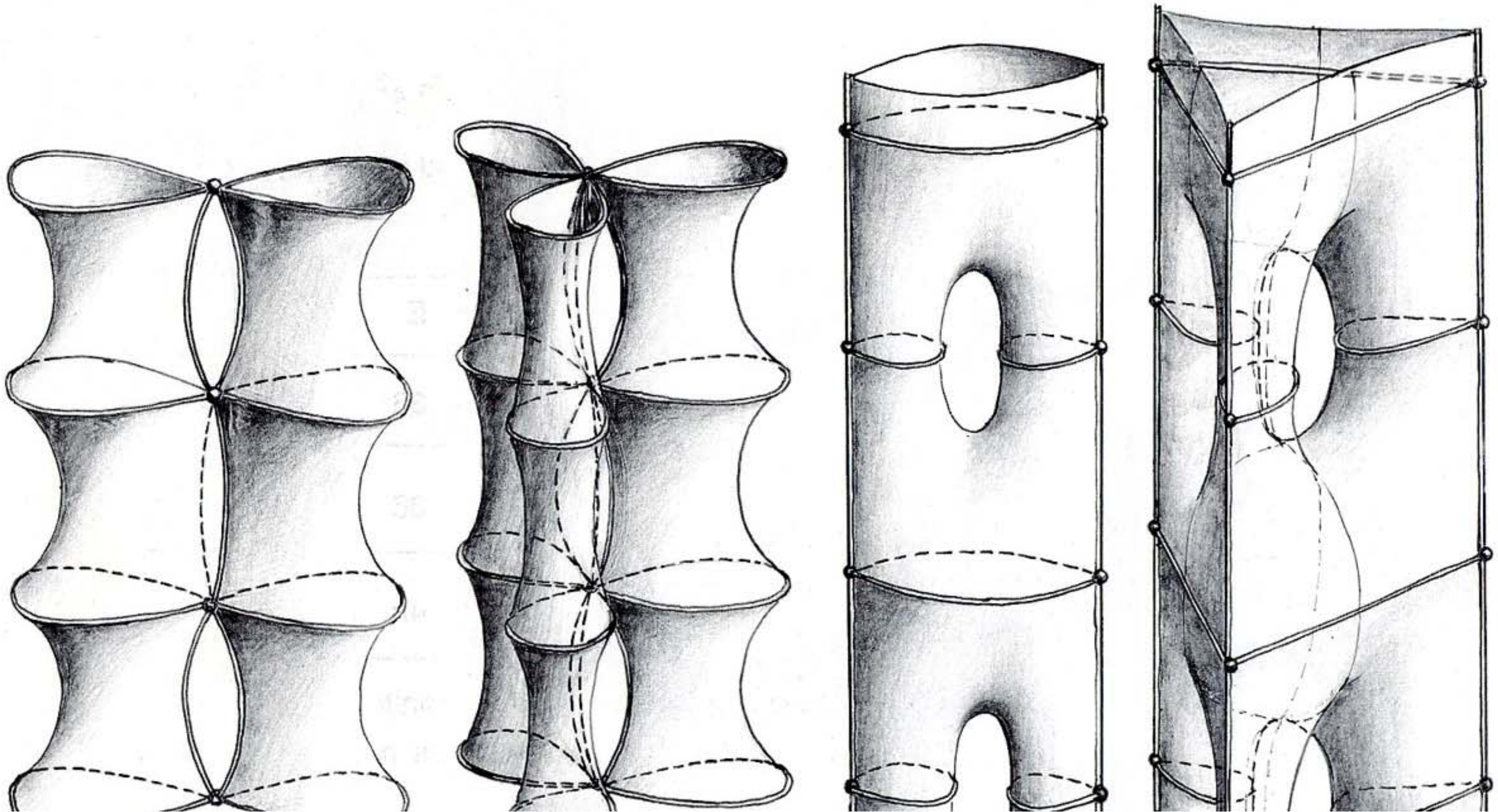


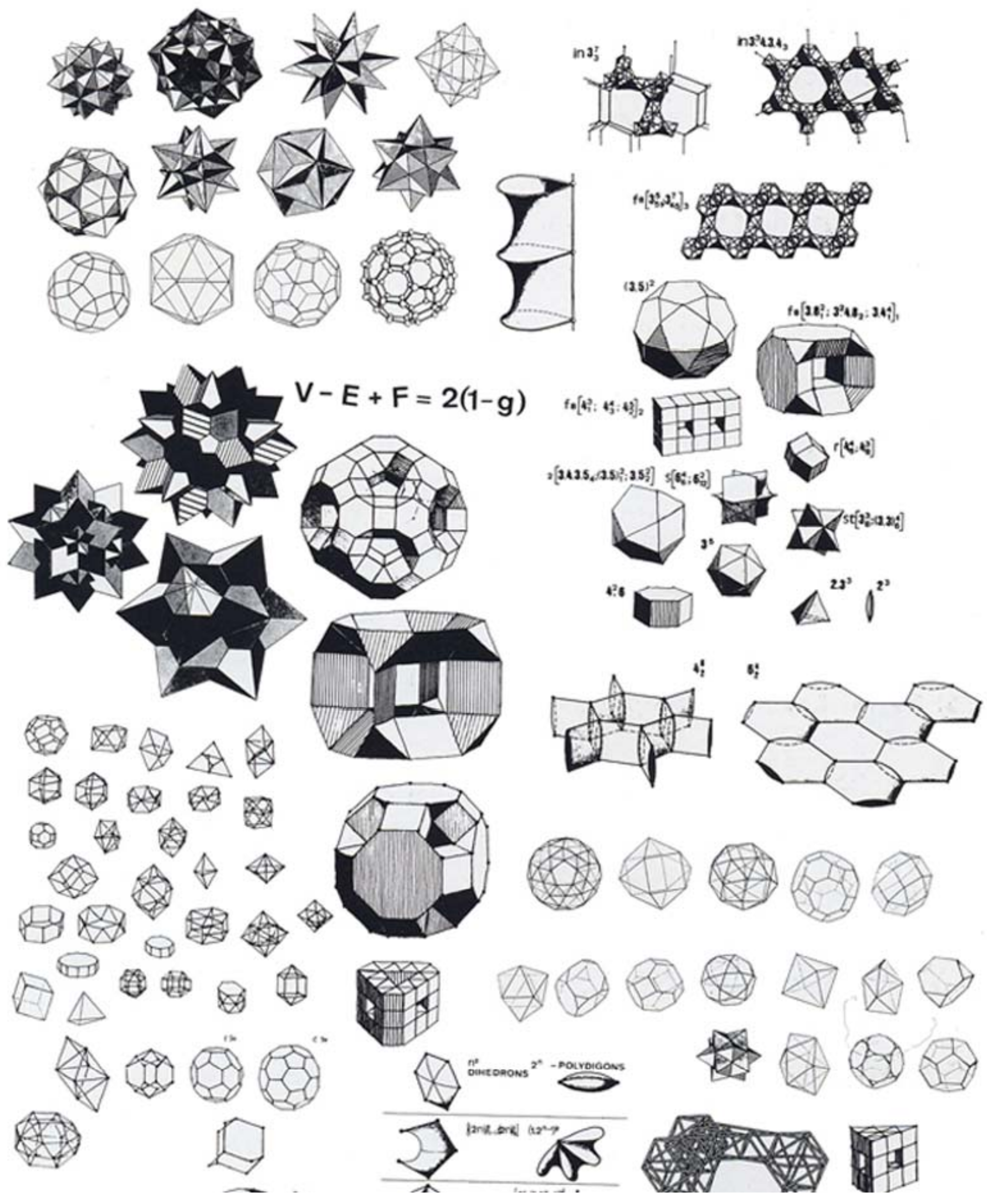


Floral polyhedra, families within the g=0 domain Aaranged according to their dual pairs.



Periodic Floral Infinite
Polyhedra.





V − E + F = 2(1−g)
DIHEDRONS
– POLYDIGONS

networks, as polyhedral tessellation configurations,
e rise to some familiar binding relations:

Pav. F = 2E = V. Val.av. ;

V (2Л –Σαav.) = 4Л(1 – g) - Descarte's theorem;

V – E + F = 2(1 – g) - Euler's theorem;

initions

olygonal region of order n, for n≥1, is a point set, topologically equivalent
circular disc with a boundary divided into n edges by set of n vertices. It
have curved edges, maybe non-planar and two edges of the same
on may be matched (Stewart) [4].

Polyhedral map drawing on a sponge surface must lead to polygonal
es which may constitute, under a suitable topological transformation, a
ne polygonal region.

olyhedron-P is a connected, unbounded 2-dimensional manifold, formed
set of simply connected polygonal regions of order n, for n≥1, arranged
hat each edge of each region is matched with exactly one other edge of
same, or another region and vertices are matched only as required by the
ching of edges. It implies that one and the same, or two, and no more
two distinct polygonal regions (faces) meet at each edge. The restriction
ertex matching in the definition means there is only one circuit of
gonal regions at each vertex of P.

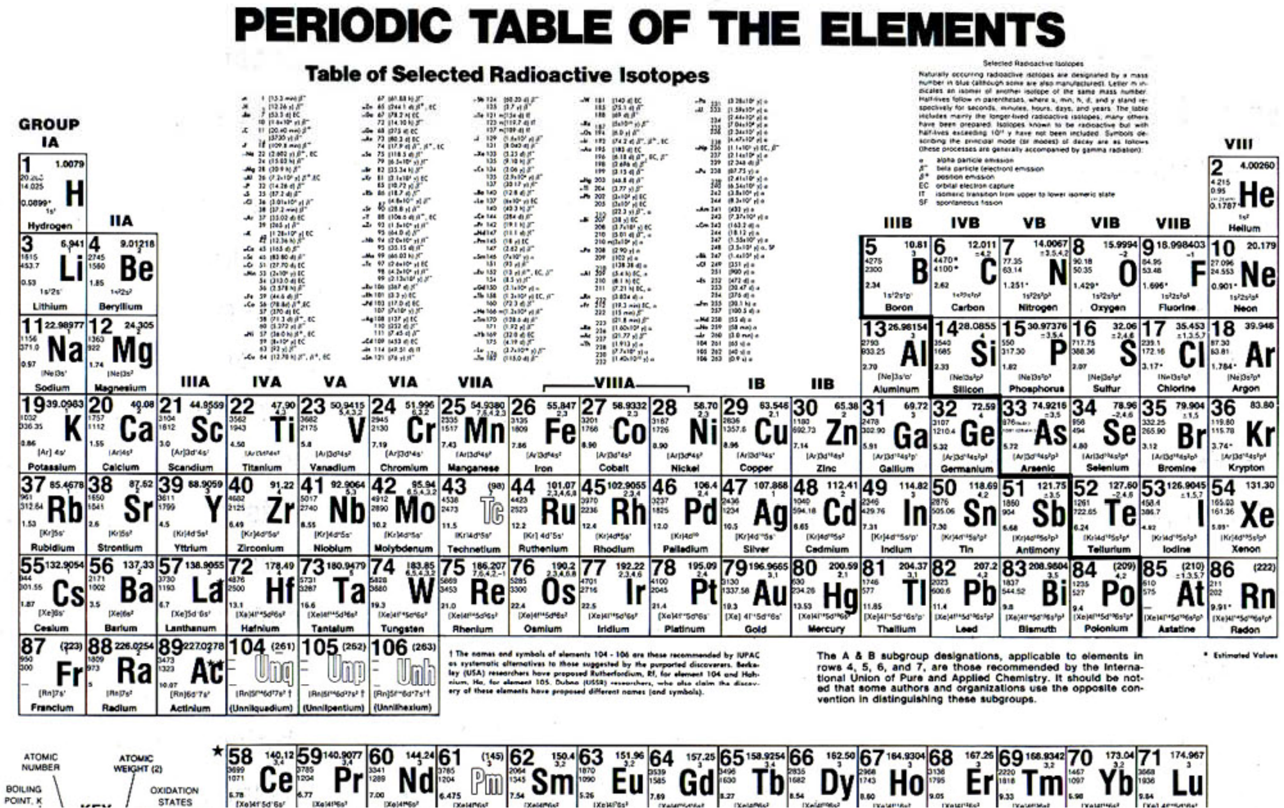
WHAT'S NEXT ?

WHERE TO LOOK FOR
MEANINGFULL NEW FORMS ?

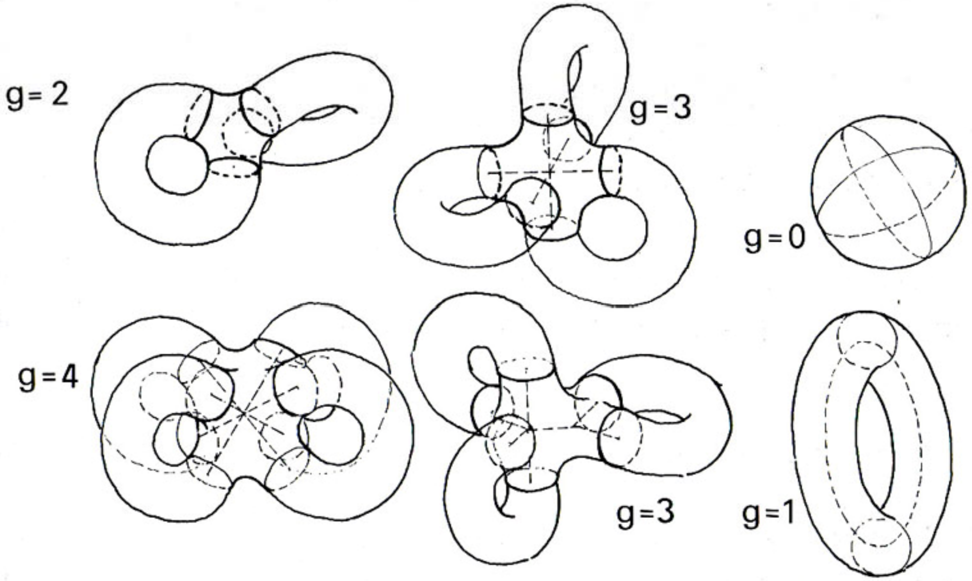
YOU DO NOT CRAWL INTO
DARK CORNERS OR HOLES
TO LOOK FOR SOMETHING
UNLESS YOU HAVE A THEORY
THAT IT MIGHT BE THERE!

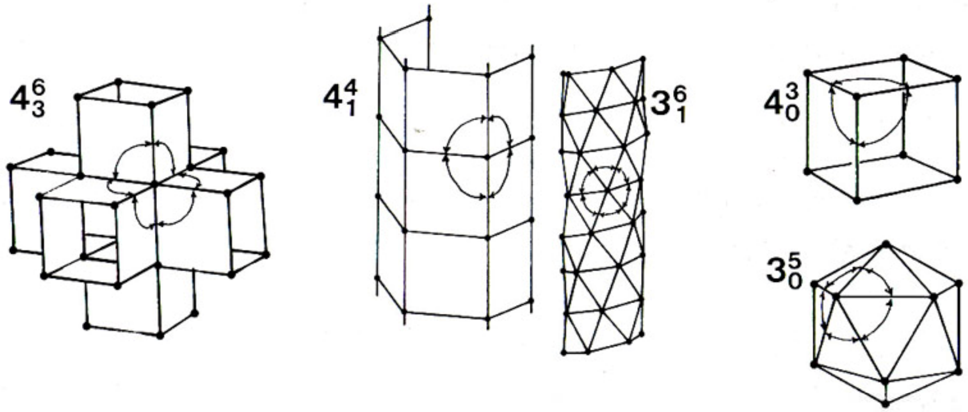
I BUILT A LOOKING GLASS
AND A THEORY WHERE

Mendeleyev's Periodic Table of the Elements.



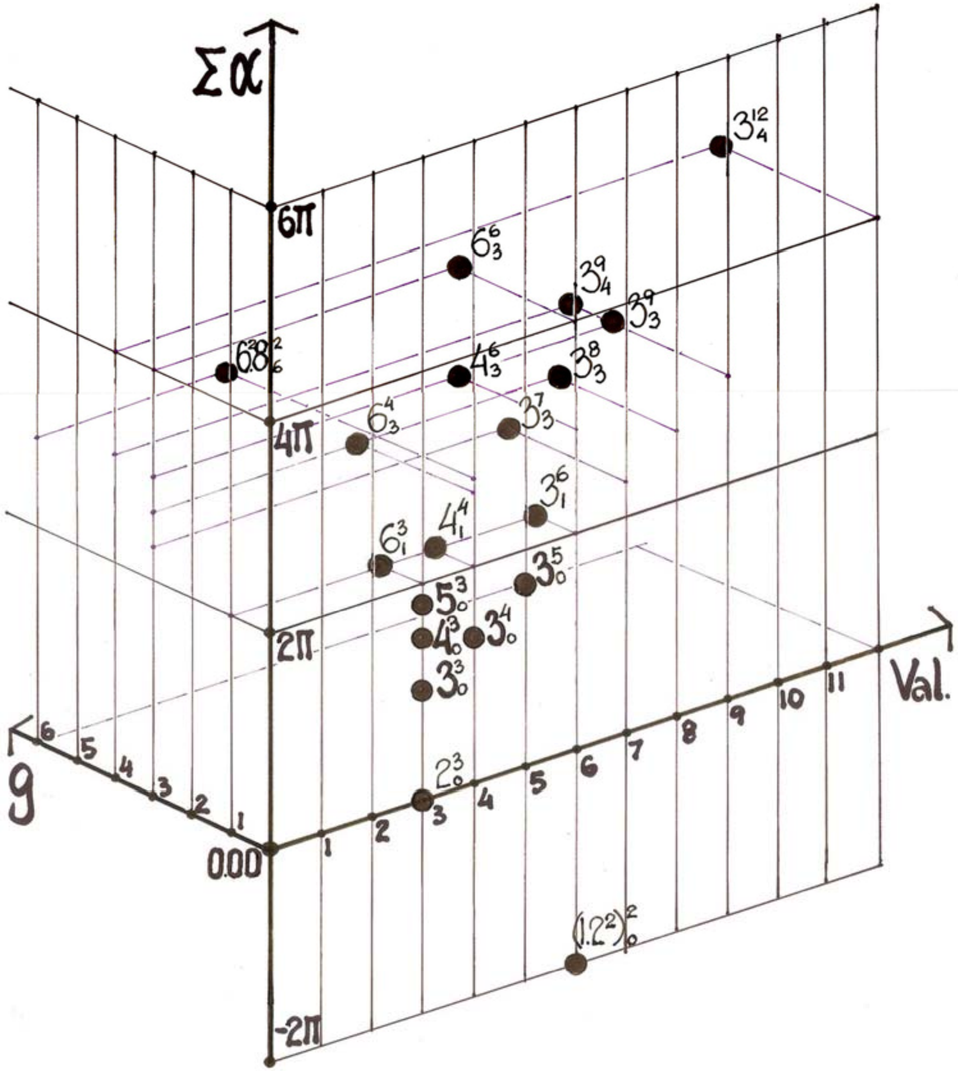
2-D manifolds surfaces, subdividing space into two complementary sub-spaces; polyhedral maps and their primary parameters.



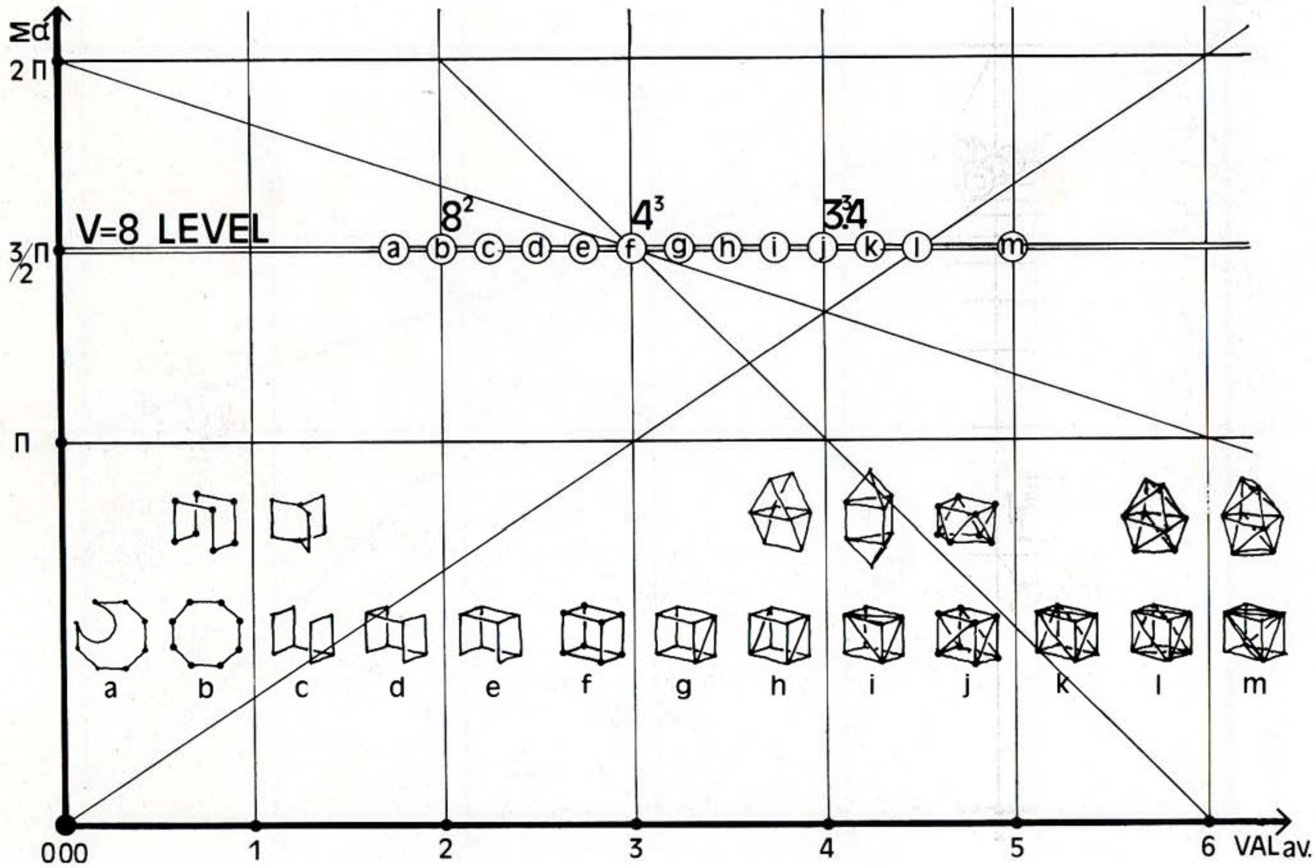


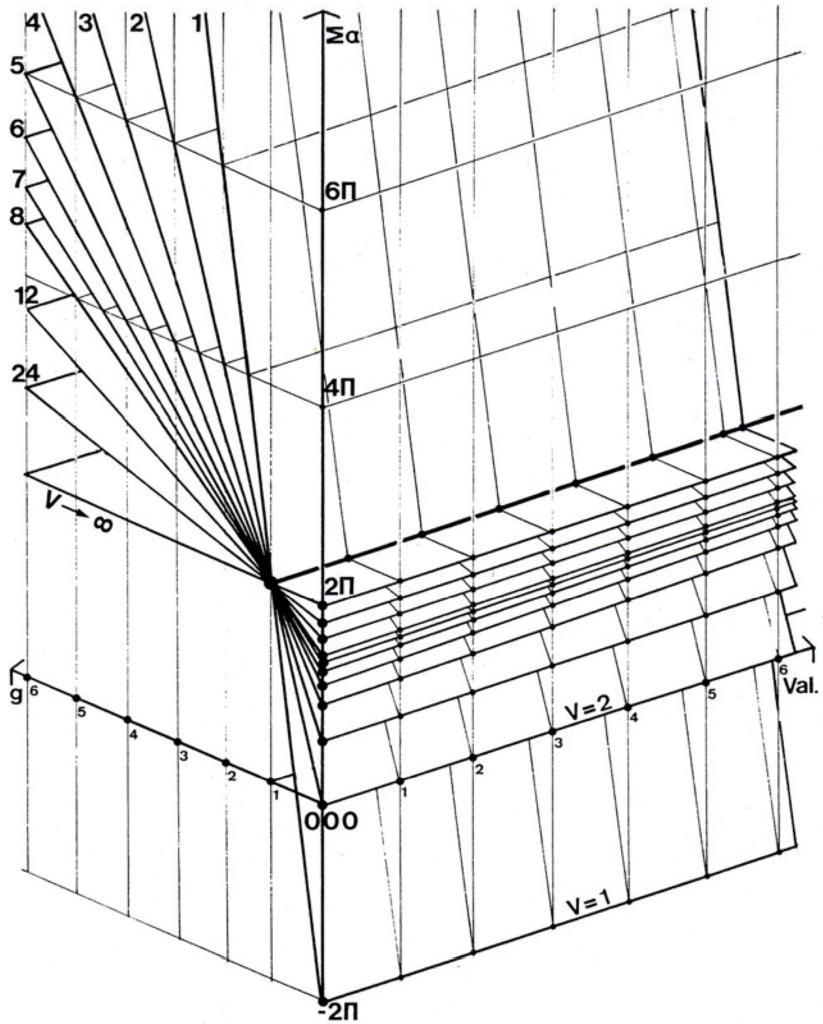
	4^3_0	3^5_0	4^4_1	3^6_1	4^6_3
$\sum\alpha$	270	300	360	360	540
Val.	3	5	4	6	6
g	0	0	1	1	3

THE PERIODIC TABLE OF THE POLYHEDRAL UNIVERSE



Polyhedra of V=8 level, within the g=0 domain.





$$V = \frac{4\pi(1-g)}{2\pi - \Sigma\alpha_{av.}}$$

Polyhedra with E=12 within the g=0 domain.

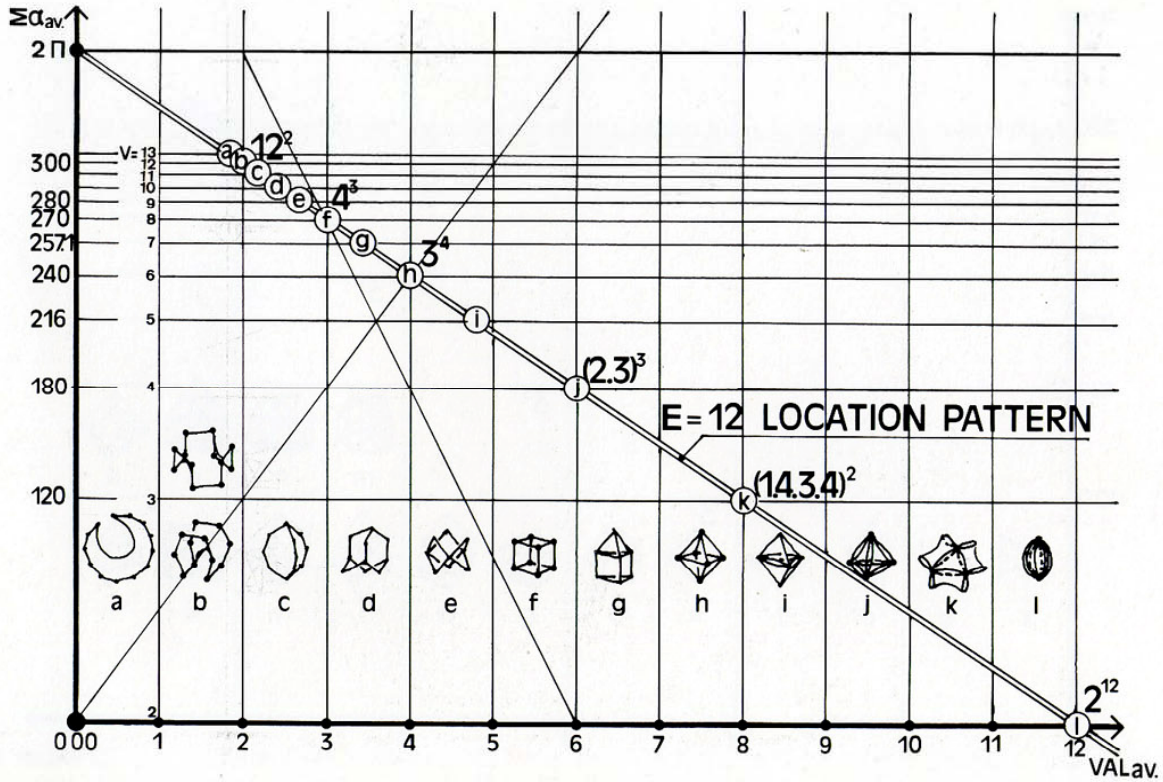
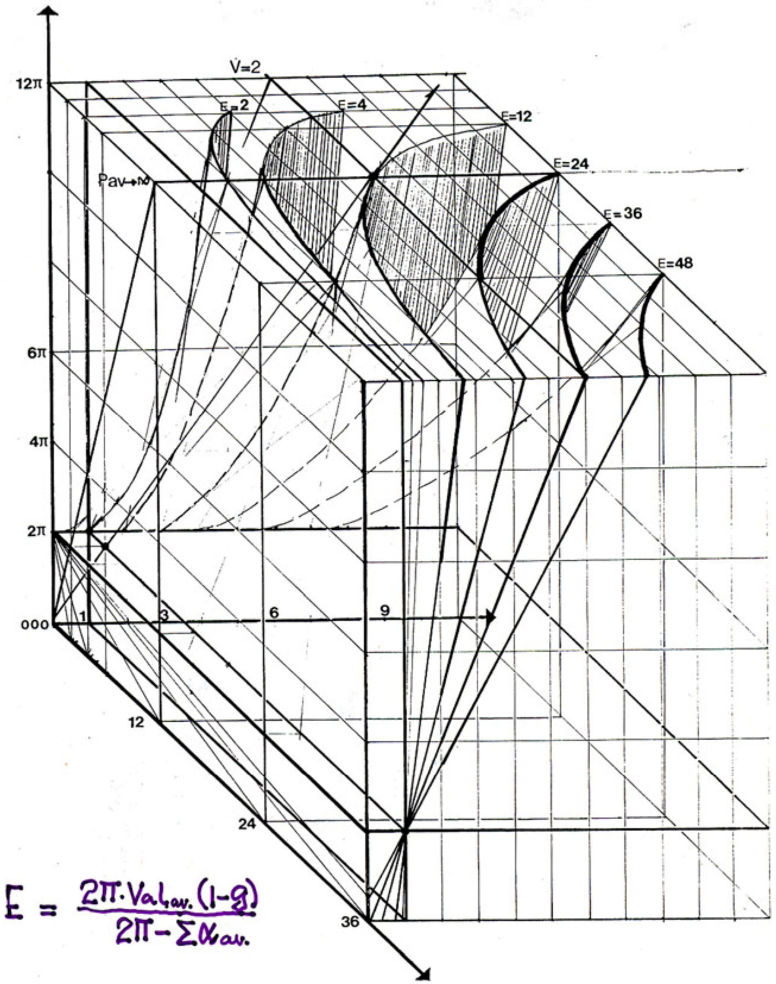
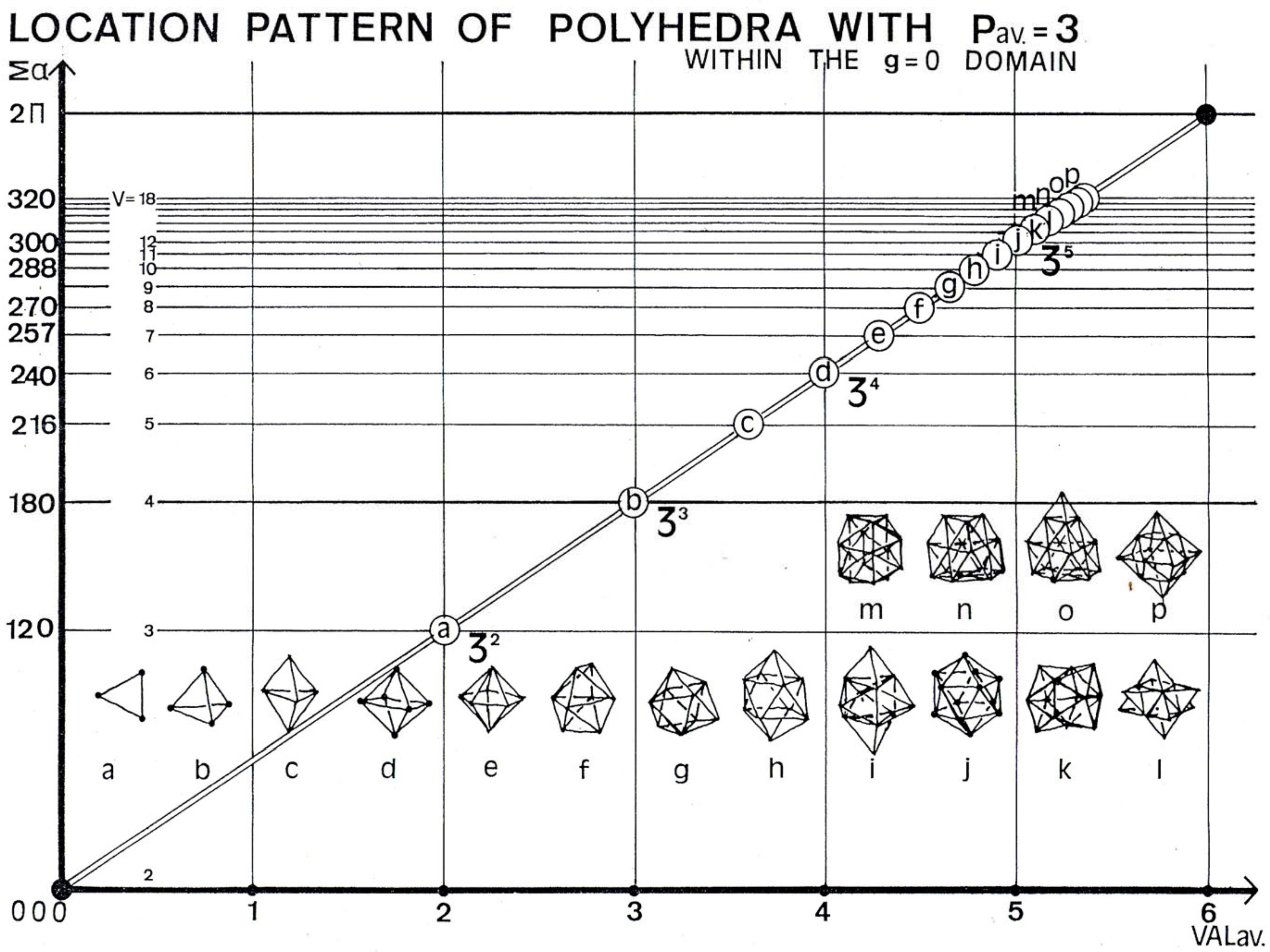


Fig. 34
Quadratic doubly curved surfaces as location patterns of polyhedra sharing the same number of edges.

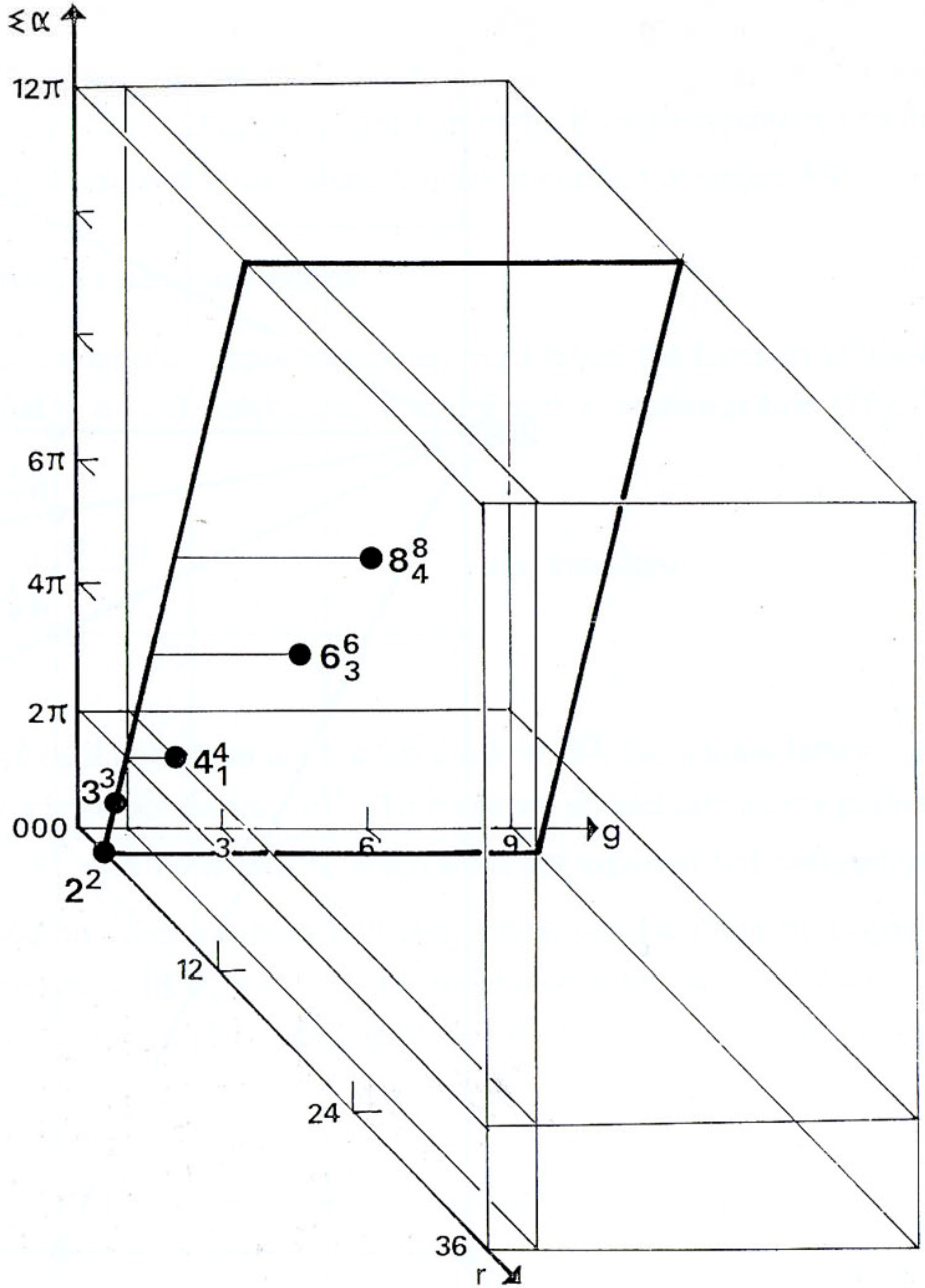


$$E = \frac{2\pi \cdot Val_{av.}(1-g)}{2\pi - \Sigma\alpha_{av.}}$$



LOCATION PATTERN OF POLYHEDRA WITH Pav. = 3
WITHIN THE g = 0 DOMAIN
Σα
2Π
320
300
288
270
257
240
216
180
120
000
V= 18
12
11
10
9
8
7
6
5
4
3
2
1
2
3
4
5
6
VALav.
3²
3³
3⁴
3⁵
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p

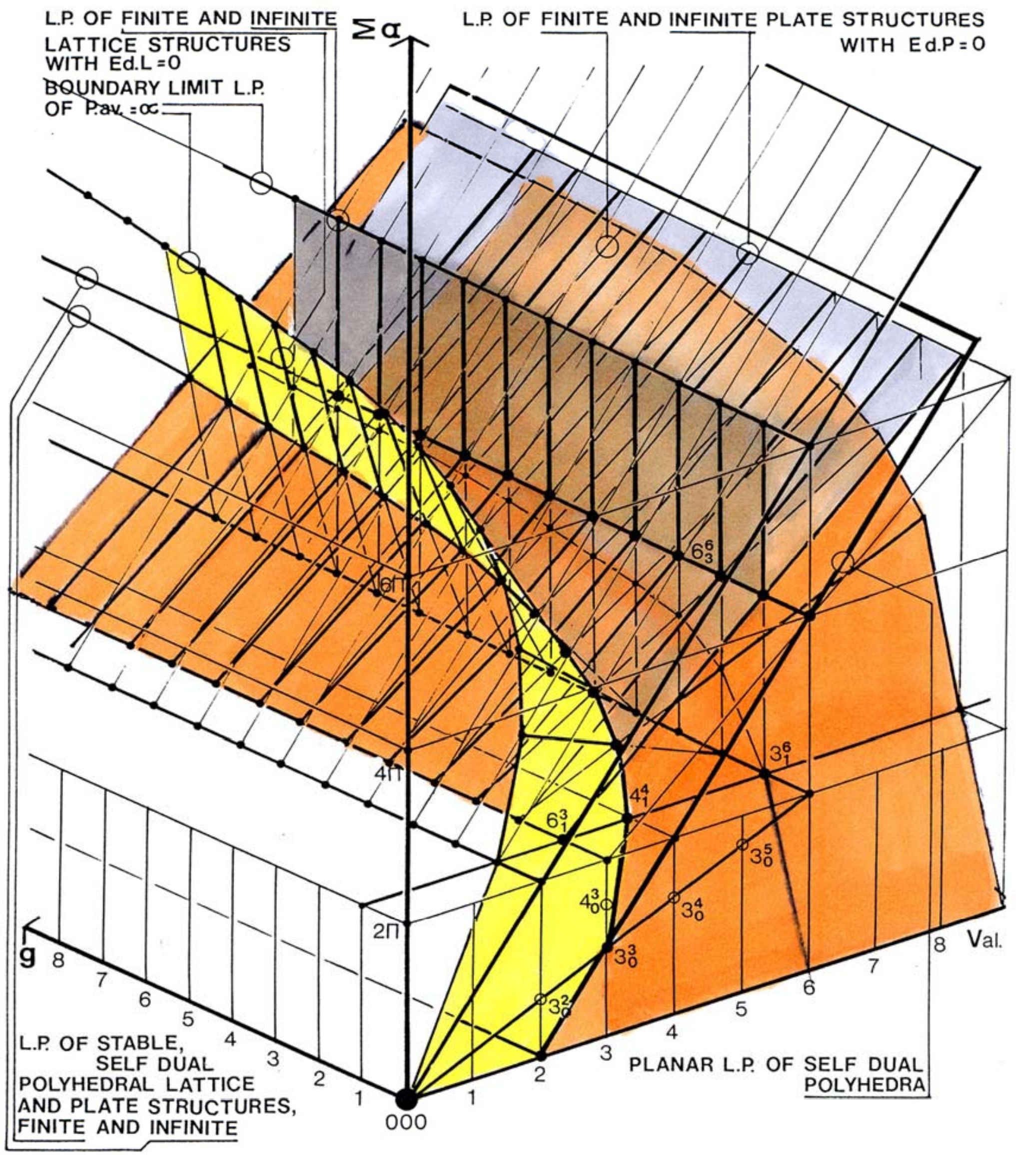
Plane location pattern
of self-dual polyhedra.

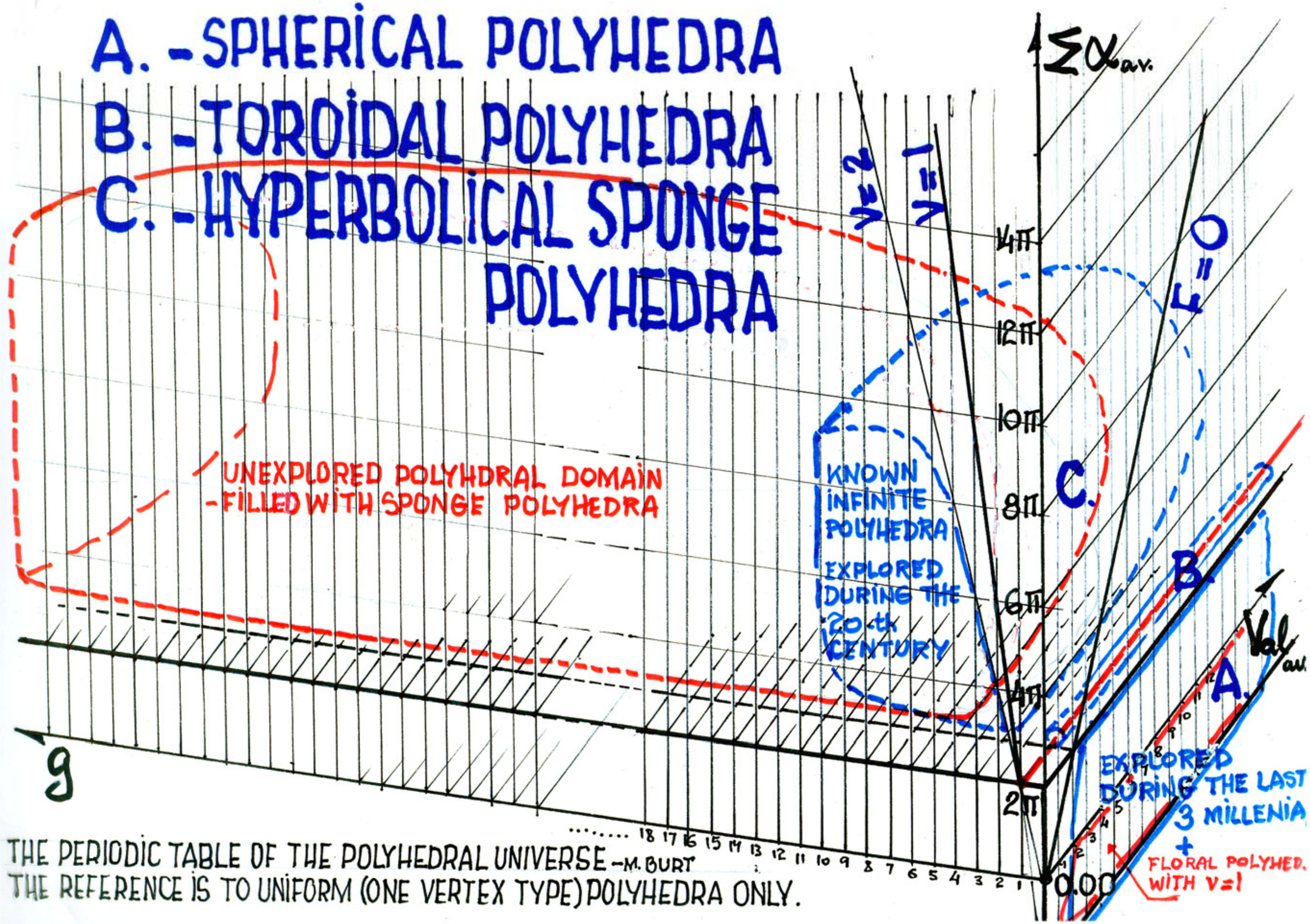


THE GRAND DIVIDES OF THE POLYHEDRAL UNIVERSE

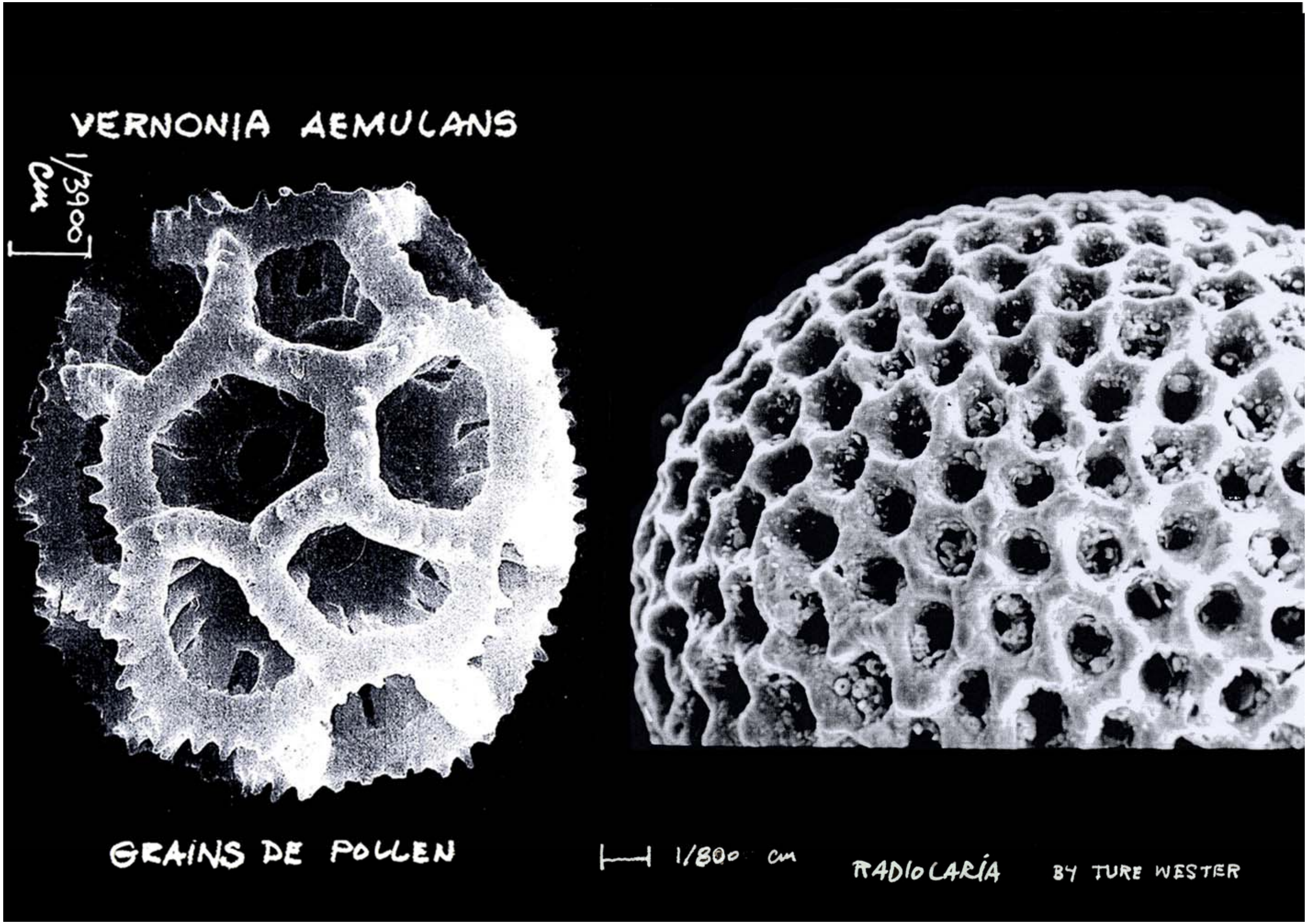
M. BURT

LOCATION PATTERN OF ALL POTENTIALLY STABLE LATTICE AND PLATE STRUCTURES





THE PERIODIC TABLE OF THE POLYHEDRAL UNIVERSE -M. BURT
THE REFERENCE IS TO UNIFORM (ONE VERTEX TYPE) POLYHEDRA ONLY.


VERNONIA AEMULANS
1/3900 cm
GRAINS DE POLLEN
1/800 cm
RADIOLARIA
BY TURE WESTER

T-1

Nature is saturated with sponge structures on every possible scale of physical-biological reality. The term was first adopted in biology: "Sponge: any member of the phylum Porifera, sessile aquatic animals, with single cavity in the body, with numerous pores. The fibrous skeleton of such an animal, remarkable for its power of sucking up water". (Wordsworth dictionary).
Of course the term applied to **'spherical sponges'. It turns out that the key characteristic of porosity is attributable to a much wider morphological phenomenon.**

$3.11.4^2.11^2.3^2.11^2.4^2.11.4^2.11^2.3^2.11^2.4^2.11.3.6.4^2.6^2.3^3.6^2.4^2.6_{601}$

$g_{(2.3.5)} = 601$

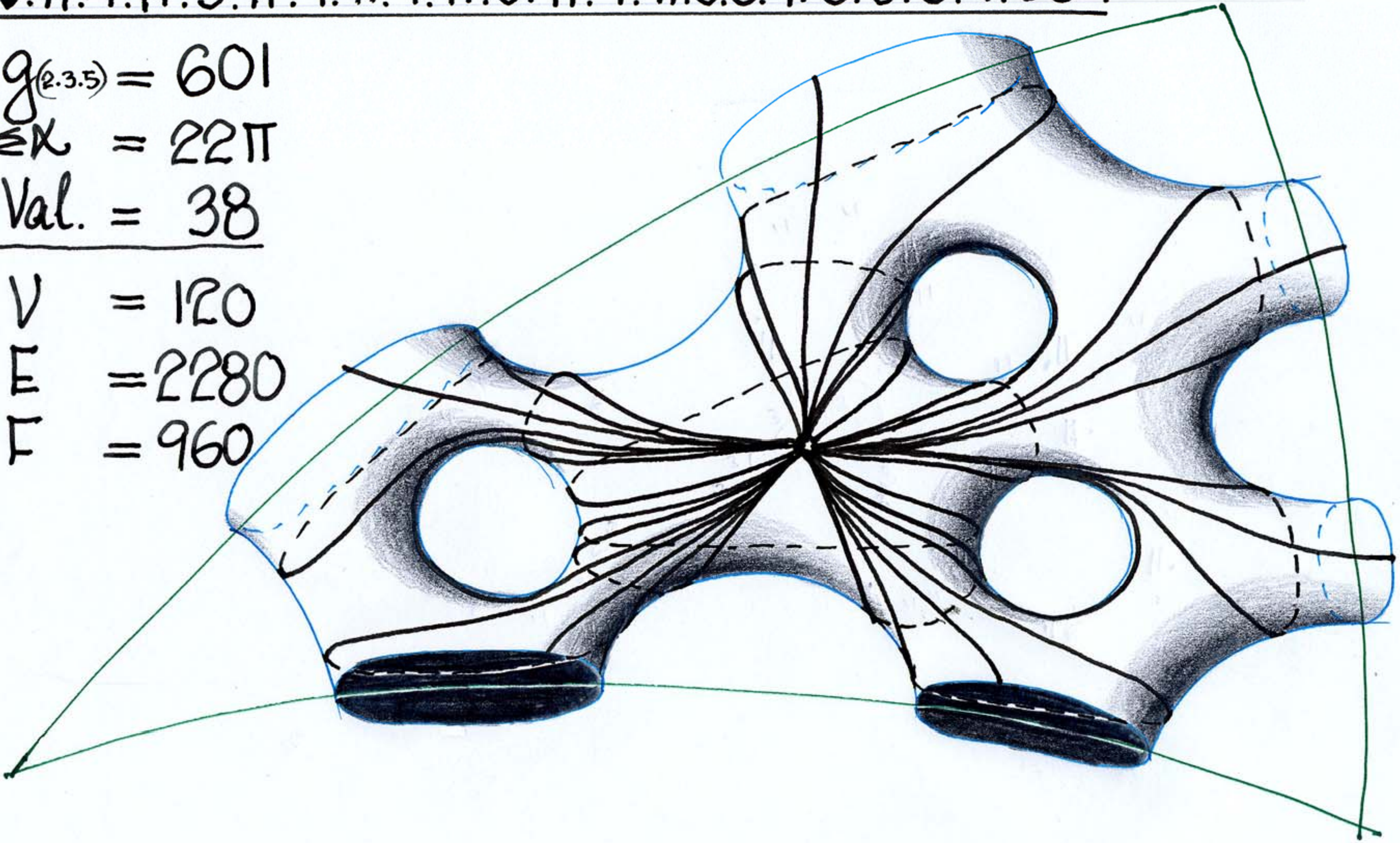
$\Sigma\alpha = 22\pi$

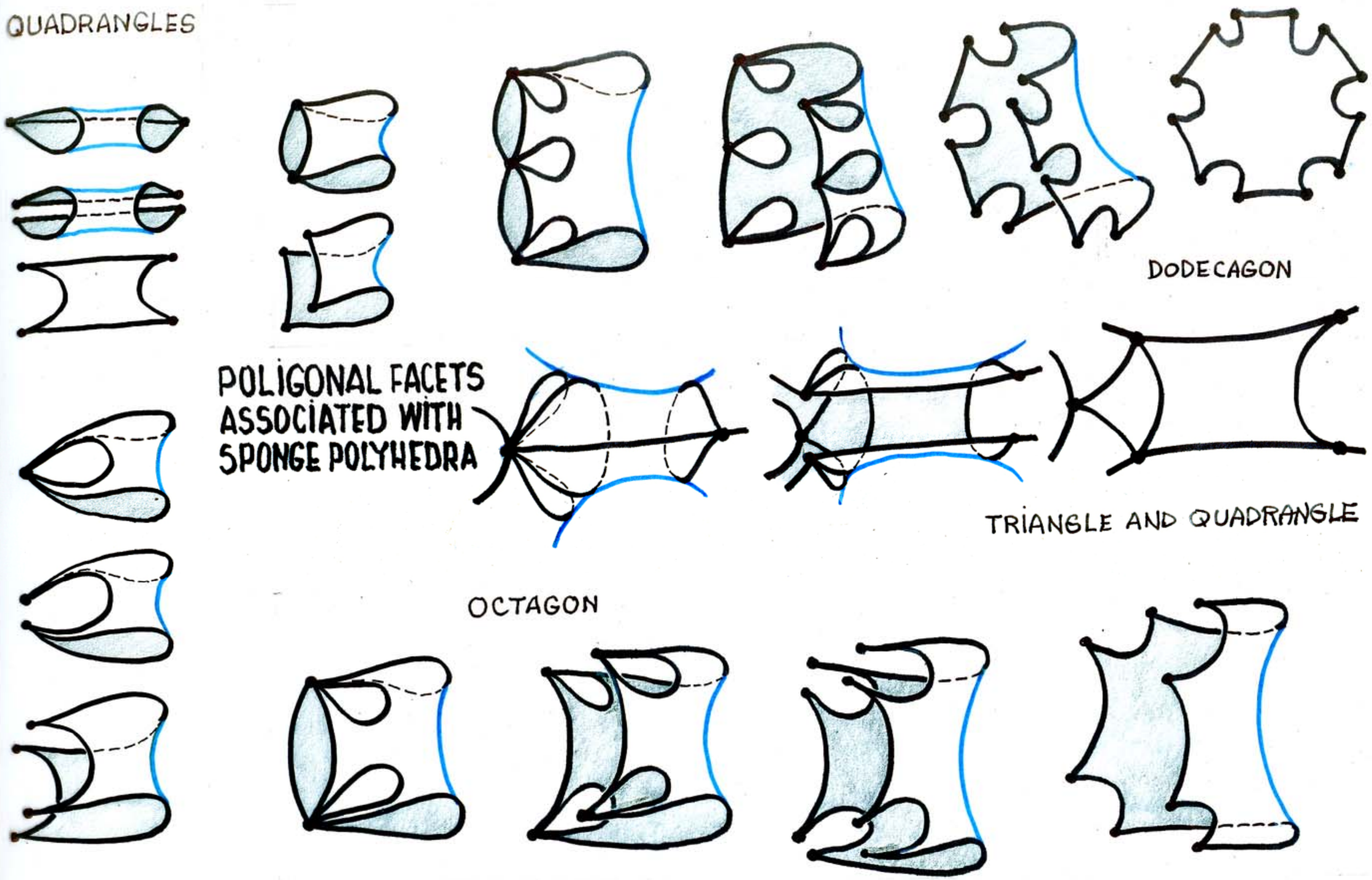
$Val. = 38$

$V = 120$

$E = 2280$

$F = 960$




QUADRANGLES
POLIGONAL FACETS ASSOCIATED WITH SPONGE POLYHEDRA
DODECAGON
TRIANGLE AND QUADRANGLE
OCTAGON

$3.4^2.3.4^2.3.8.6.4^2.6.3.4^2.3.4^2.3.6.8.13.8.6.3.4^2.3.4.3.6.4^2.6.8.3.4^2.3.4^2.3.8.4^2.8.13^2.11.13^2.3.4^2.3.4^2.3.13.4^4.13.11.4^4.11.4^4.11.19.4^4.19.3.4^2.3.4^2.3.19^2.11.19^2.4^{12}.19.3.4^4.3.4^4.3.$
$19^2.11.19^2.3.4^4.3.4^4.3.19.3.4^4.3.4^4.3.19.3.4^4.3.4^4.3.19^2.11.19^2.3.4^4.3.4^4.3.19.4^2.19.11.3.4^2.3.4^2.3.11.4^4.11.13.4^4.13.4^4.13^2.11.13^2.8.4^4.8$ 3481

$g_{(2.3.5)} = 3481$

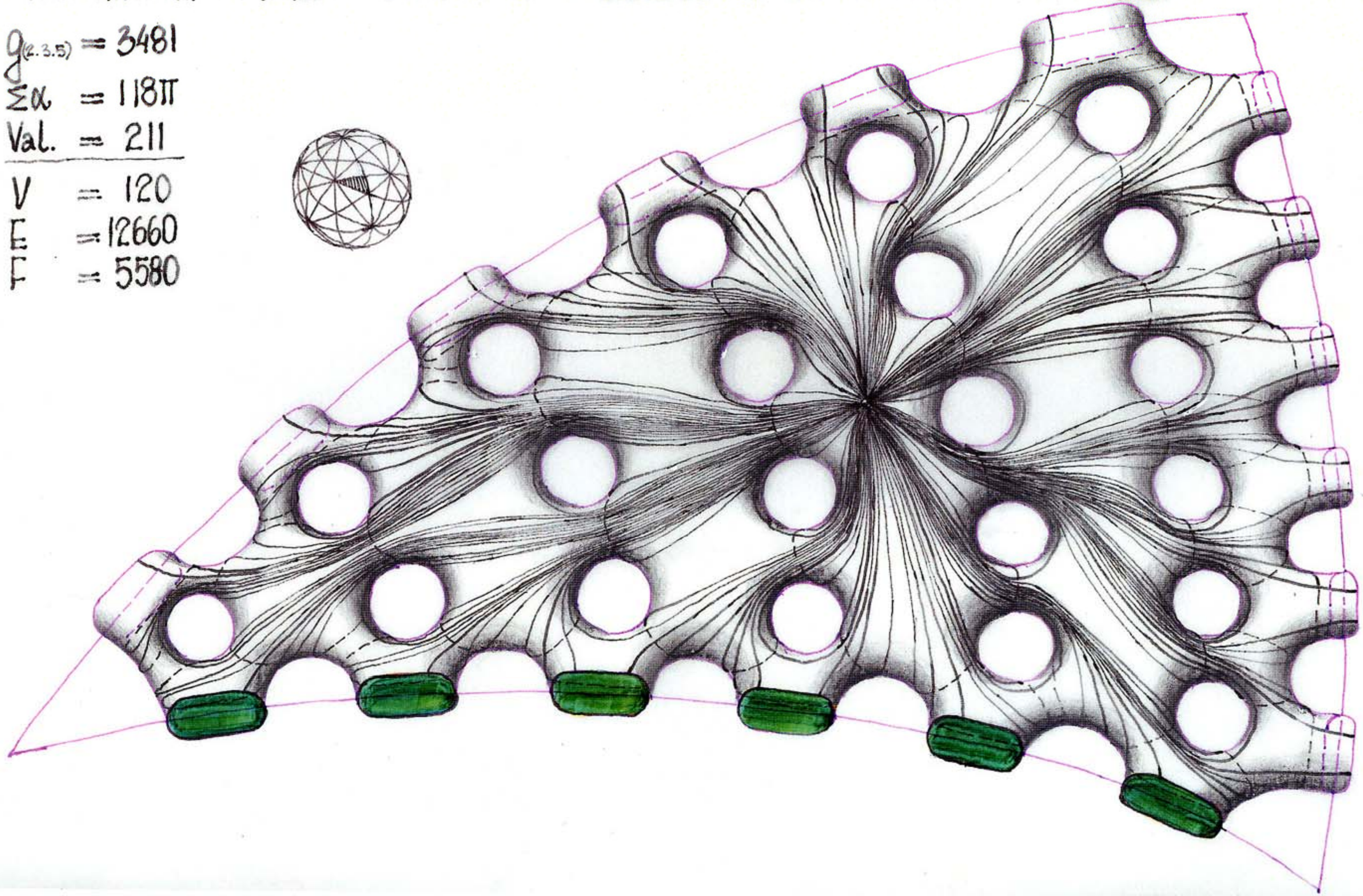
$\Sigma\alpha = 118\pi$

Val. = 211

$V = 120$

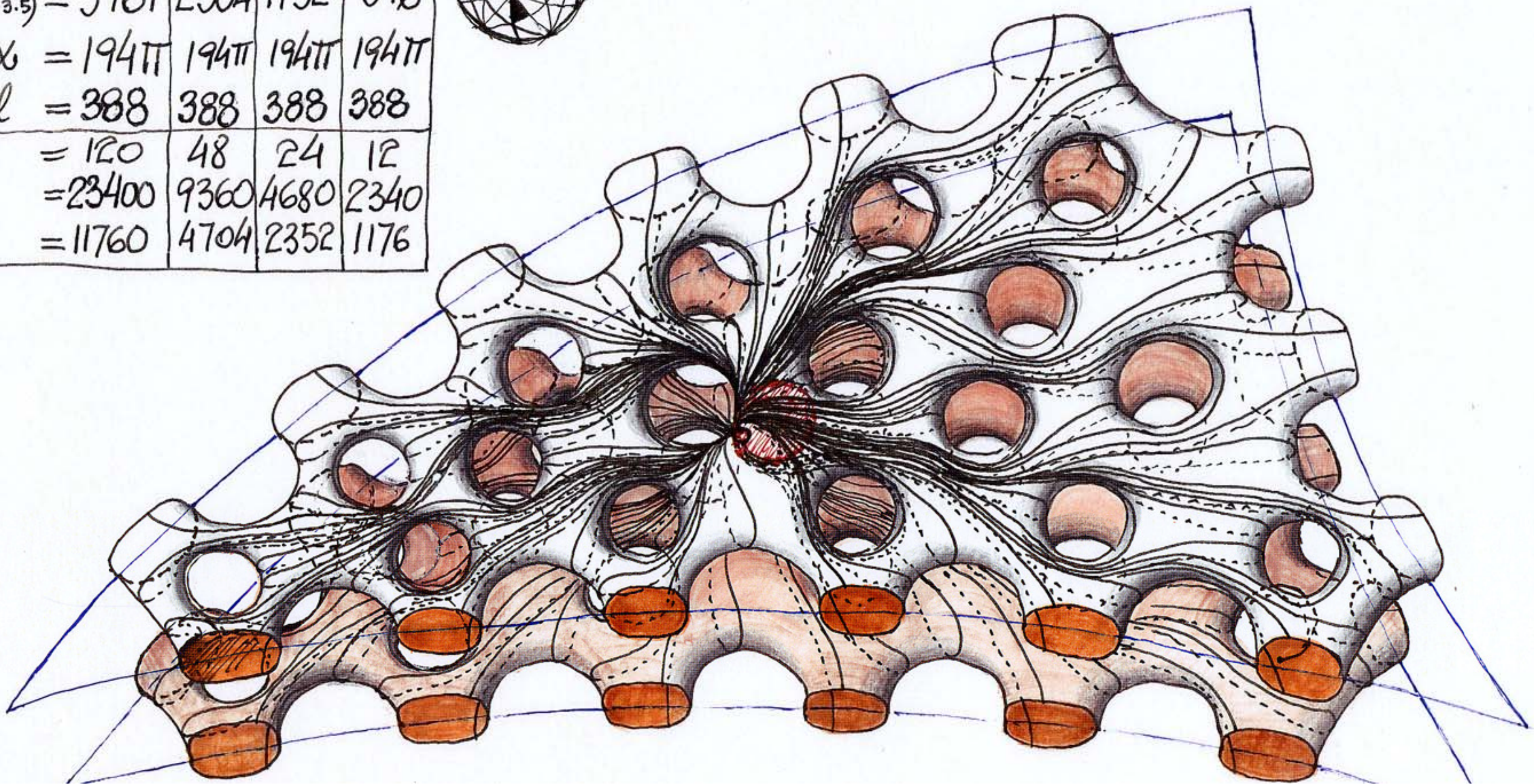
$E = 12660$

$F = 5580$



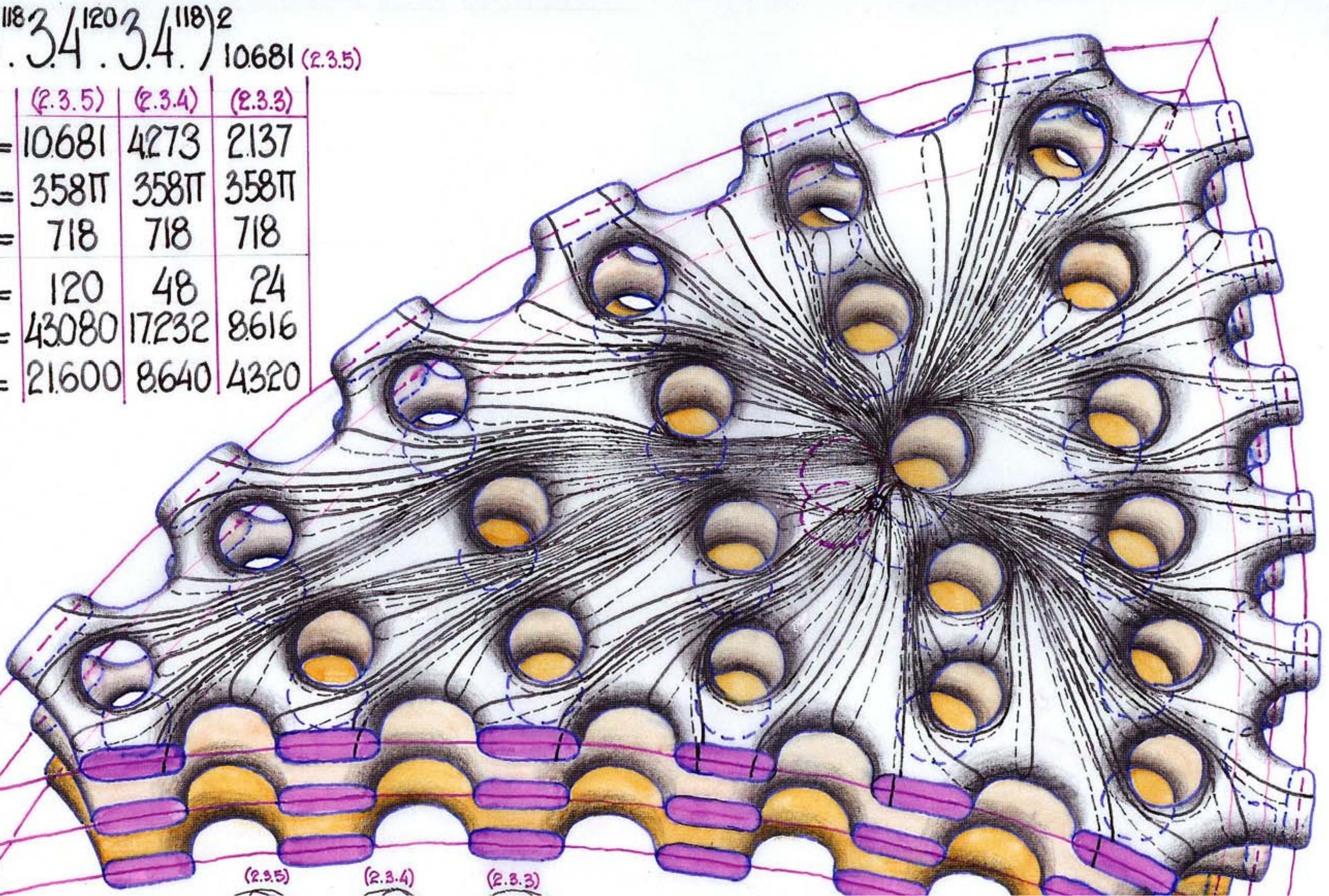
$\left(\begin{smallmatrix}194\\1\end{smallmatrix}\right)^2_{5761\,(2.3.5)}$

	(2.3.5)	(2.3.4)	(2.3.3)	(2.3.2)
(2.3.5)	= 5761	2304	1152	576
α	= 194π	194π	194π	194π
l	= 388	388	388	388
	= 120	48	24	12
	= 23400	9360	4680	2340
	= 11760	4704	2352	1176



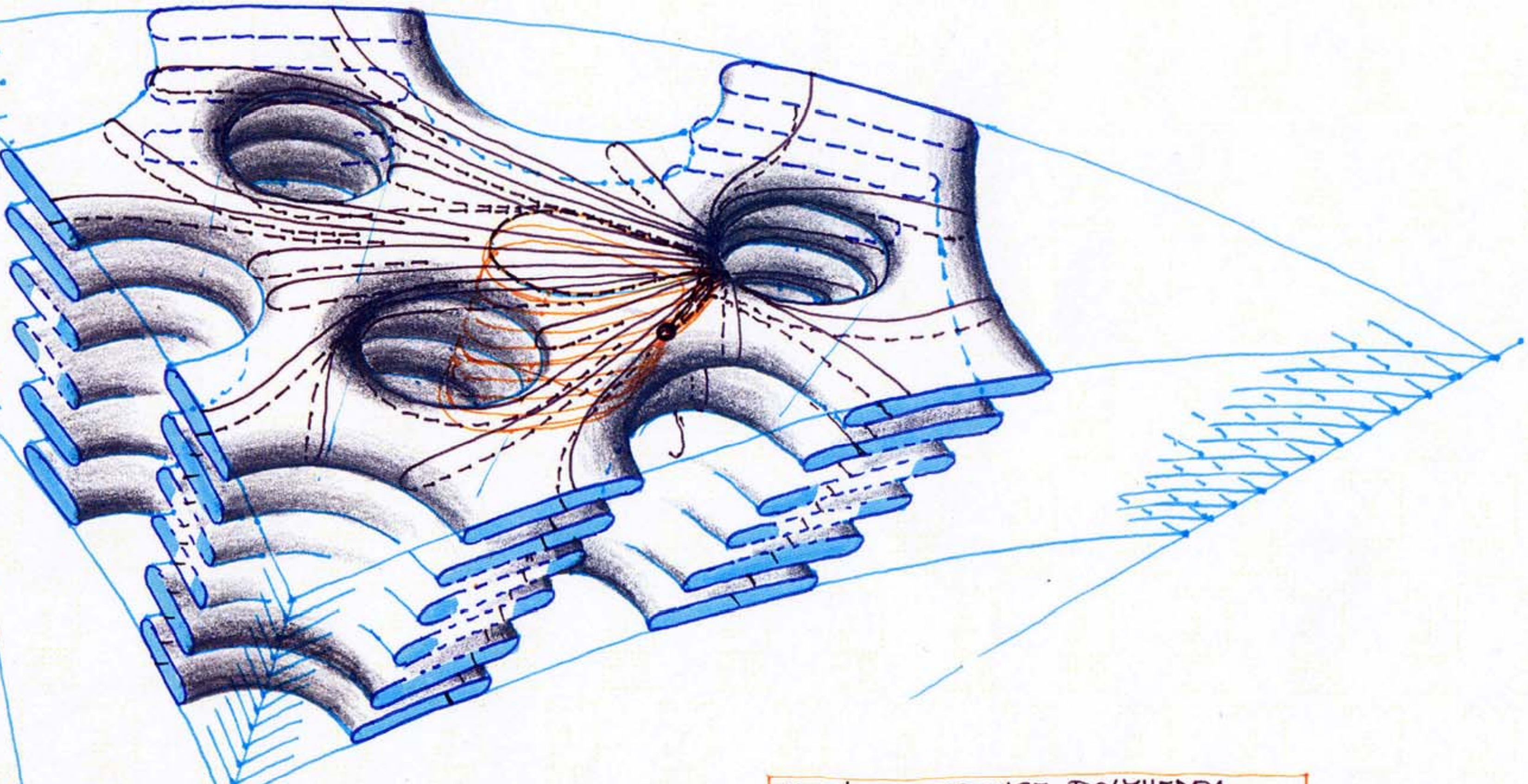
$\ldots^{118}.\,3.4^{120}.\,3.4^{118}.\,)^{2}_{10681}$ (2.3.5)

	(2.3.5)	(2.3.4)	(2.3.3)
=	10.681	4.273	2.137
=	358π	358π	358π
=	718	718	718
=	120	48	24
=	43.080	17.232	8.616
=	21.600	8.640	4.320

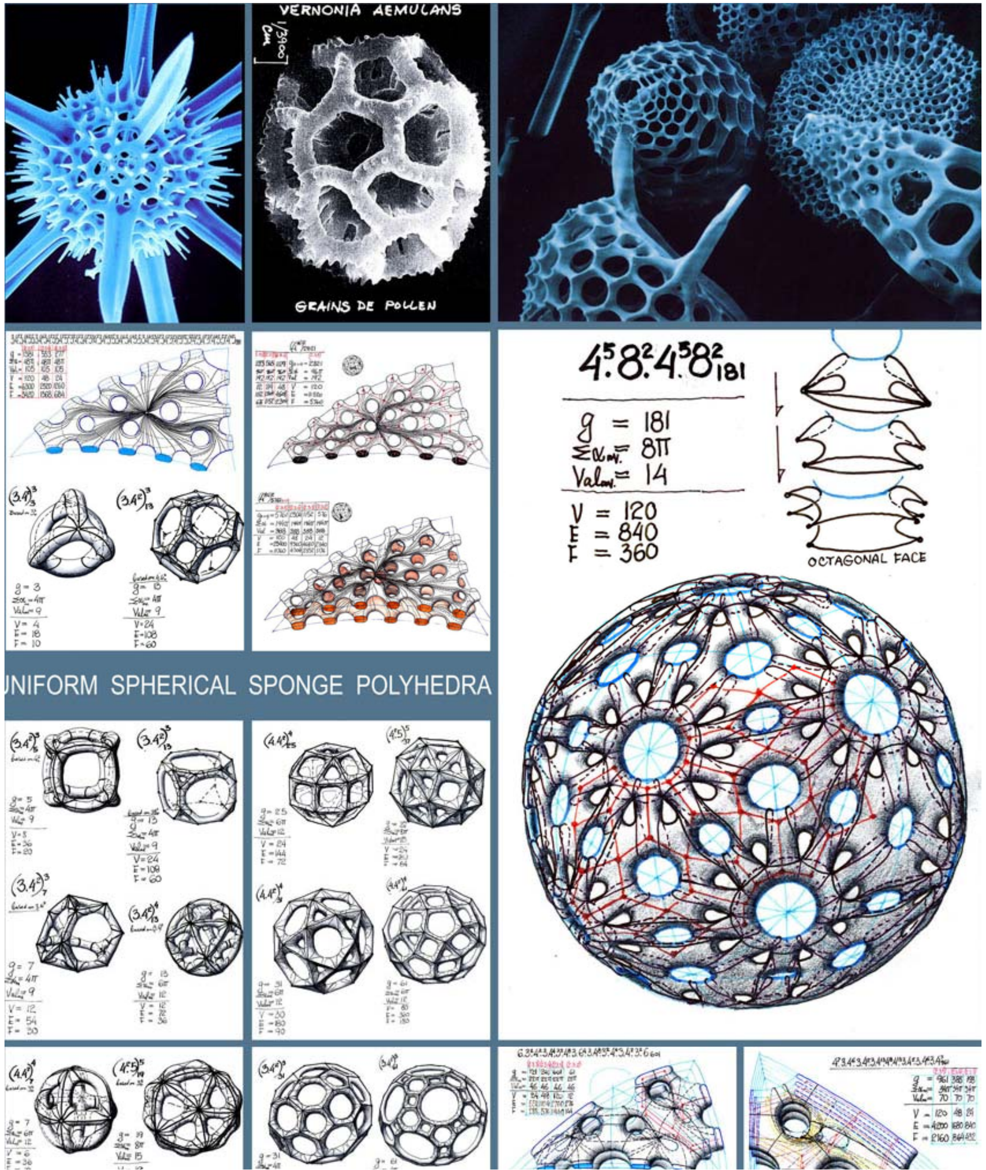


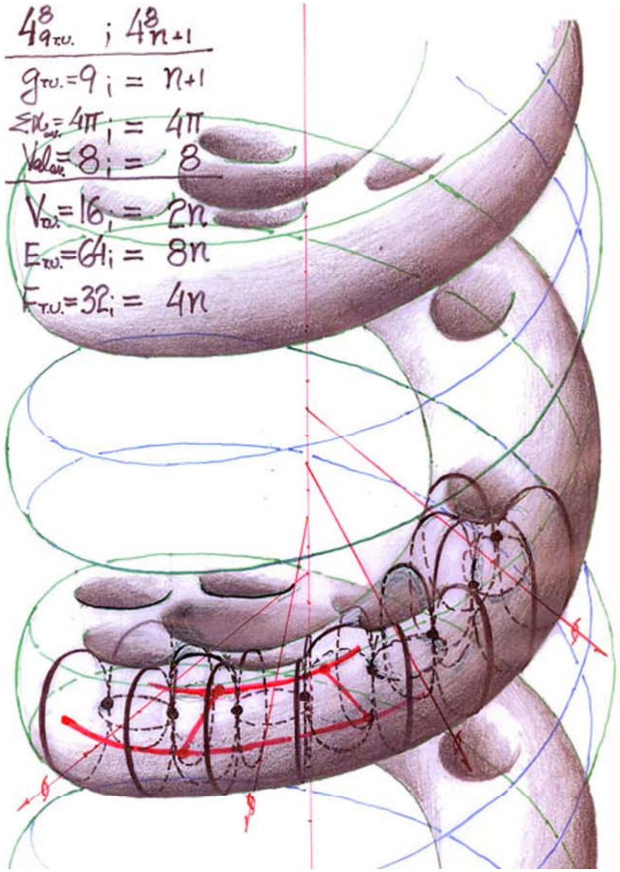
$4^{11}.3(2.4^{12}.4^{12})^{n-1}.2.3.4^{11}.4^{11})^{2}_{1321+720(n-1)}$;(2.3.5)

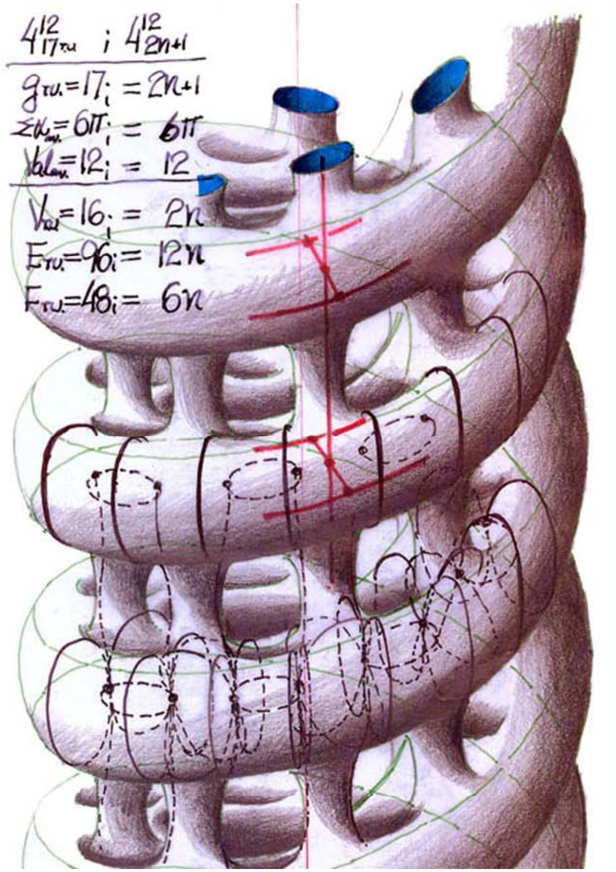
$n = 1 \ldots \infty$

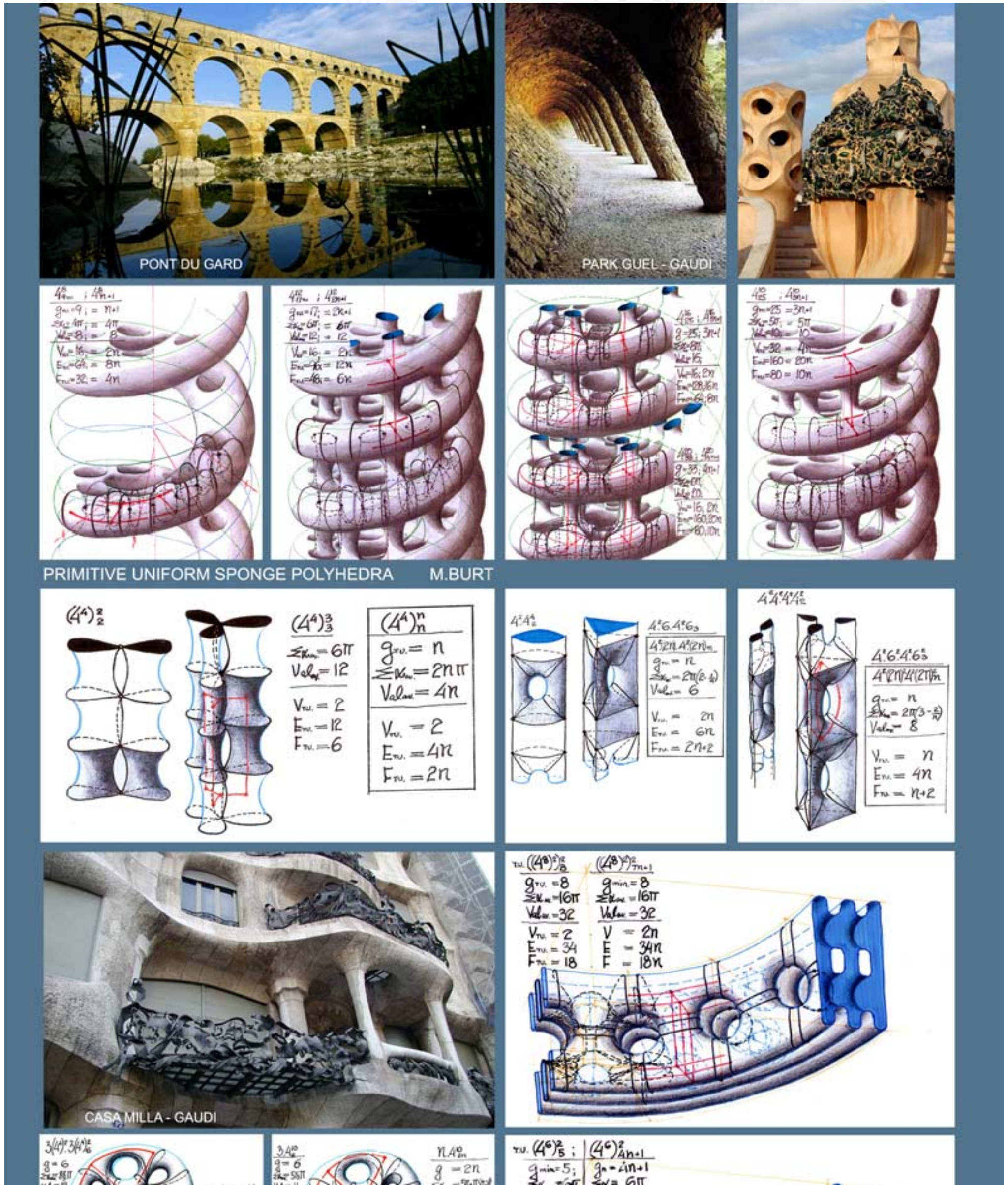


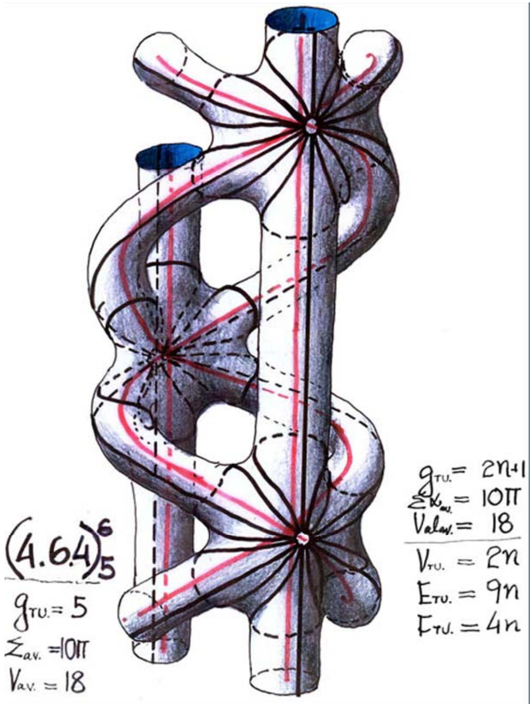
					TOROIDAL SPONGE POLYHEDRA			
GR.	(2.3.5)	(2.3.4)	(2.3.3)	(2.3.2)	$(2.3.6)_{TU.}$	$(2.4.4)_{TU.}$	$(3.3.3)_{TU.}$	2.2.m
	1321+720(n-1) $46\pi + 24\pi(n-1)$	529+288(n-1) →	265+144(n-1) →	133+72(n-1) →	133+72(n-1) →	89+48(n-1) →	67+36(n-1) →	(22+12(n-1))m+1 →

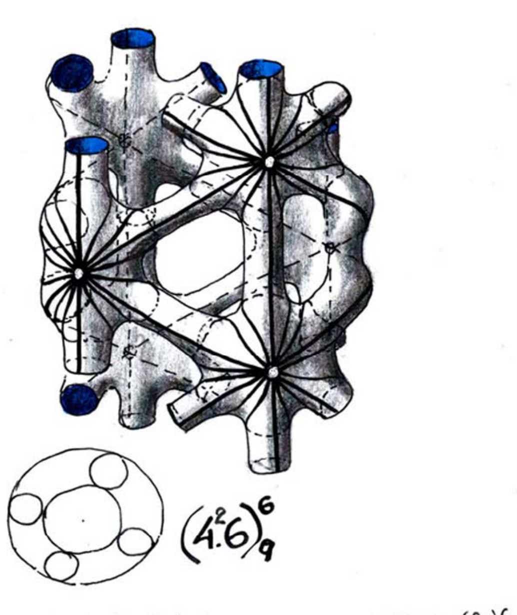

VERNONIA AEMULANS
GRAINS DE POLLEN
$4^5.8^2.4^5.8^2_{181}$
g = 181
V = 120
E = 840
F = 360
OCTAGONAL FACE
UNIFORM SPHERICAL SPONGE POLYHEDRA


4 8 9 T.U. ; 4 8 n+1
g T.U. = 9 ; = n+1
Σ = 4π ; = 4π
Valen. = 8 ; = 8
V T.U. = 16 ; = 2n
E T.U. = 64 ; = 8n
F T.U. = 32 ; = 4n


4 12 17 T.U. ; 4 12 2n+1
g T.U. = 17 ; = 2n+1
Σ = 6π ; = 6π
Valen. = 12 ; = 12
V T.U. = 16 ; = 2n
E T.U. = 96 ; = 12n
F T.U. = 48 ; = 6n


PONT DU GARD
PARK GUEL - GAUDI
PRIMITIVE UNIFORM SPONGE POLYHEDRA
M.BURT
CASA MILLA - GAUDI





PERIODIC (INFINITE) SPONGE POLYHEDRON – $(4^2.6)^6_9$
OF THE POLYHEDRAL FAMILY: $(4^2.6)^6_{g=2n+1}$

$n=2 \quad (4.^{2}5)^{5}_{19}$

$g = 19 = 9n+1$

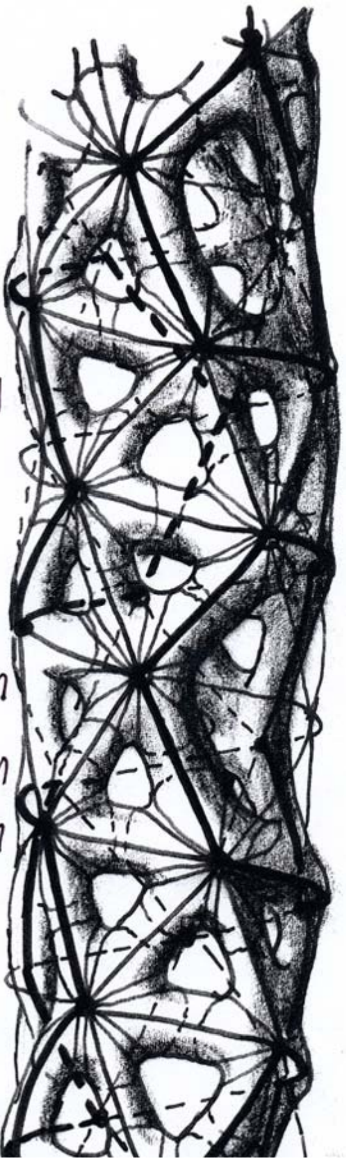
$\Sigma\alpha_{av.} = 8\pi$

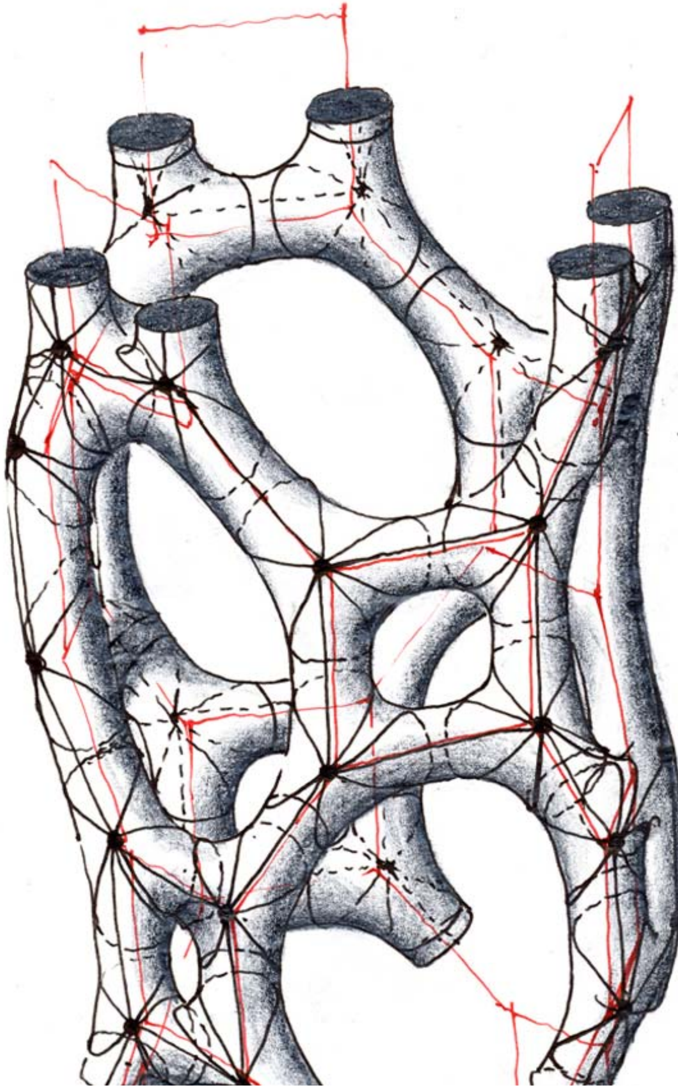
$Val_{av.} = 15$

$V_{T.U.} = 6 \times 2 = 6n$

$E_{T.U.} = 45 \times 2 = 45n$

$F_{T.U.} = 21 \times 2 = 21n$





$(3.4^2)^3_7$

$g = 7$

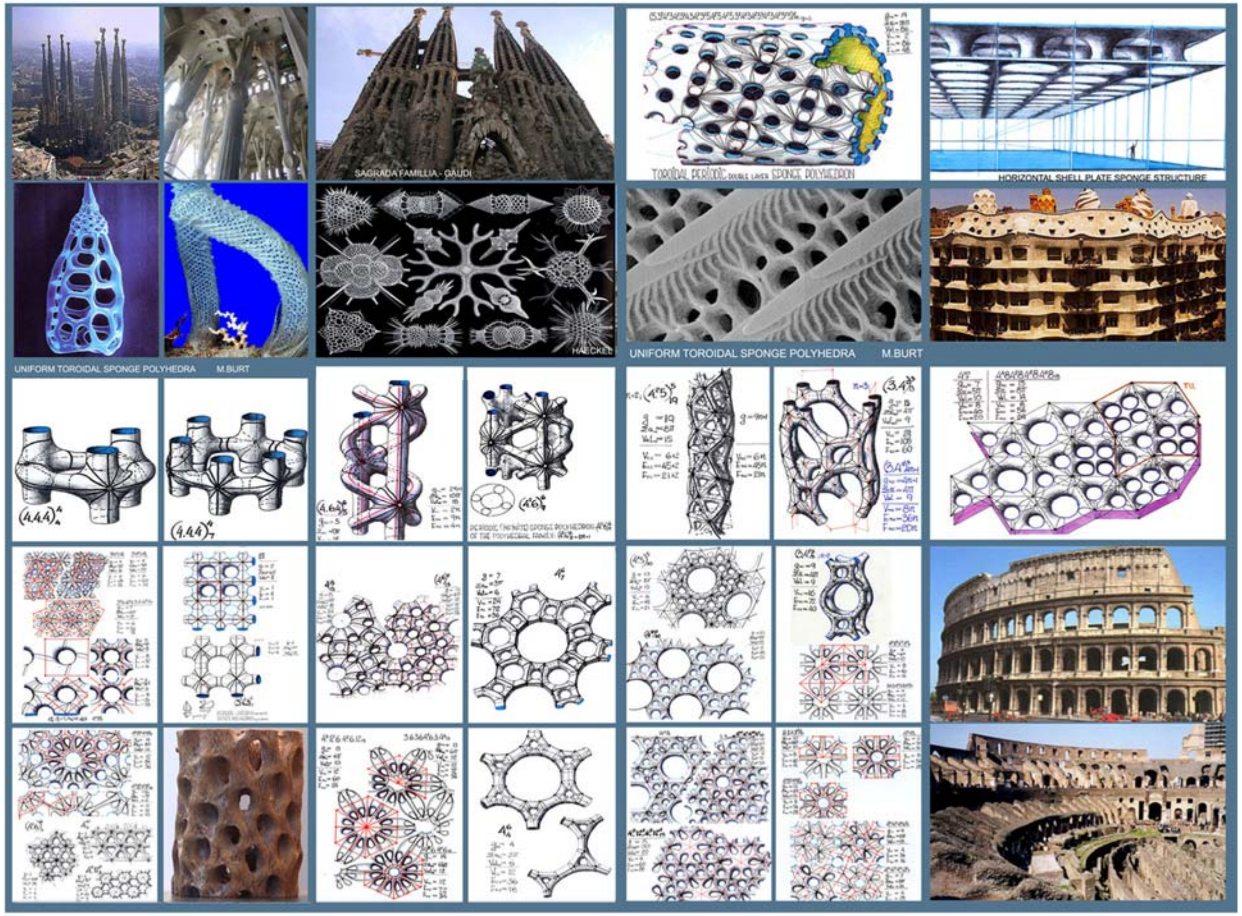
$\Sigma K_{av.} = 4\pi$

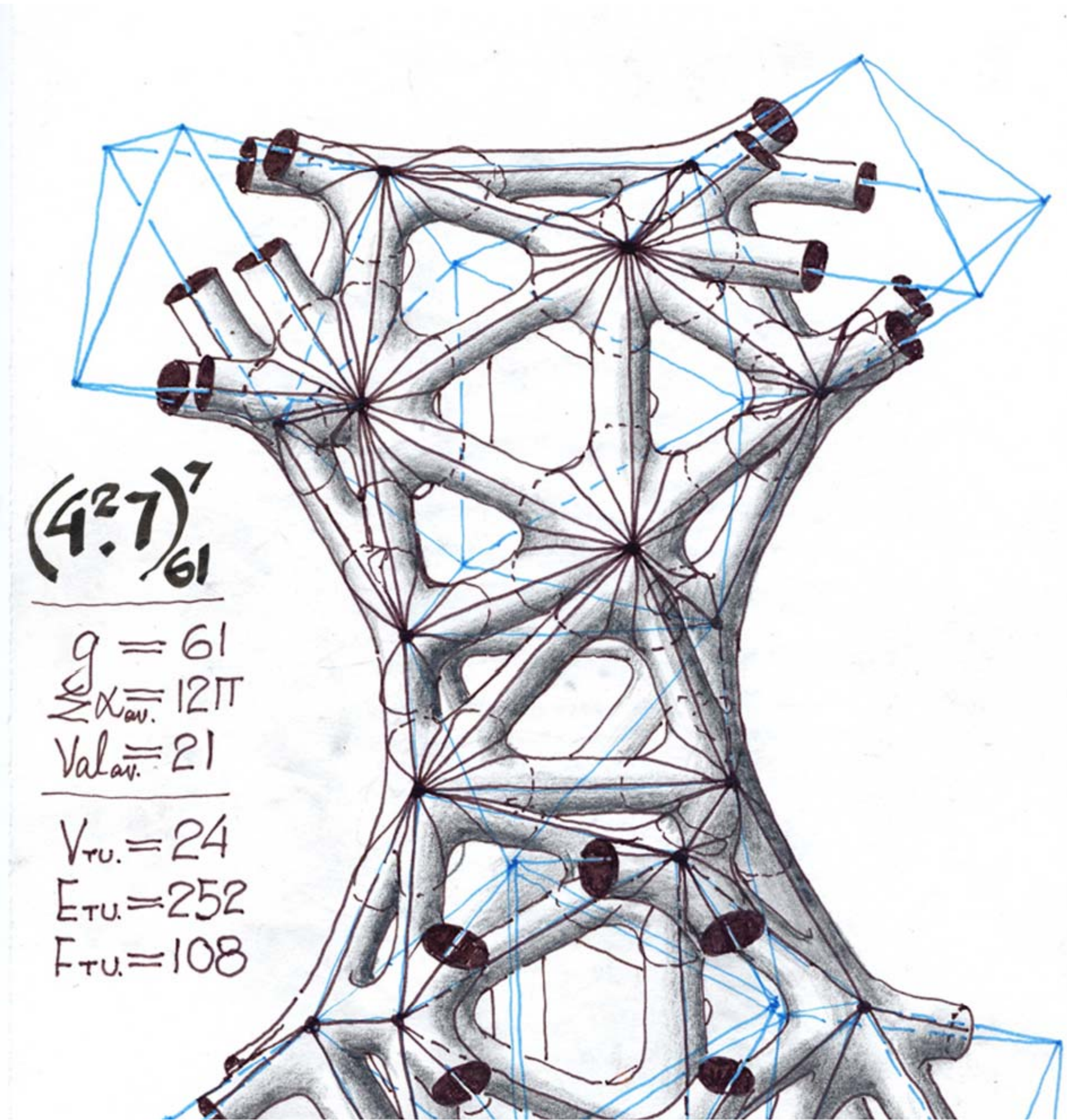
$Val_{av.} = 9$

$V_{T.U.} = 12$

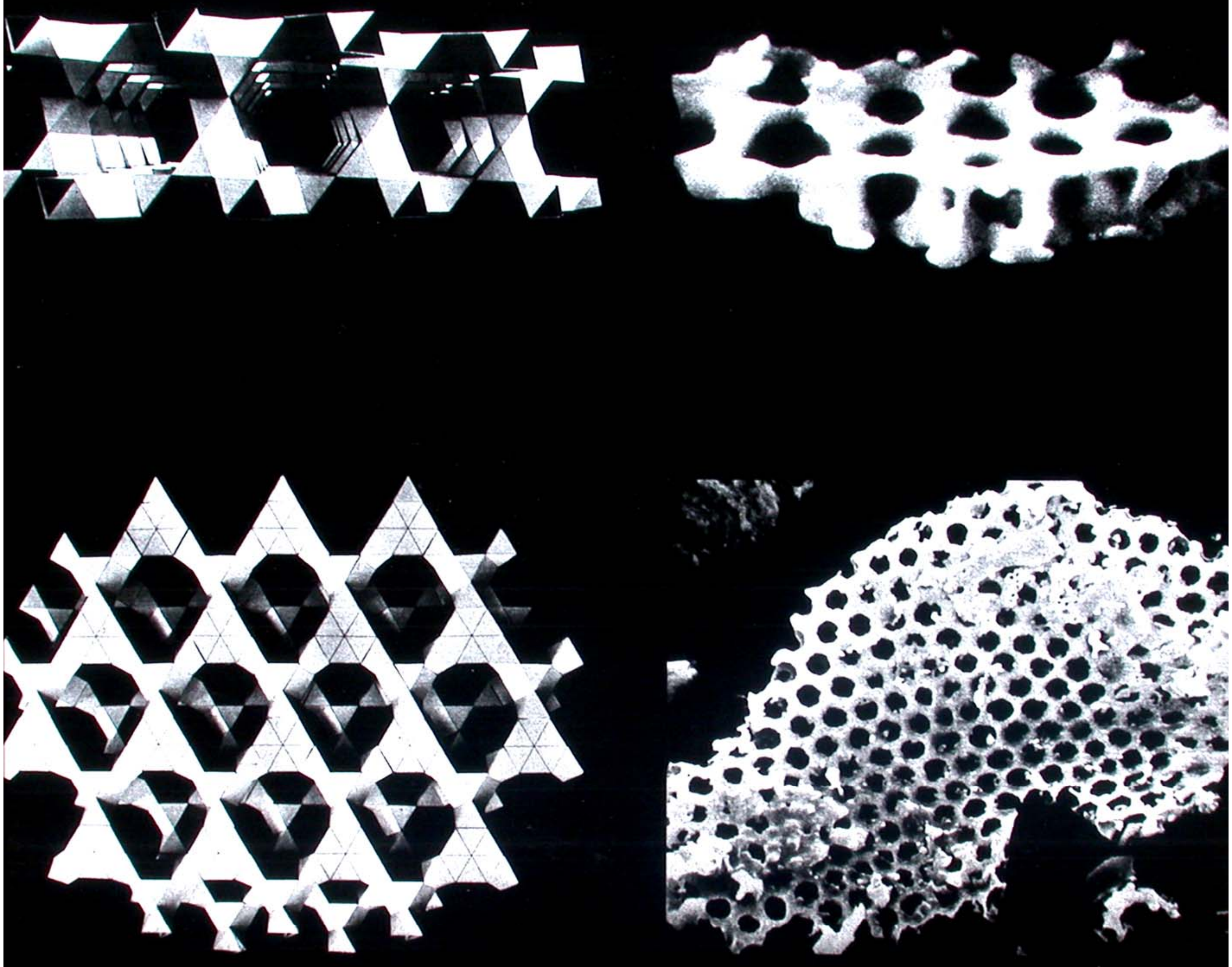
$E_{T.U.} = 54$

$F_{T.U.} = 30$


SAGRADA FAMILLIA - GAUDI
TOROIDAL PERIODIC DOUBLE LAYER SPONGE POLYHEDRON
HORIZONTAL SHELL PLATE SPONGE STRUCTURE
HAECKEL
UNIFORM TOROIDAL SPONGE POLYHEDRA M.BURT
UNIFORM TOROIDAL SPONGE POLYHEDRA M.BURT



$(4^2.7)^7_{61}$
$g = 61$
$\Sigma\alpha_{av.} = 12\pi$
$Val_{av.} = 21$
$V_{TU.} = 24$
$E_{TU.} = 252$
$F_{TU.} = 108$



$3^3.4.3.4_3$ - I.P.L PLATE

$$(4^6)^2_{97}$$

$g_{TU.} = 97$

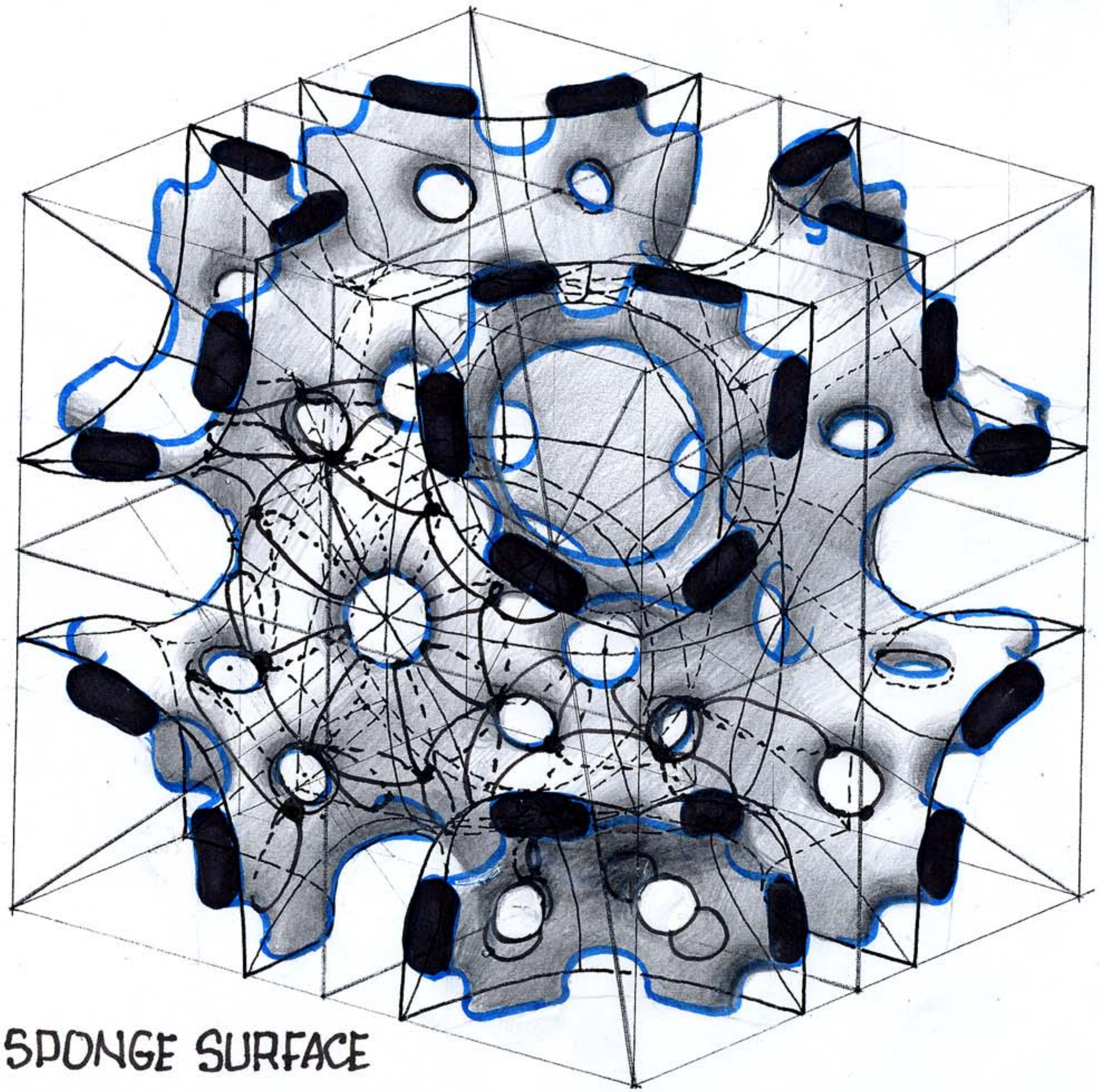
$\Sigma \delta_{av.} = 6\pi$

$Val_{av.} = 12$

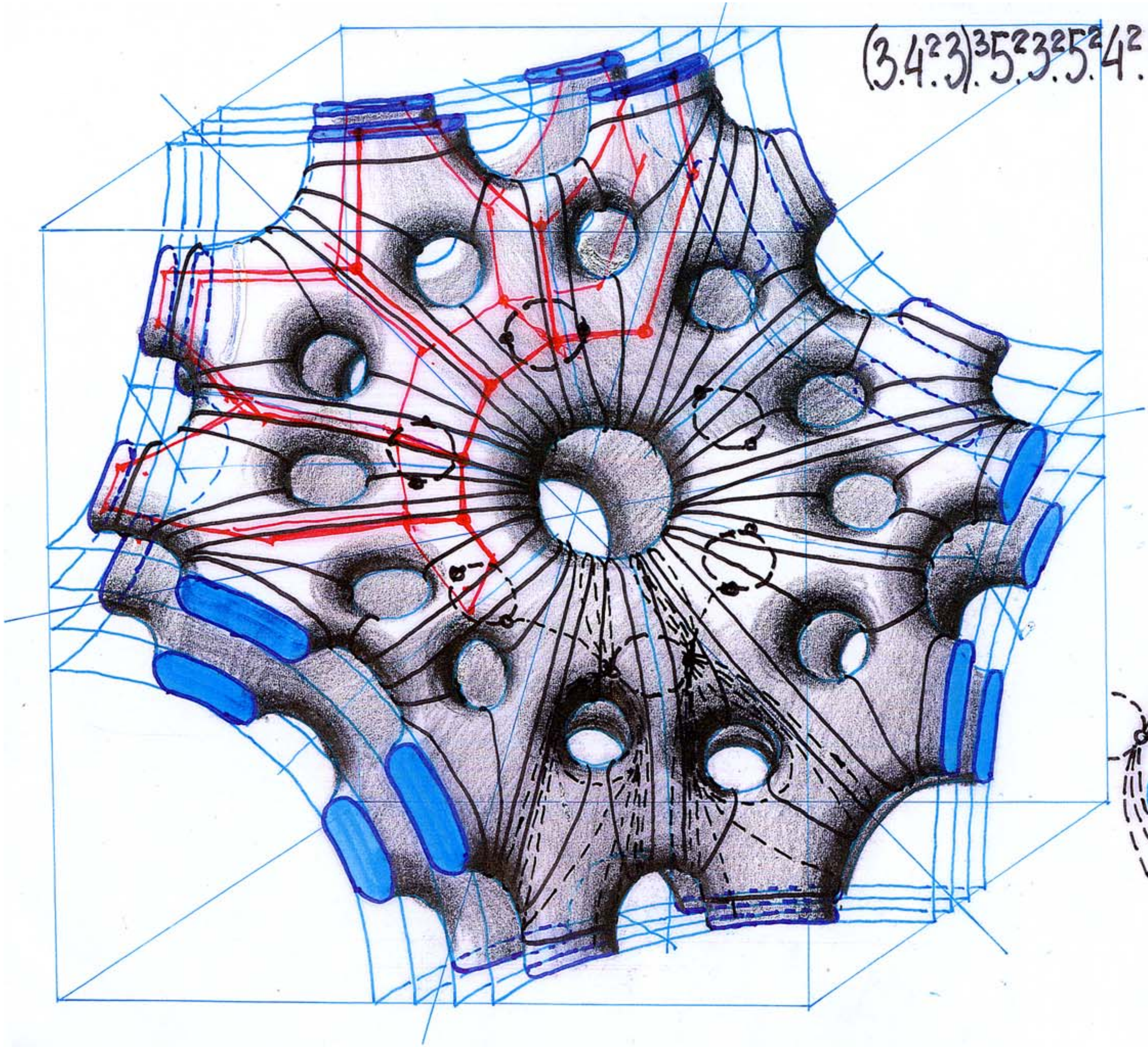
$V_{TU.} = 96$

$E_{TU.} = 576$

$F_{TU.} = 288$



HYPERBOLIC C.C SPONGE SURFACE



$$(3.4^2.3)^3.5^2.3^2.5^2.4^2.5.3.4^2.3.5.4^2.5^2.3^2.5^2_{337}$$

$g_{TU.} = 337$

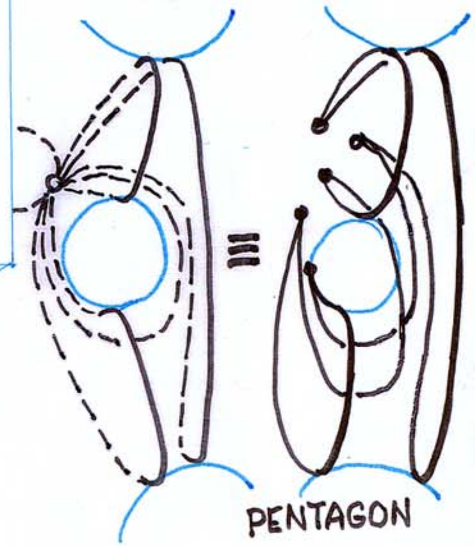
$\sum \alpha_{av.} = 16\pi$

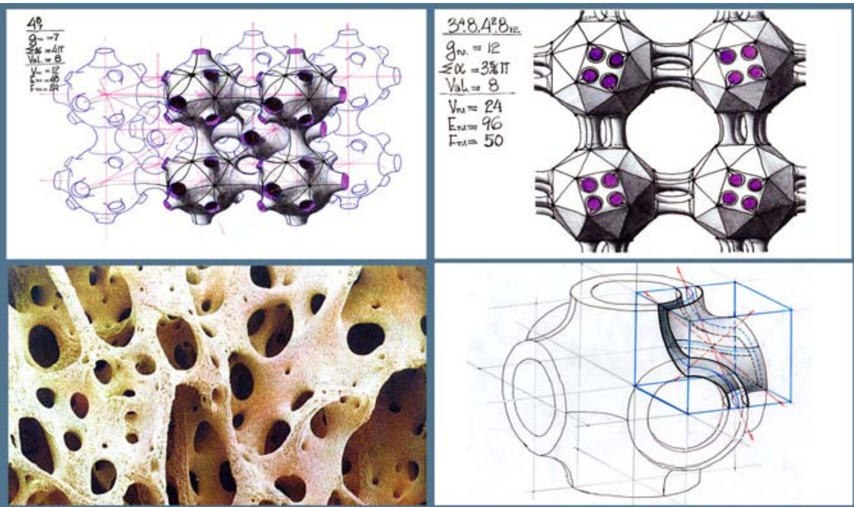
$Val_{av.} = 34$

$V_{TU.} = 96$

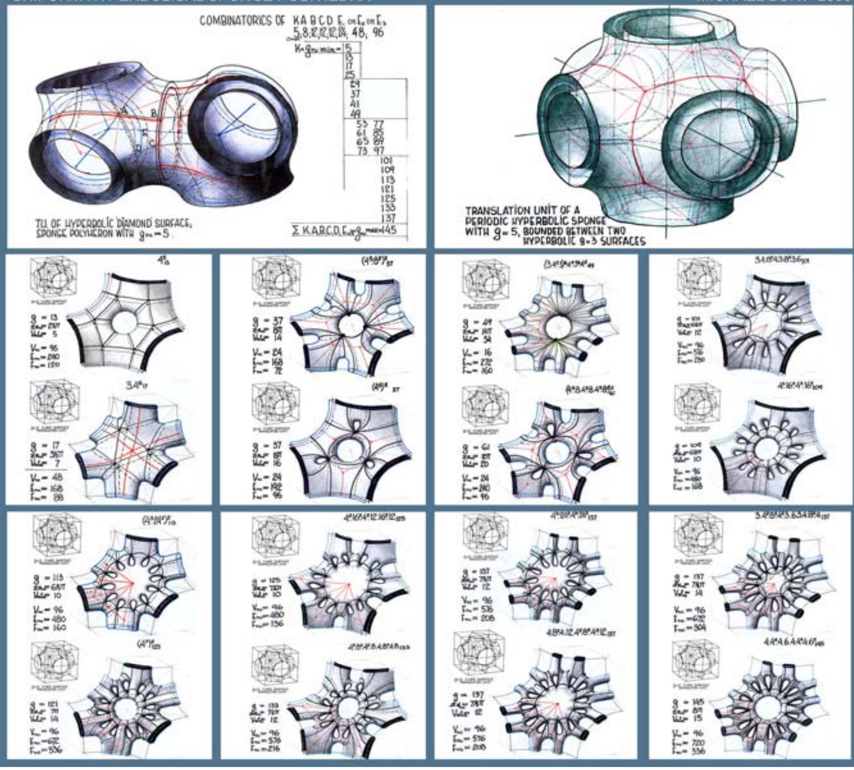
$E_{TU.} = 1632$

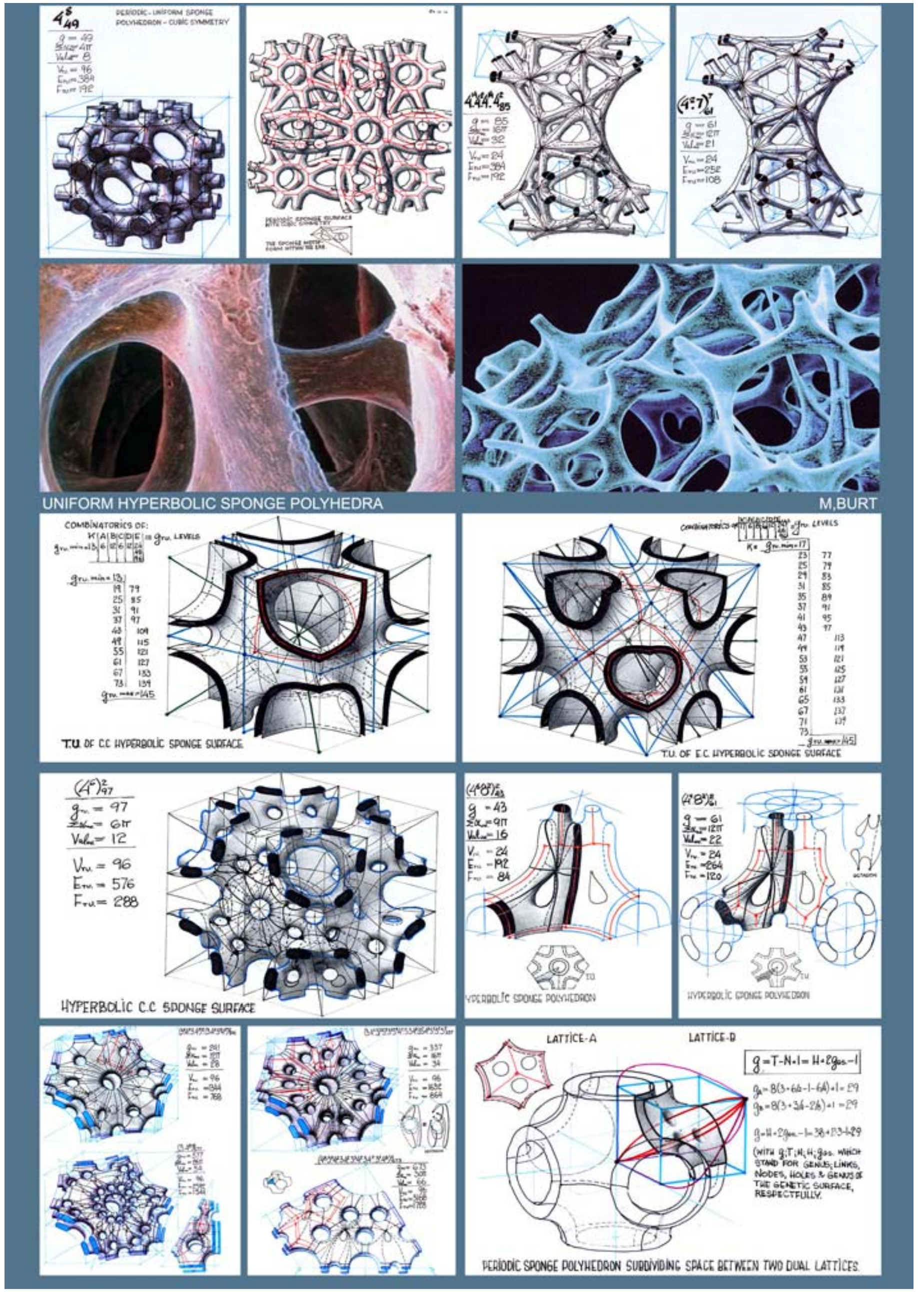
$F_{TU.} = 864$



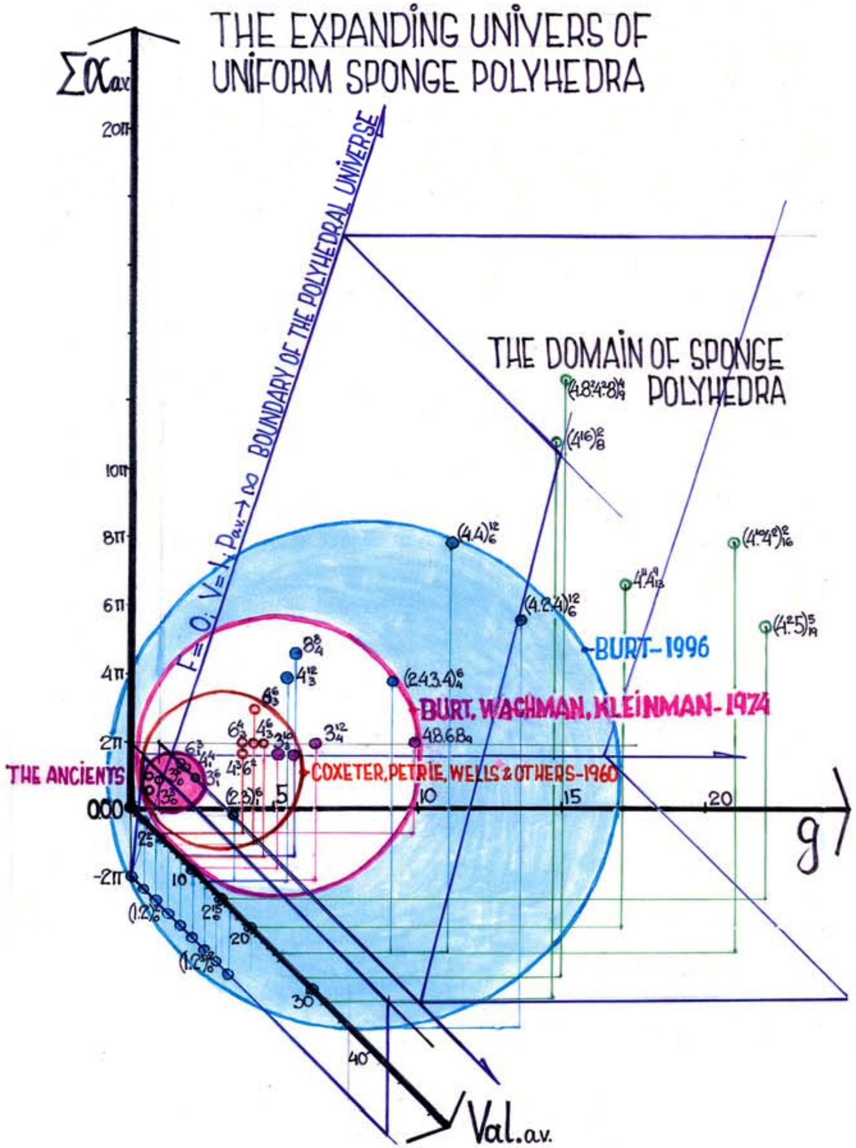


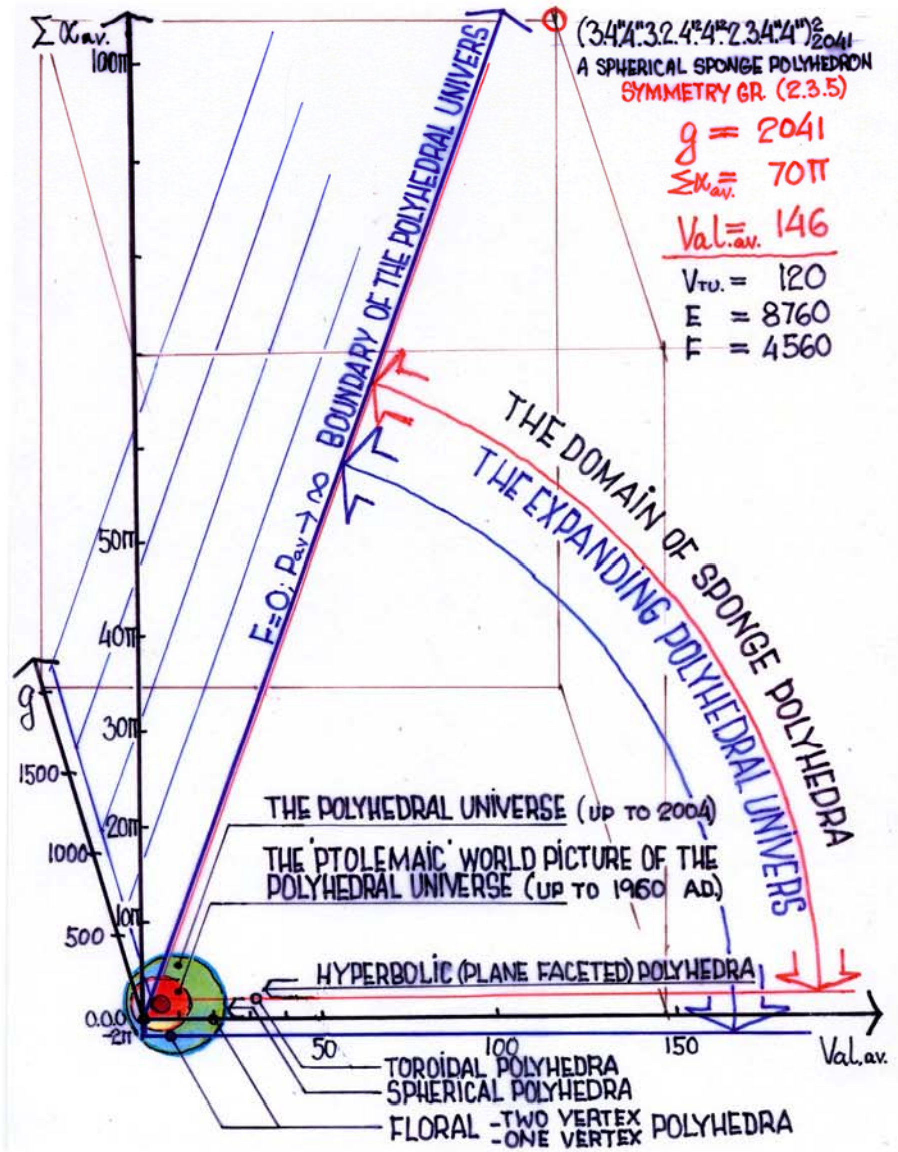
UNIFORM HYPERBOLICAL SPONGE POLYHEDRA MICHAEL BURT 2008

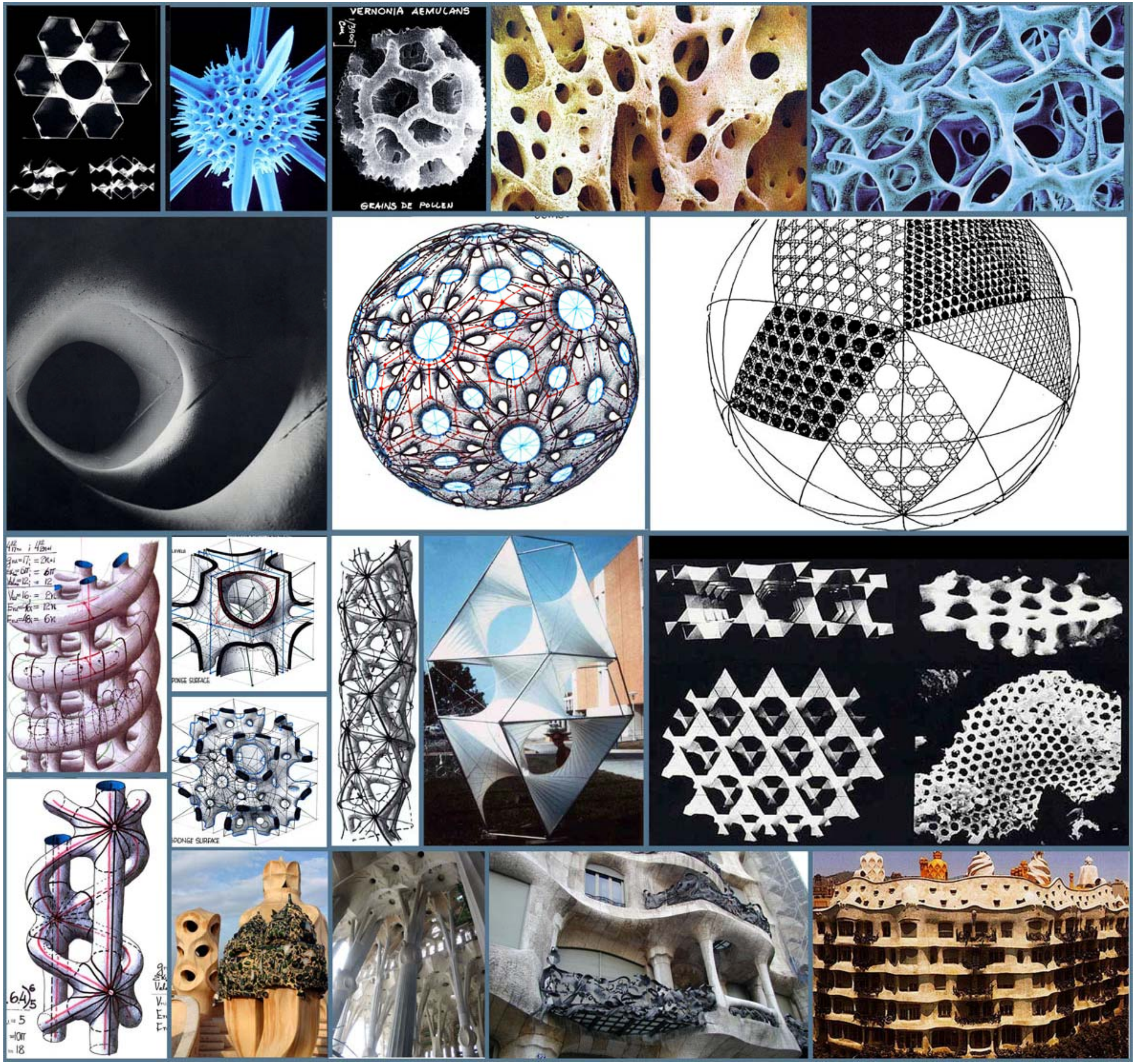


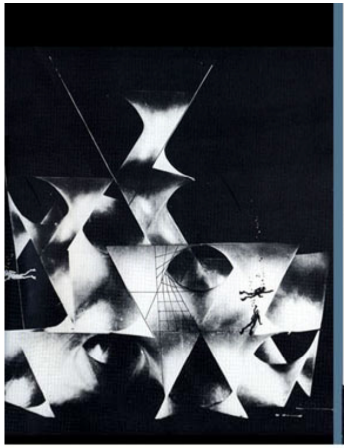


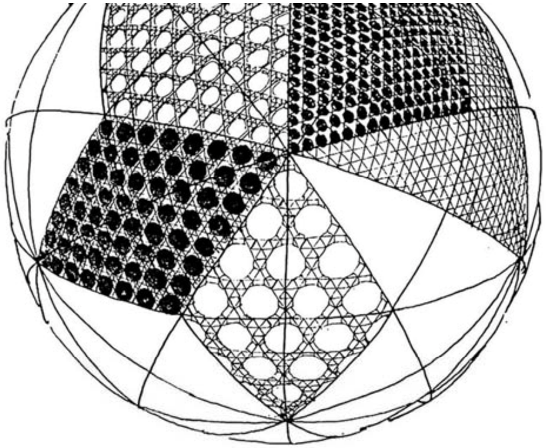
PERIODIC - UNIFORM SPONGE POLYHEDRON - CUBIC SYMMETRY
UNIFORM HYPERBOLIC SPONGE POLYHEDRA
M,BURT
COMBINATORICS OF:
T.U. OF C.C. HYPERBOLIC SPONGE SURFACE
T.U. OF E.C. HYPERBOLIC SPONGE SURFACE
HYPERBOLIC C.C. SPONGE SURFACE
HYPERBOLIC SPONGE POLYHEDRON
LATTICE-A
LATTICE-B
(WITH g;T;N;H;g WHICH STAND FOR GENUS; LINKS, NODES, HOLES & GENUS OF THE GENETIC SURFACE, RESPECTFULLY.
PERIODIC SPONGE POLYHEDRON SUBDIVIDING SPACE BETWEEN TWO DUAL LATTICES.

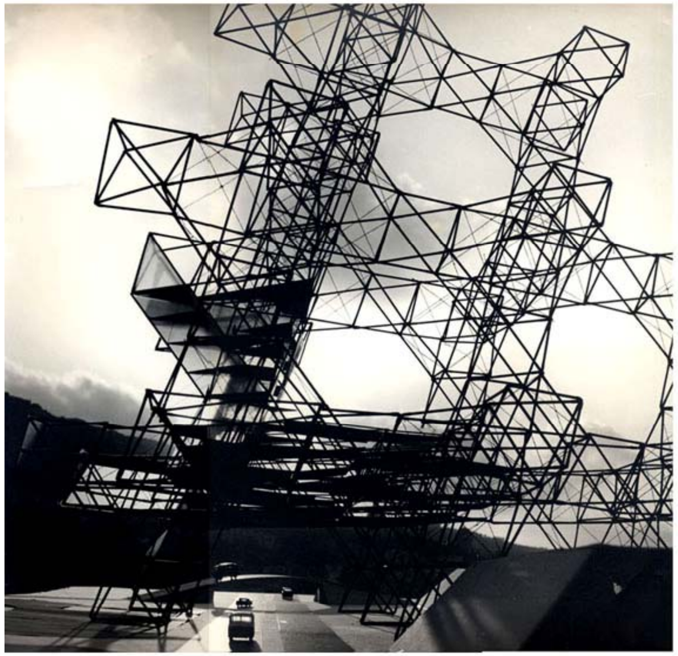

THE EXPANDING UNIVERS OF UNIFORM SPONGE POLYHEDRA
F=0; V=1; Pav.→∞ BOUNDARY OF THE POLYHEDRAL UNIVERSE
THE DOMAIN OF SPONGE POLYHEDRA
BURT-1996
BURT, WACHMAN, KLEINMAN-1974
COXETER, PETRIE, WELLS & OTHERS-1960
THE ANCIENTS
g
Val.av.
0.00
5
10
15
20
25
30
40
2π
4π
6π
8π
10π
20π
-2π


A SPHERICAL SPONGE POLYHEDRON
SYMMETRY GR. (2.3.5)
g = 2041
Val.av. = 146
V = 120
E = 8760
F = 4560
F=0; Pav.→∞ BOUNDARY OF THE POLYHEDRAL UNIVERS
THE DOMAIN OF SPONGE POLYHEDRA
THE EXPANDING POLYHEDRAL UNIVERS
THE POLYHEDRAL UNIVERSE (UP TO 2004)
THE 'PTOLEMAIC' WORLD PICTURE OF THE POLYHEDRAL UNIVERSE (UP TO 1960 AD.)
HYPERBOLIC (PLANE FACETED) POLYHEDRA
TOROIDAL POLYHEDRA
SPHERICAL POLYHEDRA
FLORAL -TWO VERTEX -ONE VERTEX POLYHEDRA
g
Val.av.
0.00
50
100
150
500
1000
1500
10π
20π
30π
40π
50π
100π
-2π

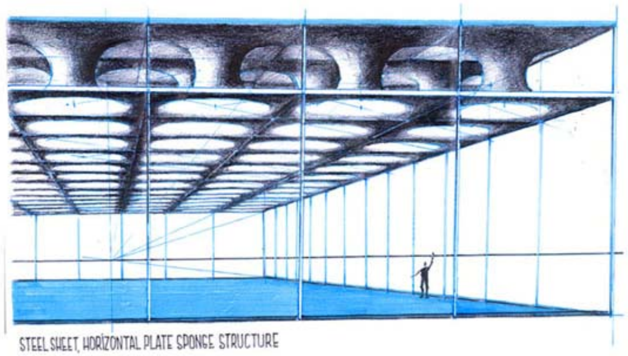

VERNONIA AEMULANS
GRAINS DE POLLEN



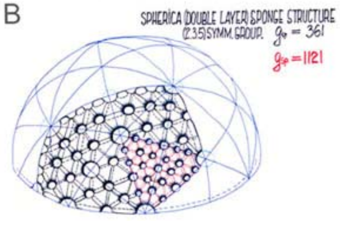




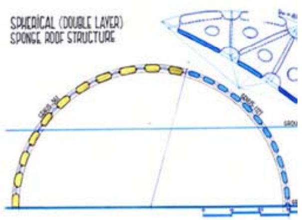
A

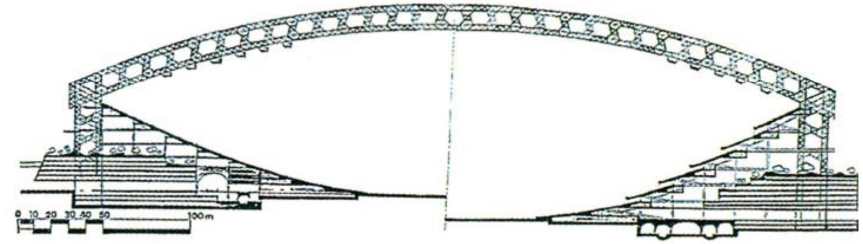

STEEL SHEET, HORIZONTAL PLATE SPONGE STRUCTURE

B


SPHERICA (DOUBLE LAYER) SPONGE STRUCTURE
(2,3,5) SYMM. GROUP. $g_0 = 361$
$g_0 = 1121$

C


SPHERICAL (DOUBLE LAYER)
SPONGE ROOF STRUCTURE


0 10 20 30 40 50 100m

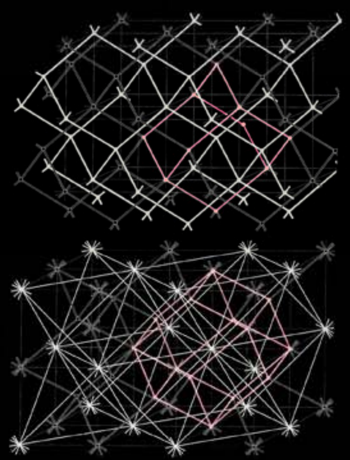
J

Any significant venture into the field of periodic sponge surfaces and polyhedra dictates a **systematic exploration of the uniform space lattice domain.**
It came as a shocking surprise to realize that in spite of the great efforts of the last three centuries or so, in the exploration of the structure of matter and space (crystallography included), **no systematic effort was committed to exhaustively explore the network domain in the "abstract realm of the theoretically imaginable".**

E

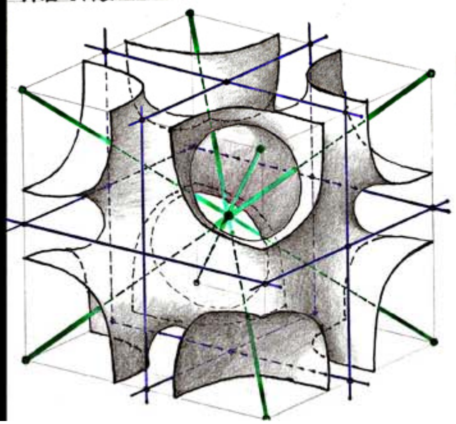
All networks come in dual pairs. Each network is uniquely determined by, and is a reciprocal of its dual (complementary) companion.
-**Every dual pair of networks is associated with a continuous hyperbolical sponge surface** which subdivides the space between the two, into two complementary sub-spaces.
This **trinity of the dual pair and the associated-reciprocal sponge surface** is the most conspicuous, all pervading geometric-topological phenomenon of our 3-D space, associated with its order and organization and more than anything else determines the way we perceive and comprehend its structure

TRINITY OF THE DUAL NETWORKS PAIR AND THE PARTITION SURFACE

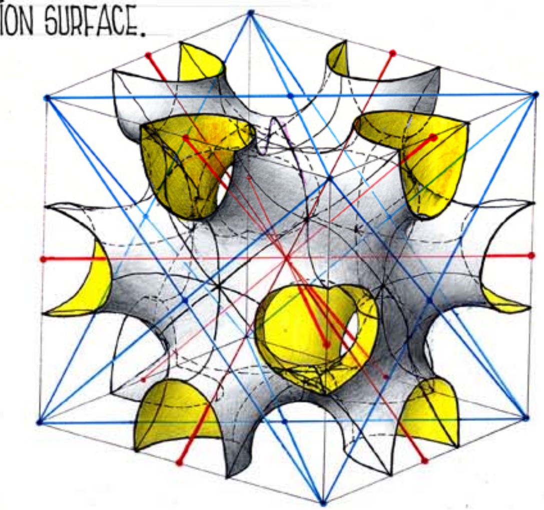


UNIFORM TETRAVALENT 'DIAMOND' LATTICE AND
UNIFORM DODECAVALENT 'OCTET' LATTICE AND DUALS.

THE TRINITY: TWO DUAL-COMPLEMENTARY NETWORKS AND THE RECIPROCAL SURFACE-PARTITION, SBDIVIDING THE SPACE BETWEEN THE TWO.



THE TRINITY OF THE DUAL PAIR OF NETWORKS AND THE ASSOCIATED PARTITION SURFACE.



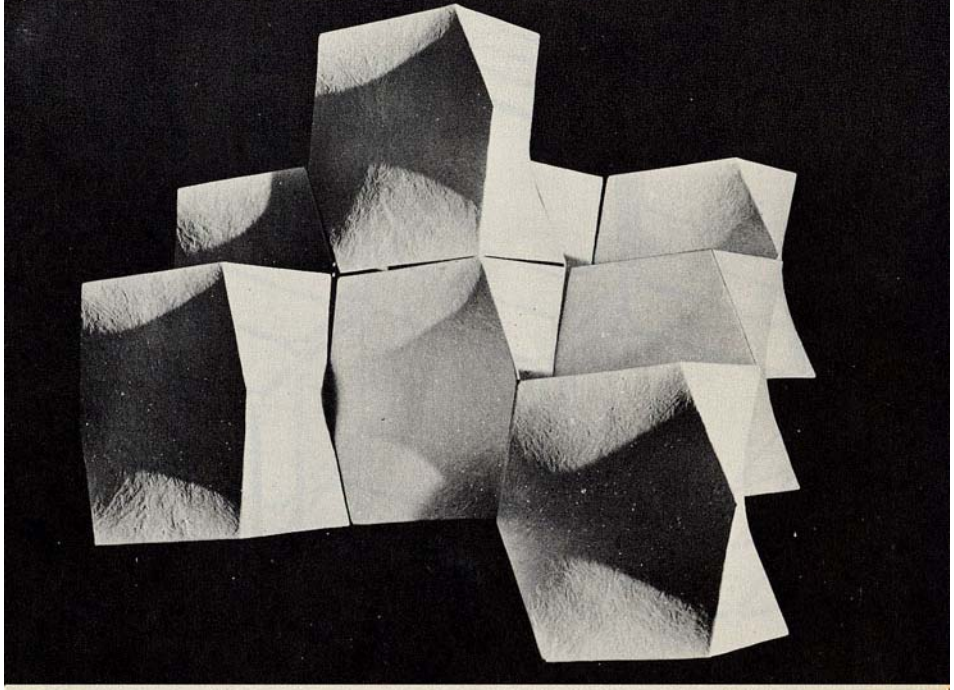
A periodic ordered space network may be generated through one of the following processes

-By an extended repetition of a locally and globally symmetrical association of vertex figures.

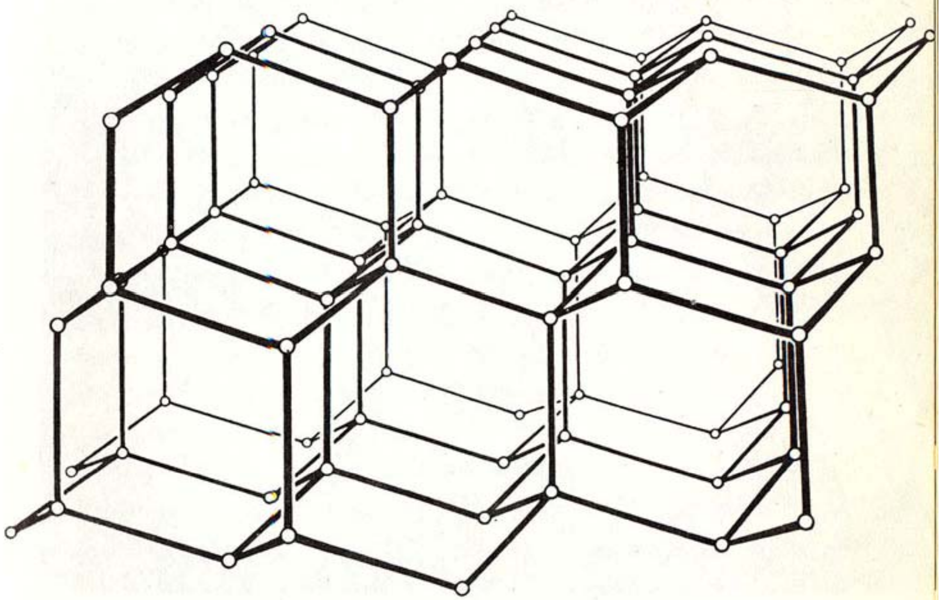
-As a result of a close (compact) packing of polyhedral cells, the vertex-edge array of which combine to form the network.

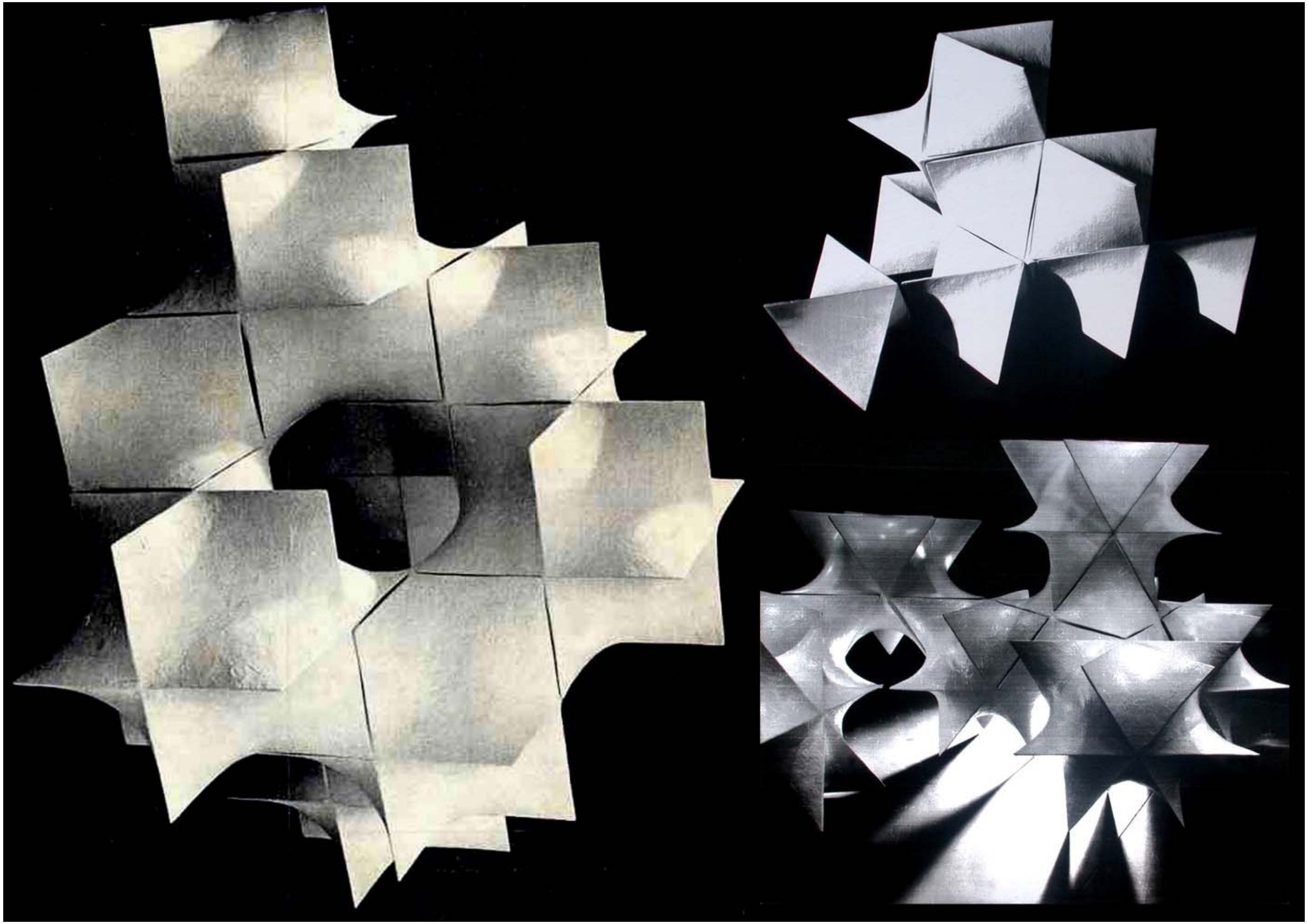
-As a result of a tessellation process of an unbounded periodic (2d-manifold) surface, spherical, toroidal or hyperbolical, leading eventually to a connected 3-D network.

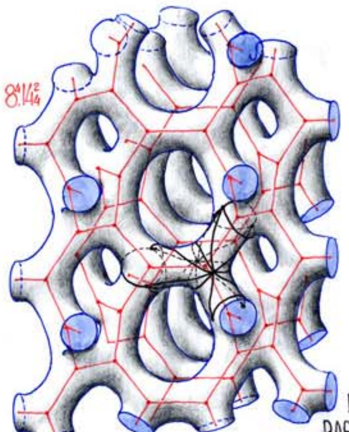
1



1



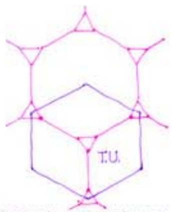




$(3^3)^3_4$

$g = 4$
$\Sigma\alpha = 3\pi$
$Val. = 9$

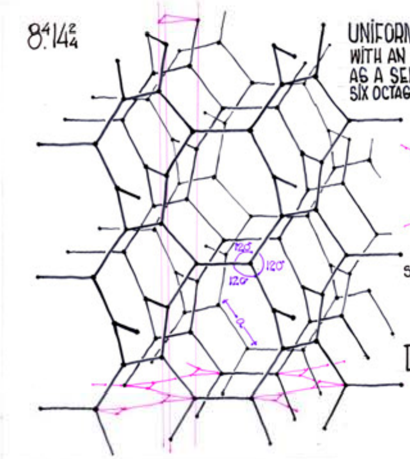
$V_{T.U.} = 12$
$E_{T.U.} = 54$
$F_{T.U.} = 36$



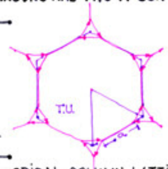
UNIFORM SPONGE POLYHEDRON

PERIODIC SPONGE SURFACE ON THE BASIS OF A UNIFORM TRIVALENT LATTICE

$8^4.14^2_4$

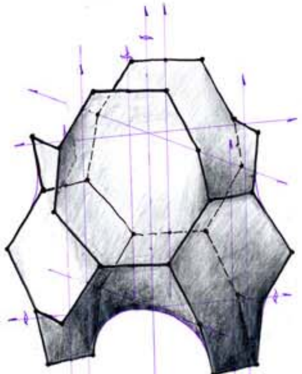


UNIFORM TRIVALENT SPACE LATTICE WITH AN OCTAHEDRAL SADDLE POLYHEDRON AS A SELF CLOSE PACKING SOLID, WITH SIX OCTAGONS AND TWO 14-GON FACES.



SPIRAL COLUMN LATTICE

$Den_{8^4.14^2_4} = 0.254558441\ a/a^3$

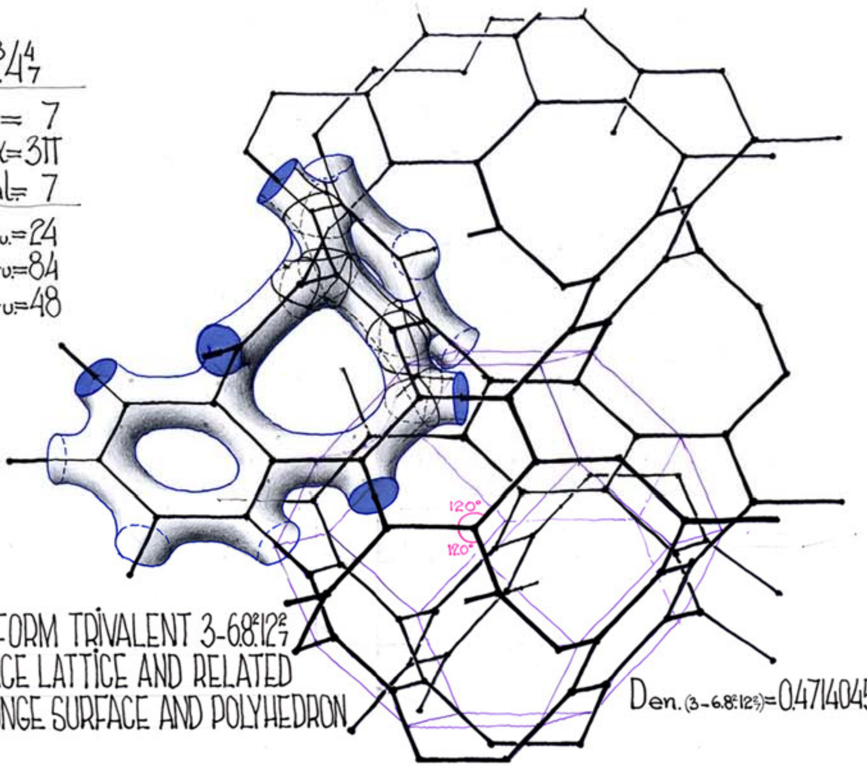


SELF CLOSE PACKING SADDLE POLYHEDRON GENERATING UNIFORM TRIVALENT SPACE LATTICE-$8^4.14^2_4$

$3^3.4^4_7$

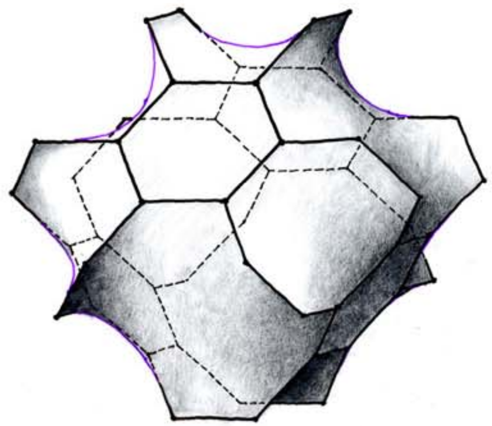
$g = 7$
$\Sigma\alpha = 3\pi$
$Val. = 7$

$V_{T.U.} = 24$
$E_{T.U.} = 84$
$F_{T.U.} = 48$

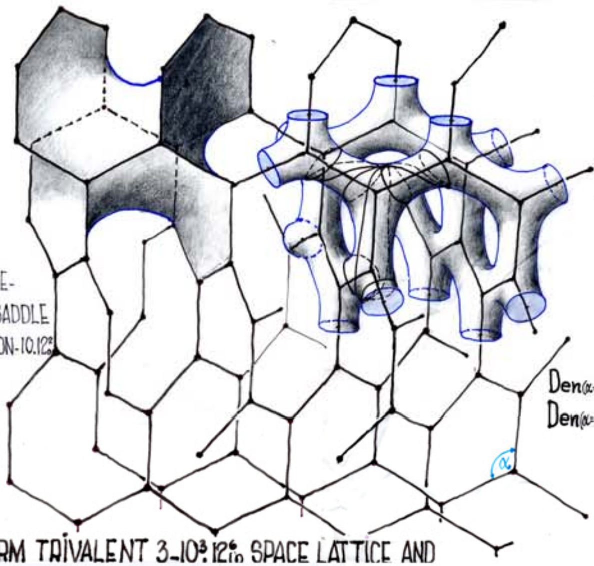


UNIFORM TRIVALENT $3-6.8^2.12^2_7$ SPACE LATTICE AND RELATED SPONGE SURFACE AND POLYHEDRON

$Den._{(3-6.8^2.12^2_7)} = 0.47140452\ a/a^3$



SELF CLOSE-PACKING SADDLE POLYHEDRON, WITH 6, 8 & 12-GONS, GENERATING THE UNIFORM TRIVALENT $3-6.8^2.12^2_7$ SPACE-LATTICE.



SELF CLOSE-PACKING SADDLE POLYHEDRON-10.12²₁₀

$(3^3.4^2)^2_{10}$

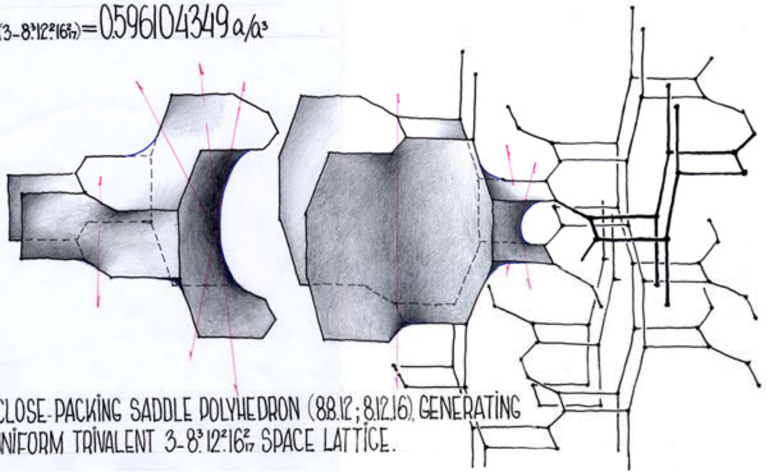
$g_{TU.} = 10$
$\Sigma\alpha = 4\pi$
$Val. = 10$

$V_{TU.} = 18$
$E_{TU.} = 90$
$F_{TU.} = 54$

$Den_{(\alpha=120^\circ)} = 0.769800358\ a/a^3$
$Den_{(\alpha=109^\circ28'17'')} = 0.730710452\ a/a^3$

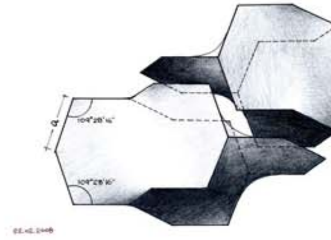
UNIFORM TRIVALENT 3-10³ 12⁴₁₀ SPACE LATTICE AND

$Den_{(3-8^3 12^2 16^2_{17})} = 0.596104349\ a/a^3$

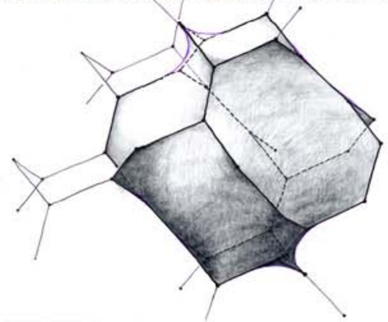


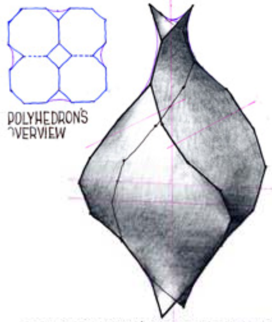
SELF CLOSE-PACKING SADDLE POLYHEDRON (8.8.12 ; 8.12.16), GENERATING THE UNIFORM TRIVALENT 3-8³ 12² 16²₁₇ SPACE LATTICE.

DECA-TETRAHEDRON, A SELF CLOSE-PACKING SADDLE-POLYHEDRON, GENERATING THE UNIFORM TRIVALENT SPACE LATTICE-10³₅.

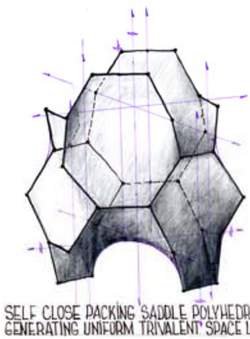


SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE- 6.10⁵₄



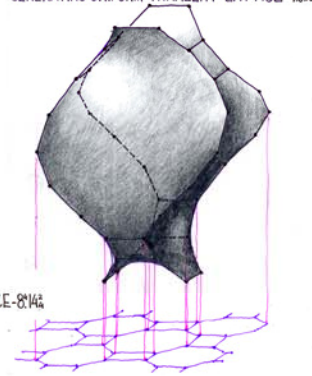


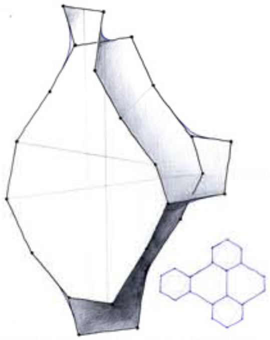
SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE-4.14⁵₃



SELF CLOSE PACKING SADDLE POLYHEDRON GENERATING UNIFORM TRIVALENT SPACE LATTICE-8⁴.14⁴₄

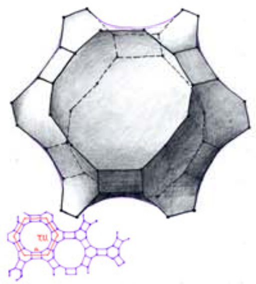
SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE-4.8.16⁵₃

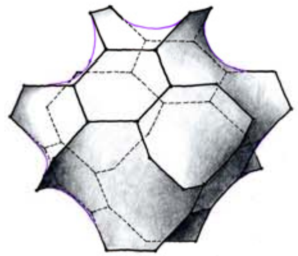




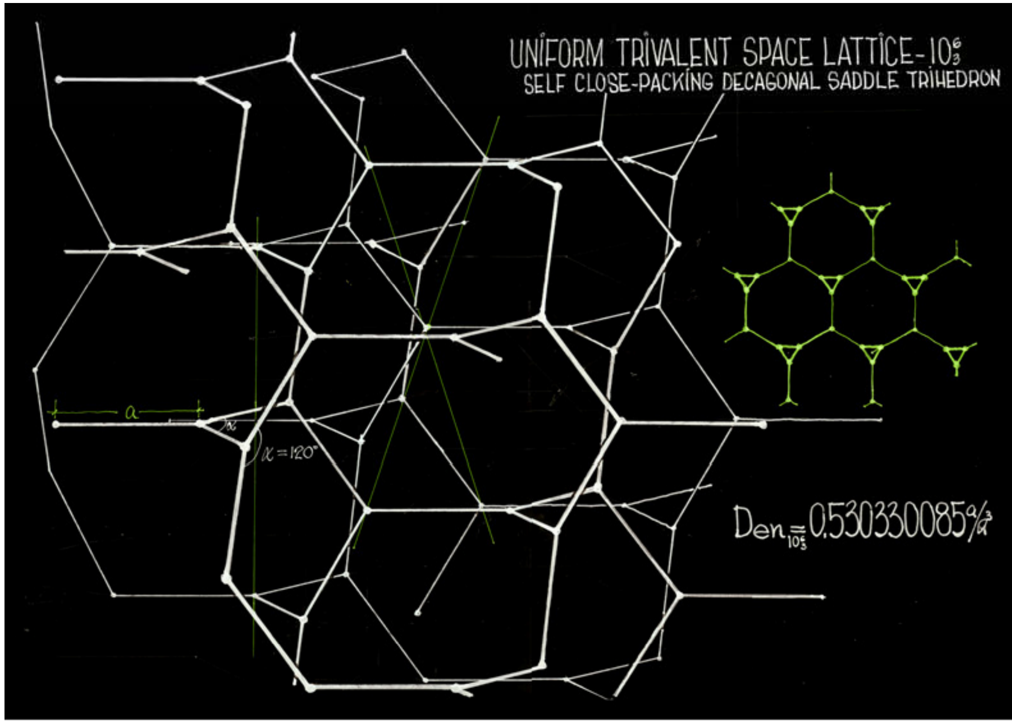
SELF CLOSE-PACKING SADDLE POLYHEDRON, GENERATING THE UNIFORM TRIVALENT 3-12⁴.14⁴₅ SPACE LATTICE.

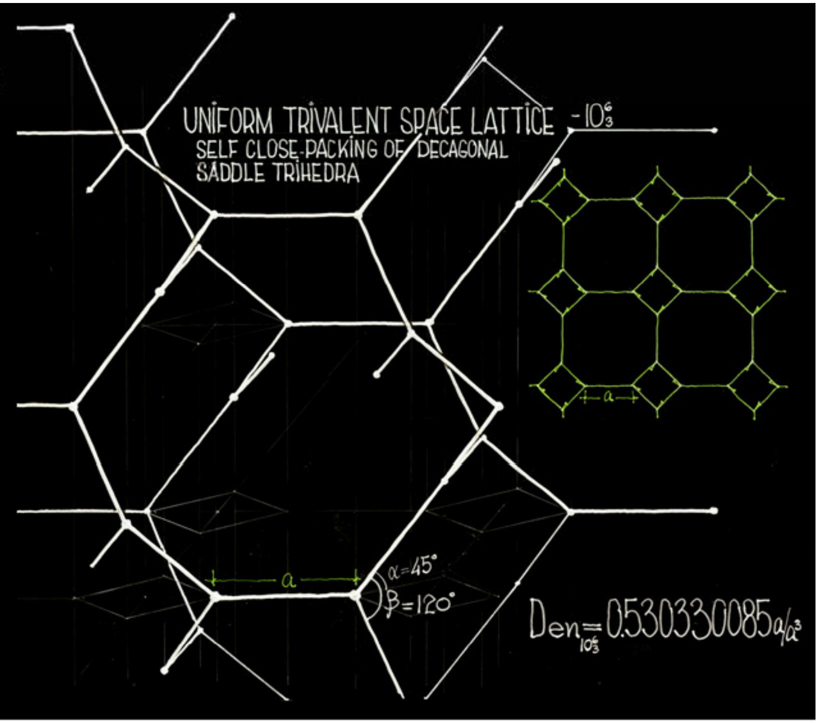
SELF CLOSE-PACKING SADDLE-POLYHEDRON GENERATING UNIFORM TRIVALENT LATTICE - THE 4.12⁴₇

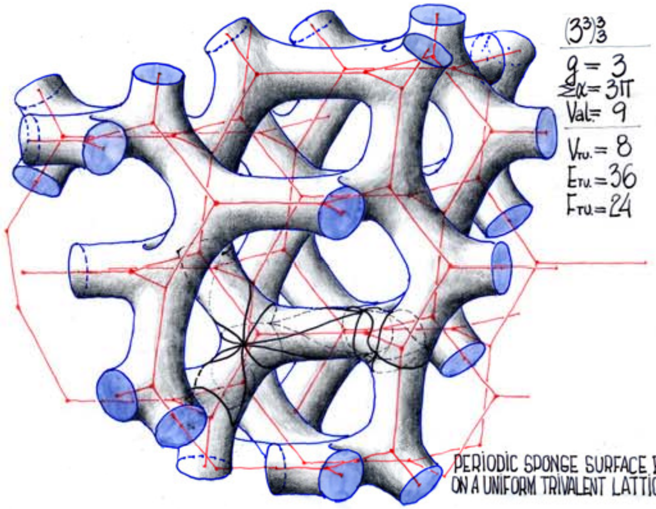




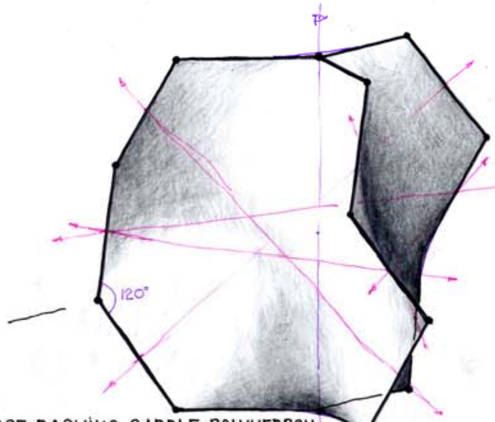
(ING SADDLE POLYHEDRON WITH 6,8 & 12-GONS, UNIFORM TRIVALENT 3-6.8².12²₇ SPACE LATTICE.



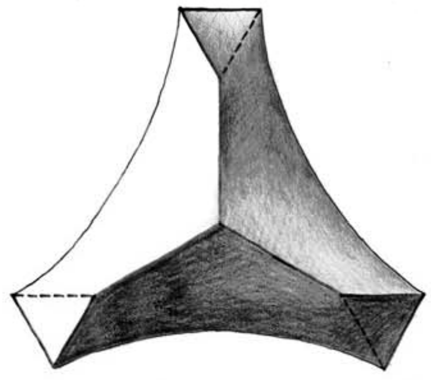




PERIODIC SPONGE SURFACE BASED ON A UNIFORM TRIVALENT LATTICE-10^6_3

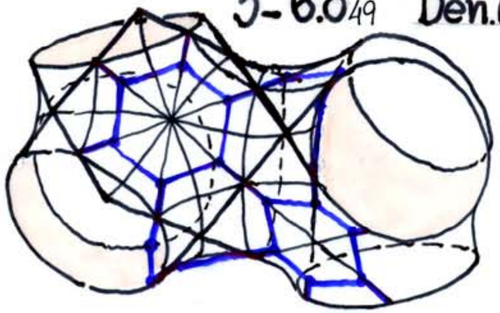


SELF CLOSE PACKING SADDLE POLYHEDRON GENERATING UNIFORM TRIVALENT SPACE LATTICE - THE-10^6_3 -DECAGONAL TRIHEDRON.



SELF CLOSE PACKING SADDLE POLYHEDRON WITH 3 DODECAGONAL FACES, GENERATING THE 3-10^6_3 LATTICE

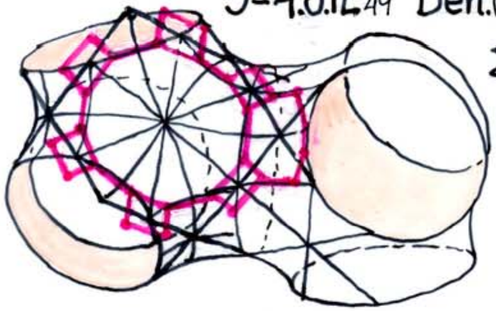
$3\text{-}6.8^2_{49}$ $Den._{(3\text{-}6.8^2_{49})} = 0.47140452\ a/a^3$



$\Sigma\alpha = 390°$

$Con_{T.U.} \equiv (g_{T.U.}) = 49$

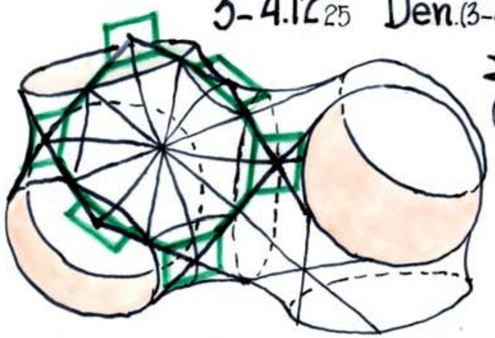
$3\text{-}4.8.12_{49}$ $Den._{(3\text{-}4.8.12_{49})} = 0.240236673\ a/a^3$



$\Sigma\alpha = 375°$

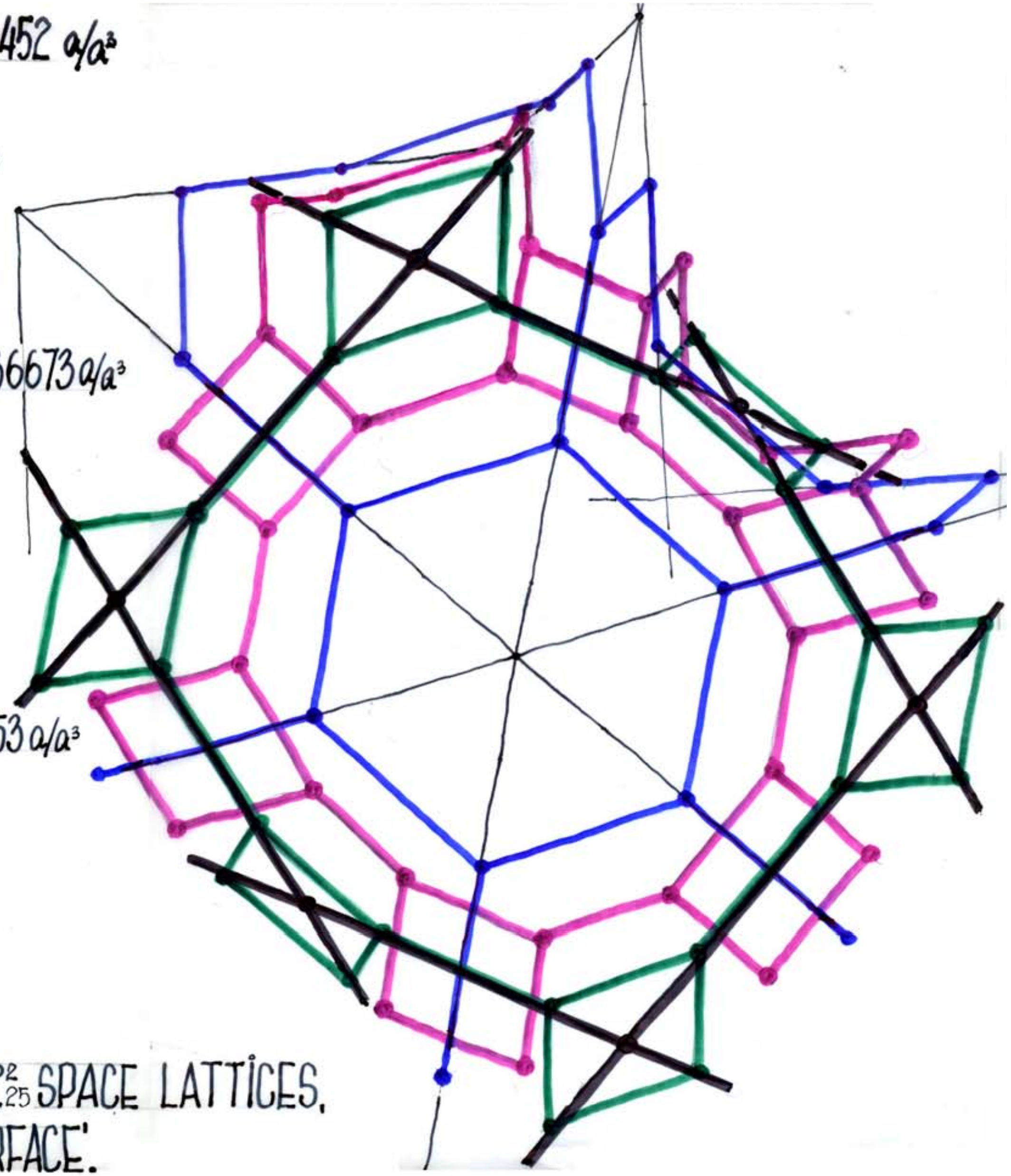
$Con_{T.U.} \equiv (g_{T.U.}) = 49$

$3\text{-}4.12^2_{25}$ $Den._{(3\text{-}4.12^2_{25})} = 0.319805153\ a/a^3$



$\Sigma\alpha = 390°$

$Con_{T.U.} \equiv (g_{T.U.}) = 25$



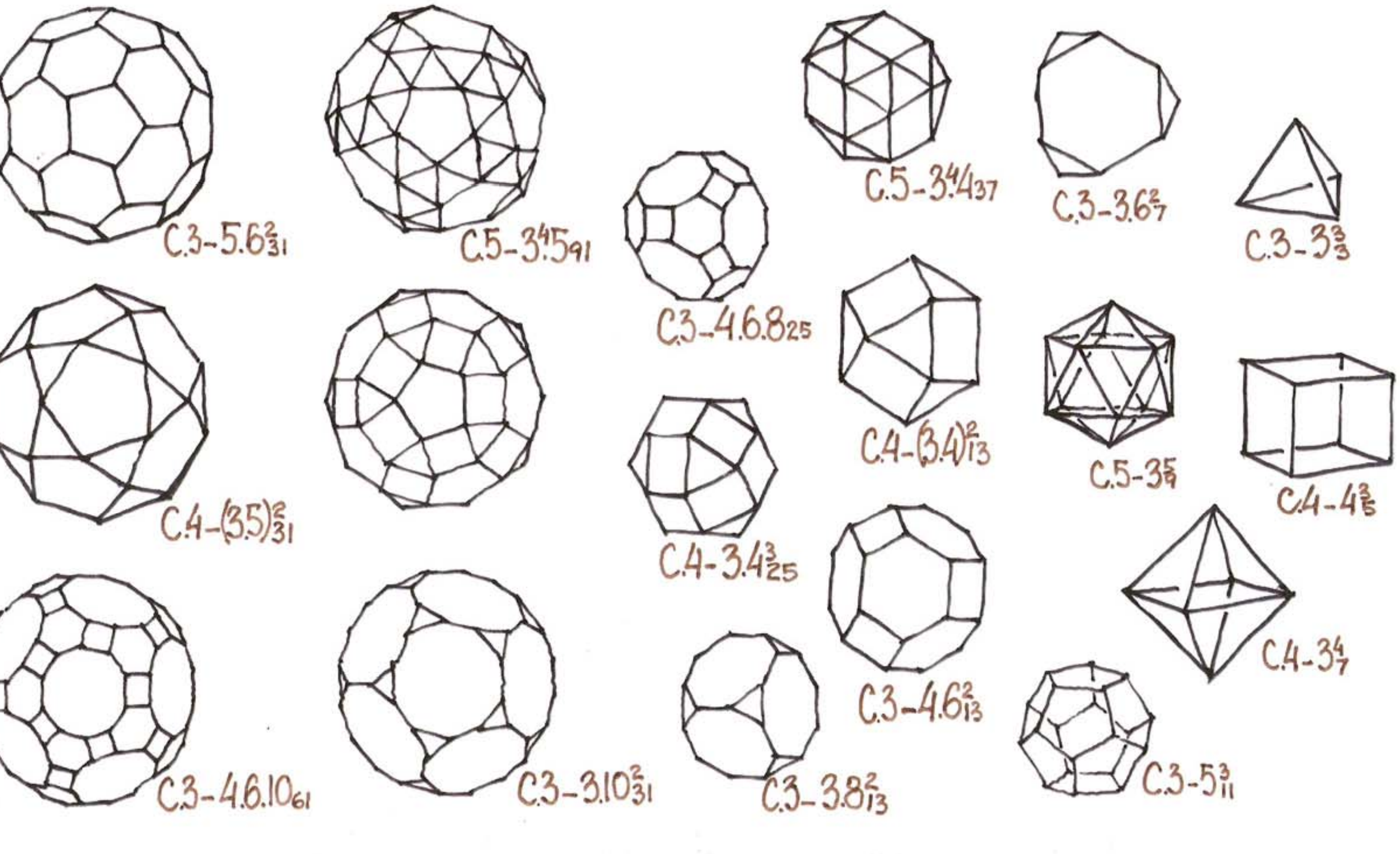
UNIFORM TRIVALENT $3\text{-}6.8^2_{49}$; $3\text{-}4.8.12_{49}$; $3\text{-}4.12^2_{25}$ SPACE LATTICES. ALL TESSELATIONS OF THE 'DIAMOND SURFACE'.

G

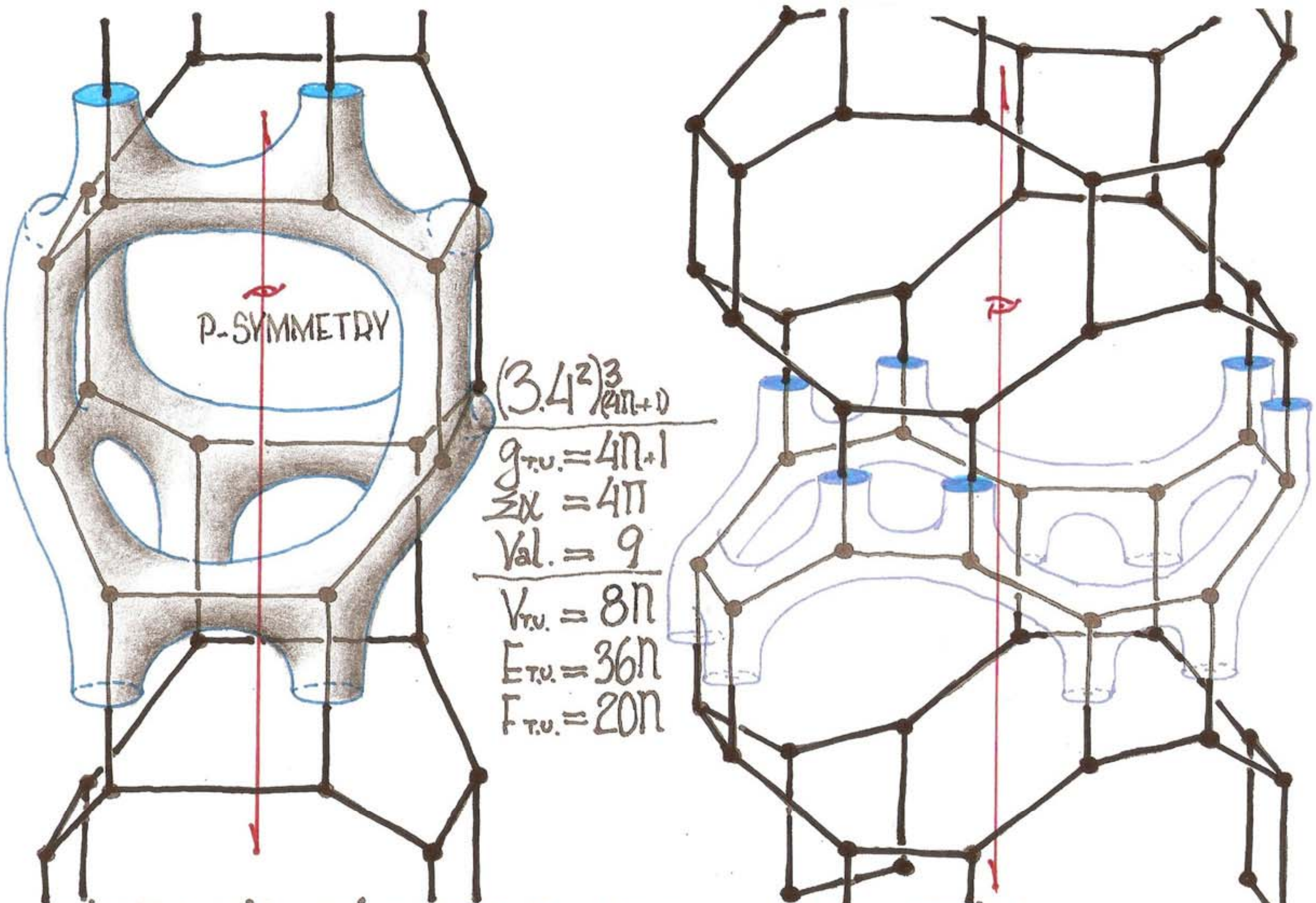
The vertex figure characteristics (geometric-symmetrical and topological) of a given network, tightly correspond to the topological-symmetrical characteristics of the close-pack cells of it's dual. By proxy it may be stated that all constituents of a given 'trinity' (the dual networks pair and the associated sponge surface) act under the same topological – Symmetrical regime.

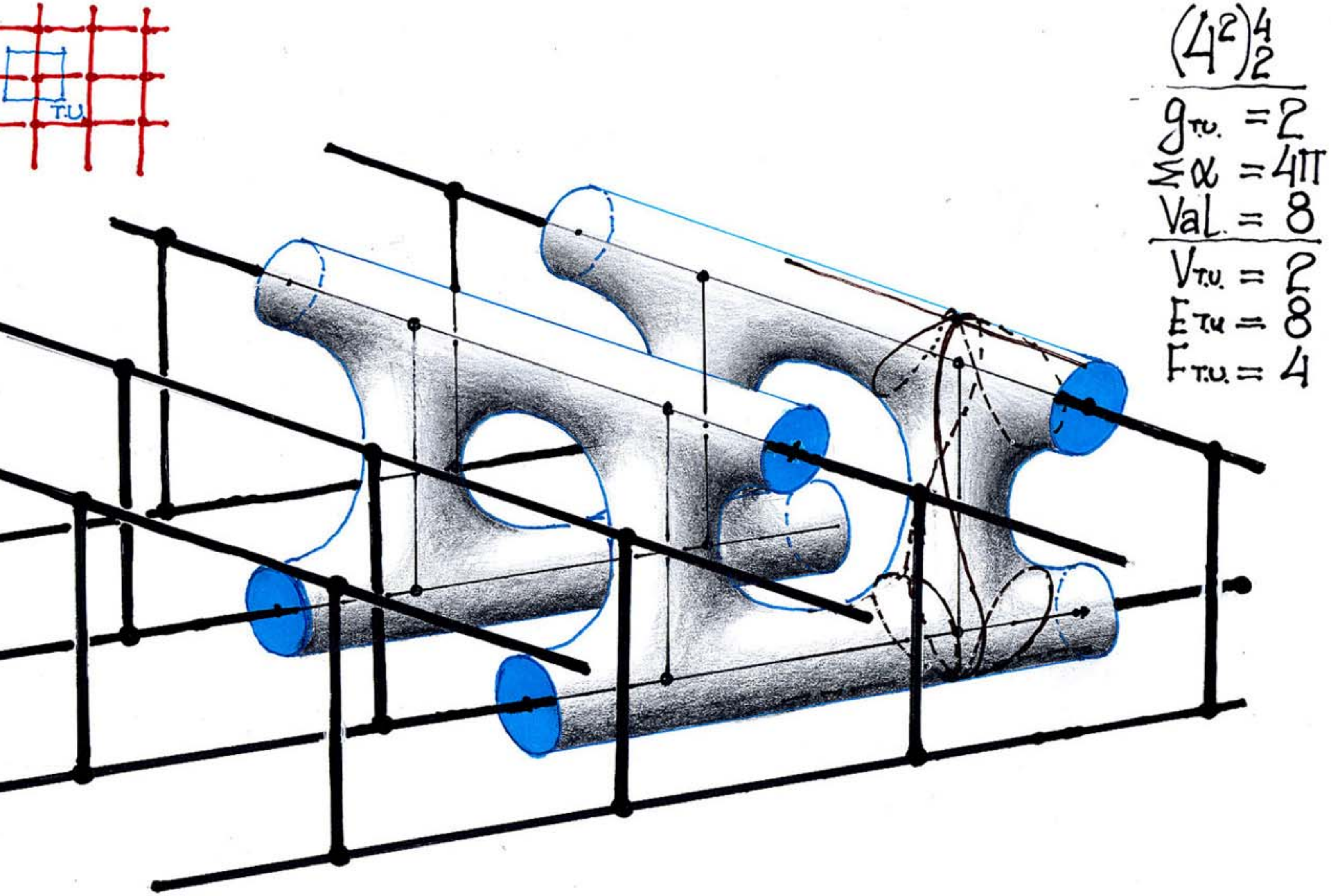
-Connectivity value (C) of the two continuous dual network graphs is one and the same for both, and is the same as genus-(g) value of the associated sponge surface:

C=L-N+1=g (with L&N as the number of Line-edges and vertex-Nodes respectively

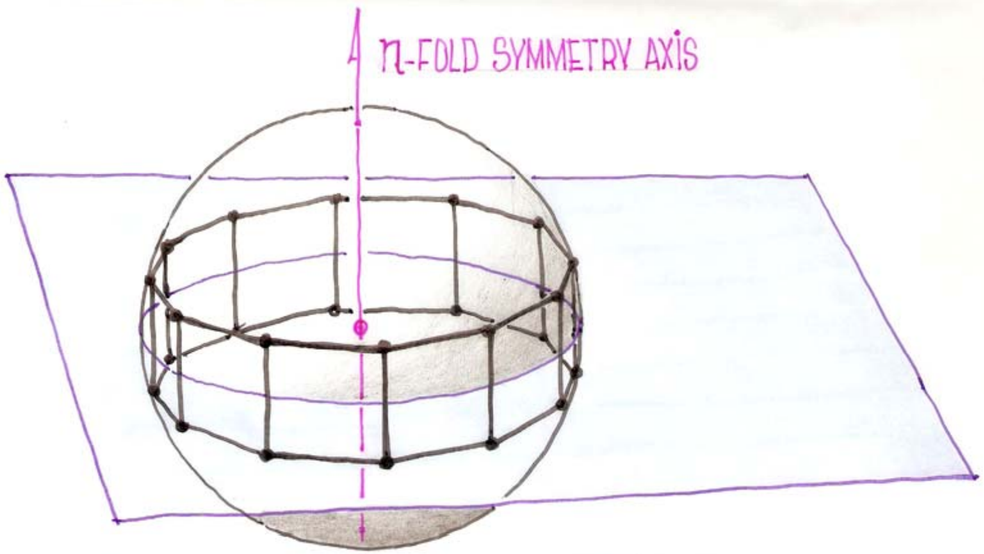


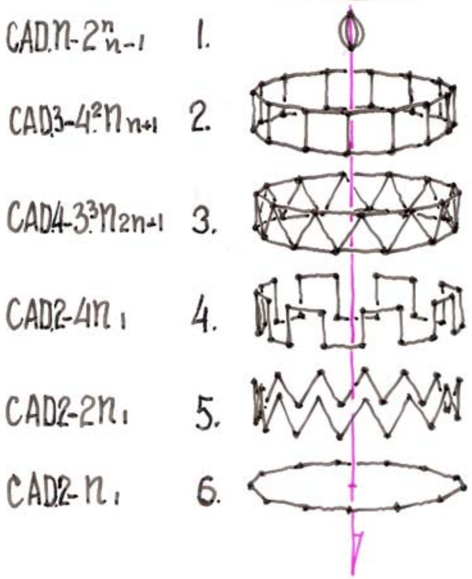
NIFORM CENTROIDAL NETWORKS. THE VERTICES OF WHICH ARE EQUI DISTANT FROM A


P-SYMMETRY
(3.4²)³(4n+1)
g T.U. = 4n+1
Σx = 4n
Val. = 9
V T.U. = 8n
E T.U. = 36n
F T.U. = 20n



...FORM DOUBLE LAYER TRIVALENT R² O² SPACE LATTICE AND THE



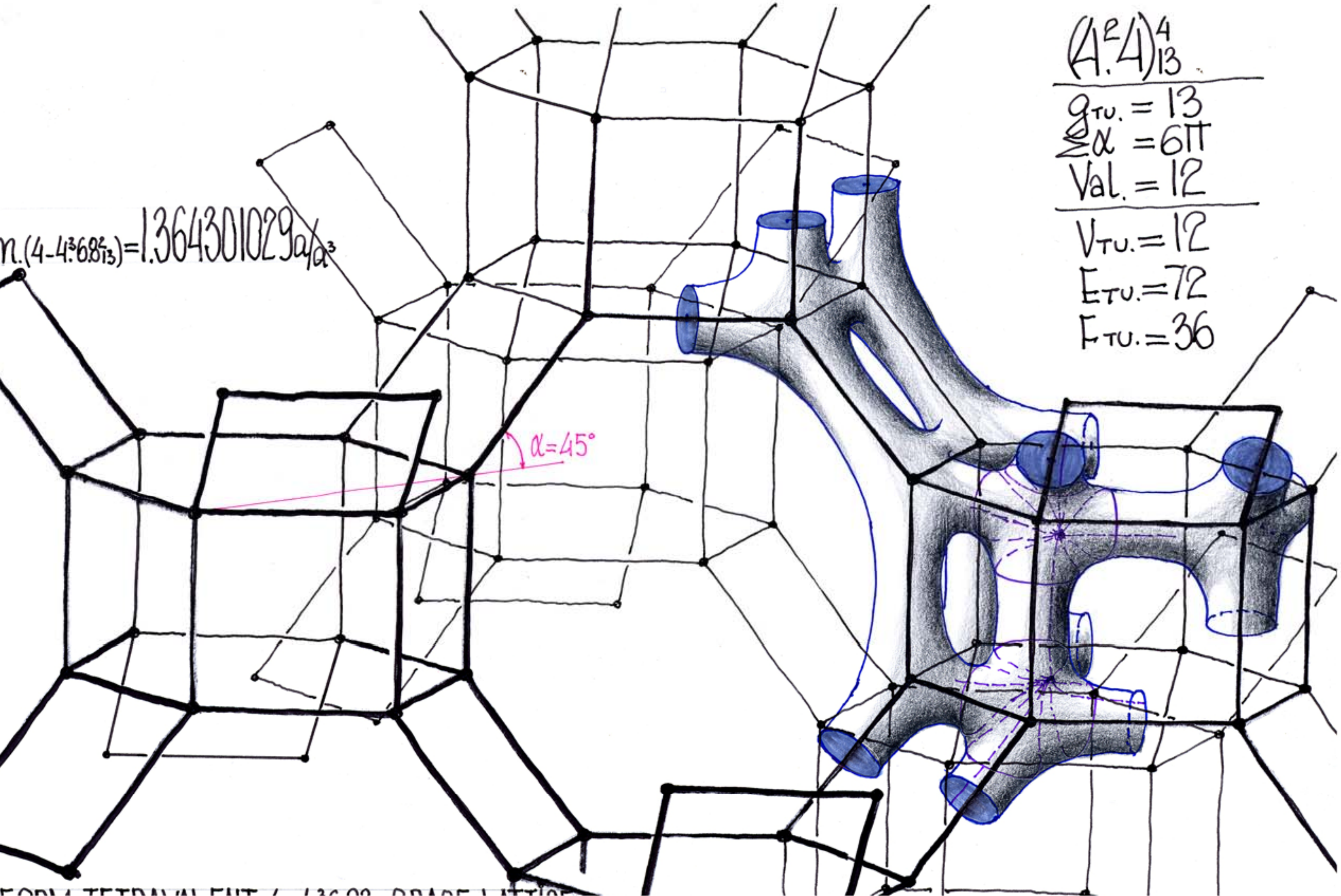


POLYDIGONS

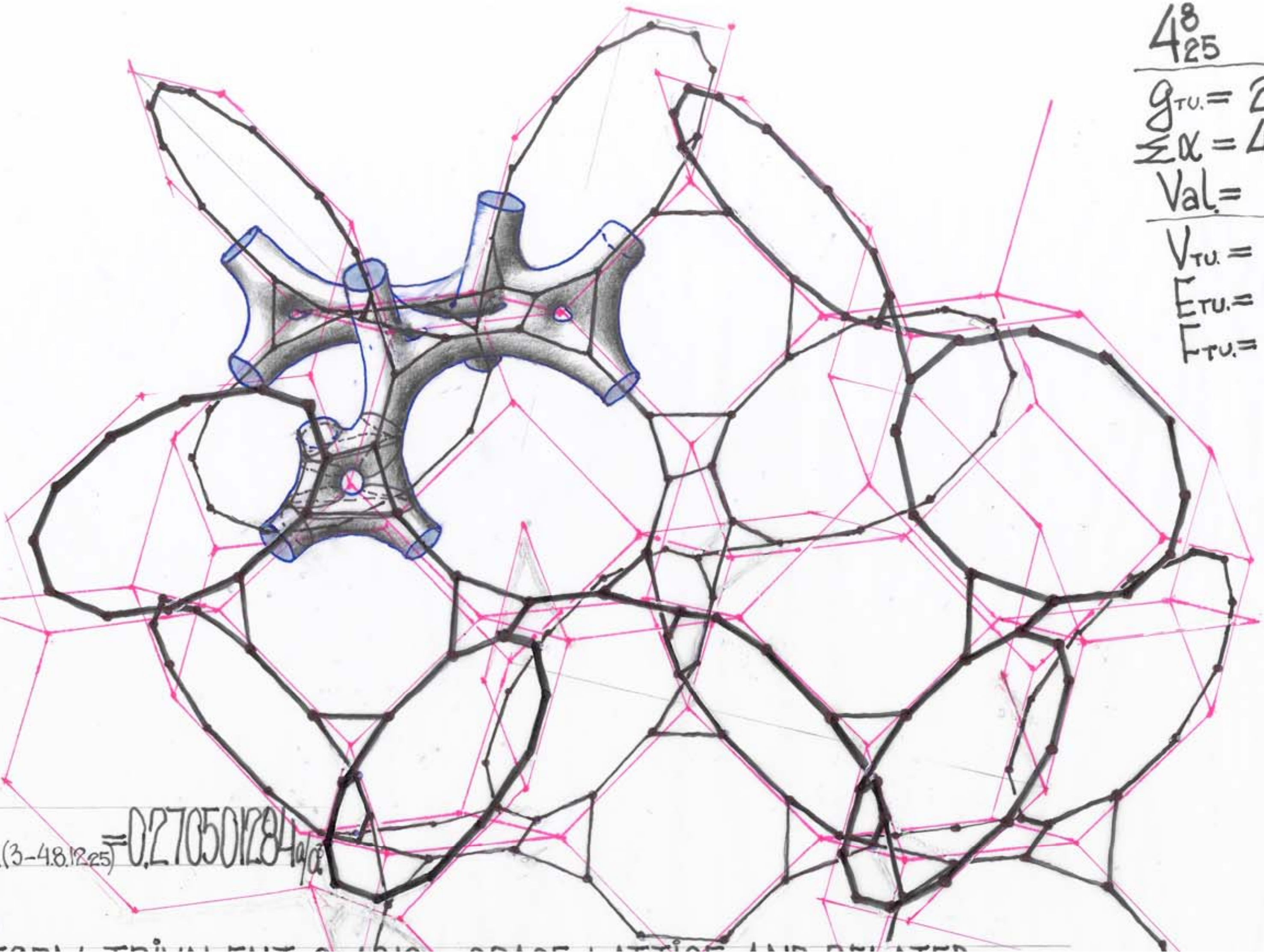
ALL VERTICES OF THE BAND NETWORKS ARE EQUIDISTANT FROM A FIXED CENTRE-POINT, AXIS AND A PLANE SURFACE.

THE CAD CATEGORY GROUP INCLUDES INFINITE NUMBER OF MEMBERS.

DISTINCTIVE GROUP OF PRIMITIVE UNIFORM BAND-



n.(4-4³6̄8̄13)=1.364301029a/a³
(4².4) 4 13
gTU. = 13
Σα = 6π
Val. = 12
VTU. = 12
ETU. = 72
FTU. = 36
α=45°



4^{8}_{25}

$g_{T.U.} = 25$

$\Sigma\alpha = 4\pi$

Val. = 8

$V_{T.U.} = 48$

$E_{T.U.} = 192$

$F_{T.U.} = 96$

= 6

= $2\frac{5}{6}\pi$

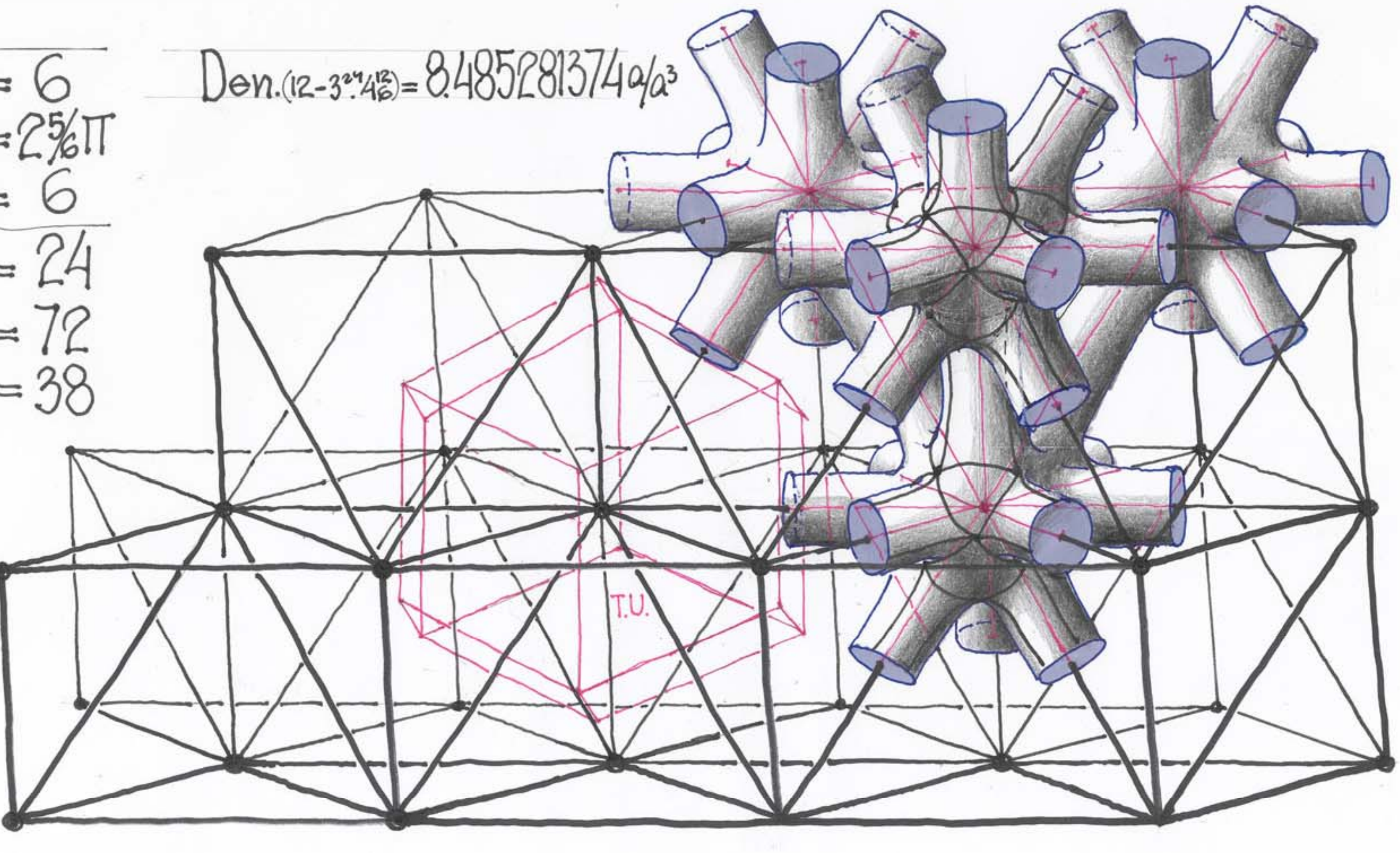
= 6

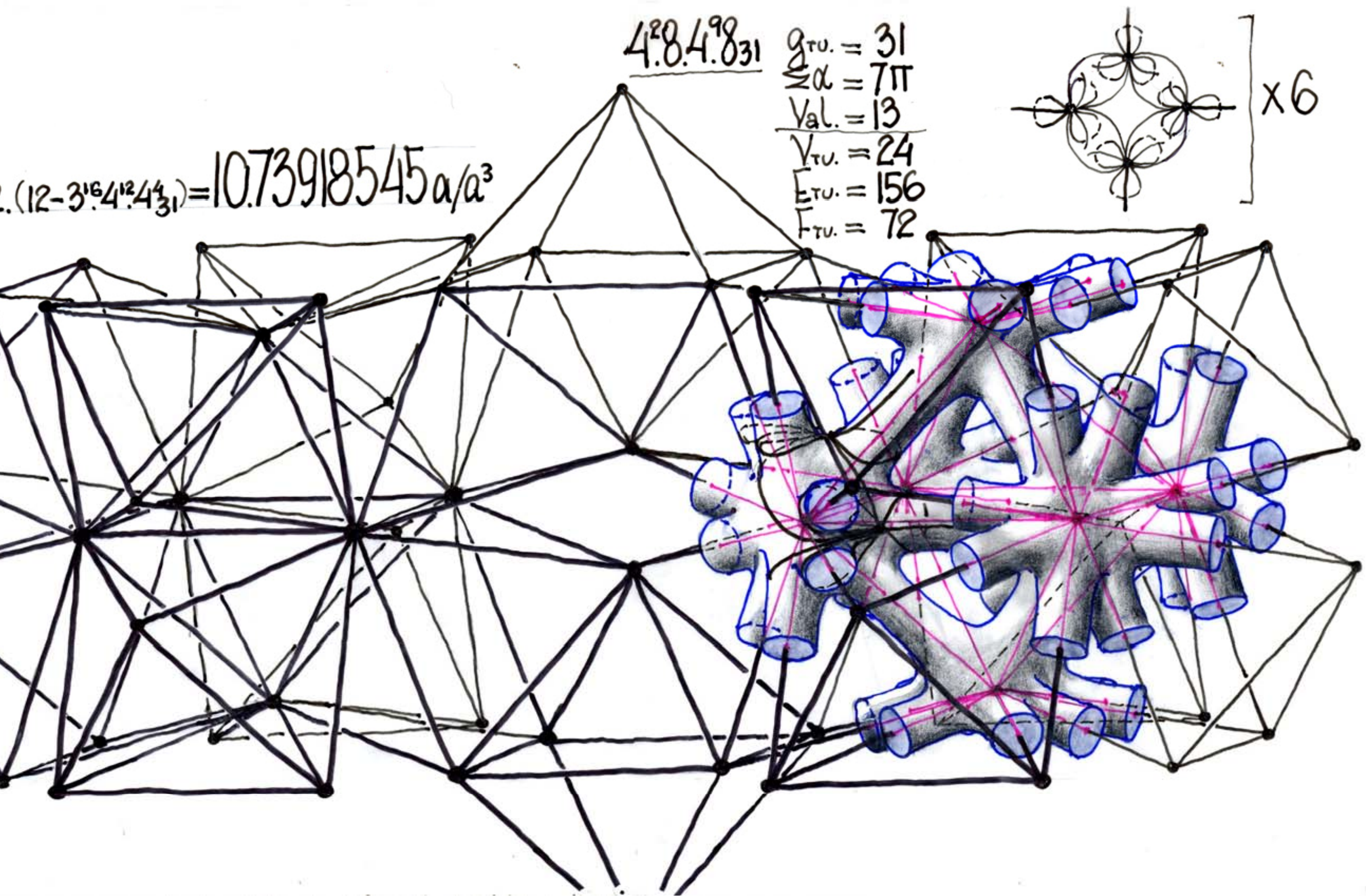
= 24

= 72

= 38

$Den._{(12-3^{24}.4^{12}_{6})} = 8.485281374\ a/a^3$





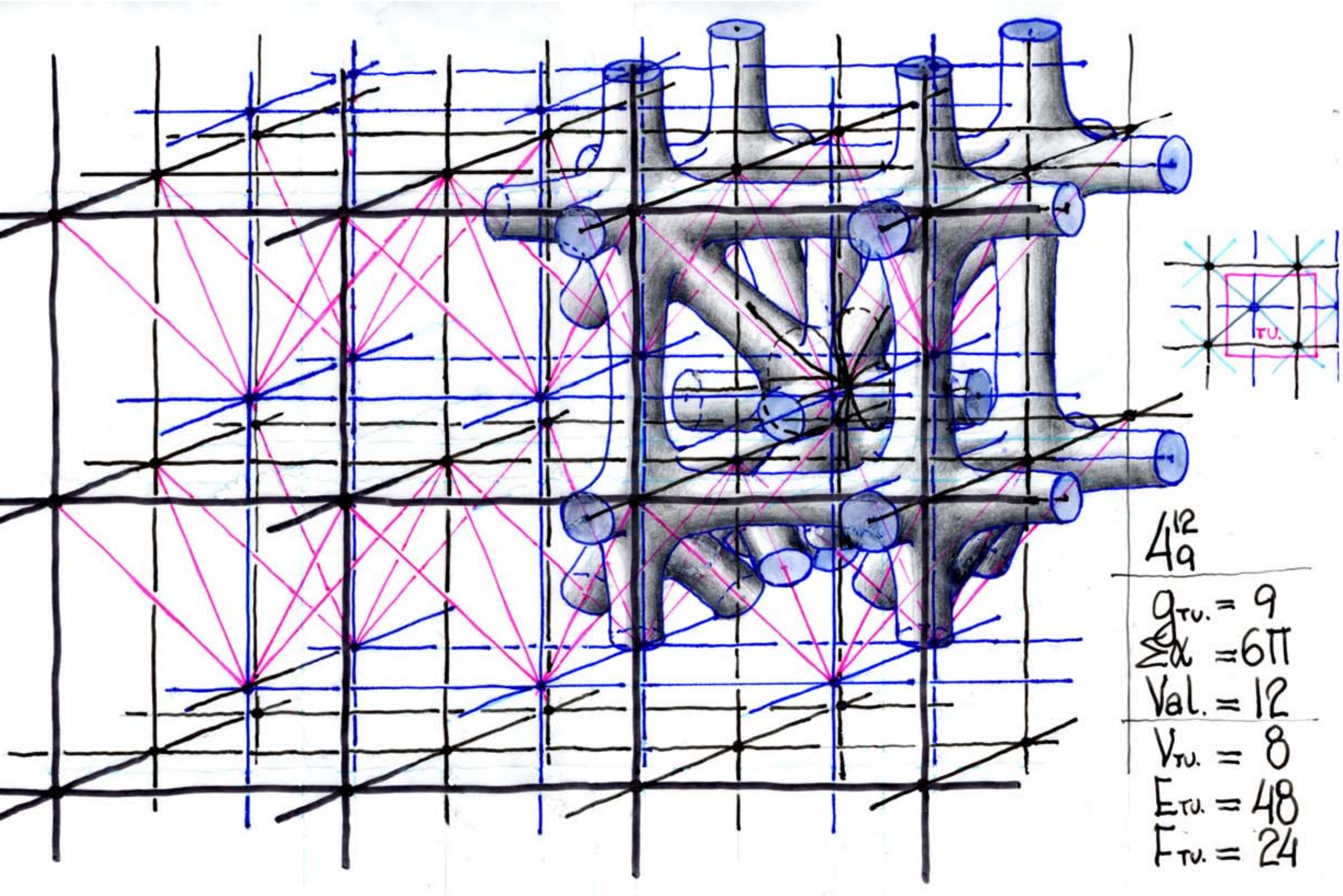
.(12-3¹⁶.4¹².4⁴₃₁)=10.739185 45 a/a³
4²⁸.4⁹⁸₃₁
gTU. = 31
Σα = 7π
Val. = 13
VTU. = 24
ETU. = 156
FTU. = 72
x6

P

niform Dodecavalent and higher valency Space Lattices or: how far alency and spatial density values can go.

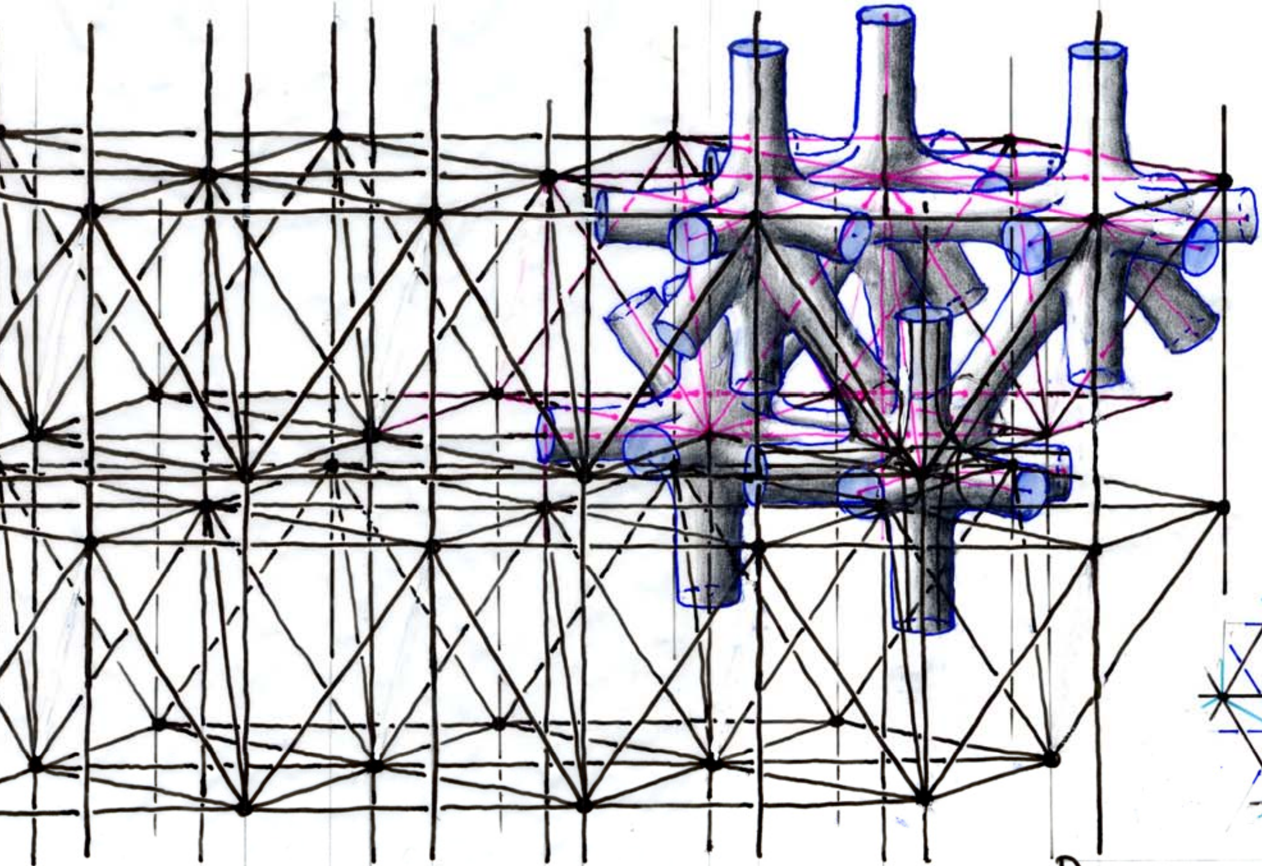
niform dodecavalent 'octet' based space lattices exist in more than one pological version, but all come to same spatial density of **8,485281374 /a^3.**

he infinite sponge polyhedron **$3^{12}{}_4$** gives rise to a uniform dodecavalent **2-3^{12}.$4^{20}{}_{31}$)** space lattice, the density of which **is 10,73918545a/a^3 (!)**



ORM DECAVALENT 10-$3^{12}4^{12}_{9}$ SPACE LATTICE

Den (10-$3^{12}4^{12}$) = 10.000



4^{10}_{10}

$g_{TU.} = 10$

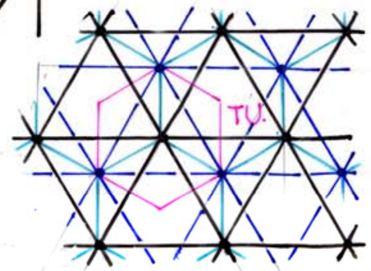
$\sum\alpha = 5\pi$

$Val. = 10$

$V_{TU.} = 12$

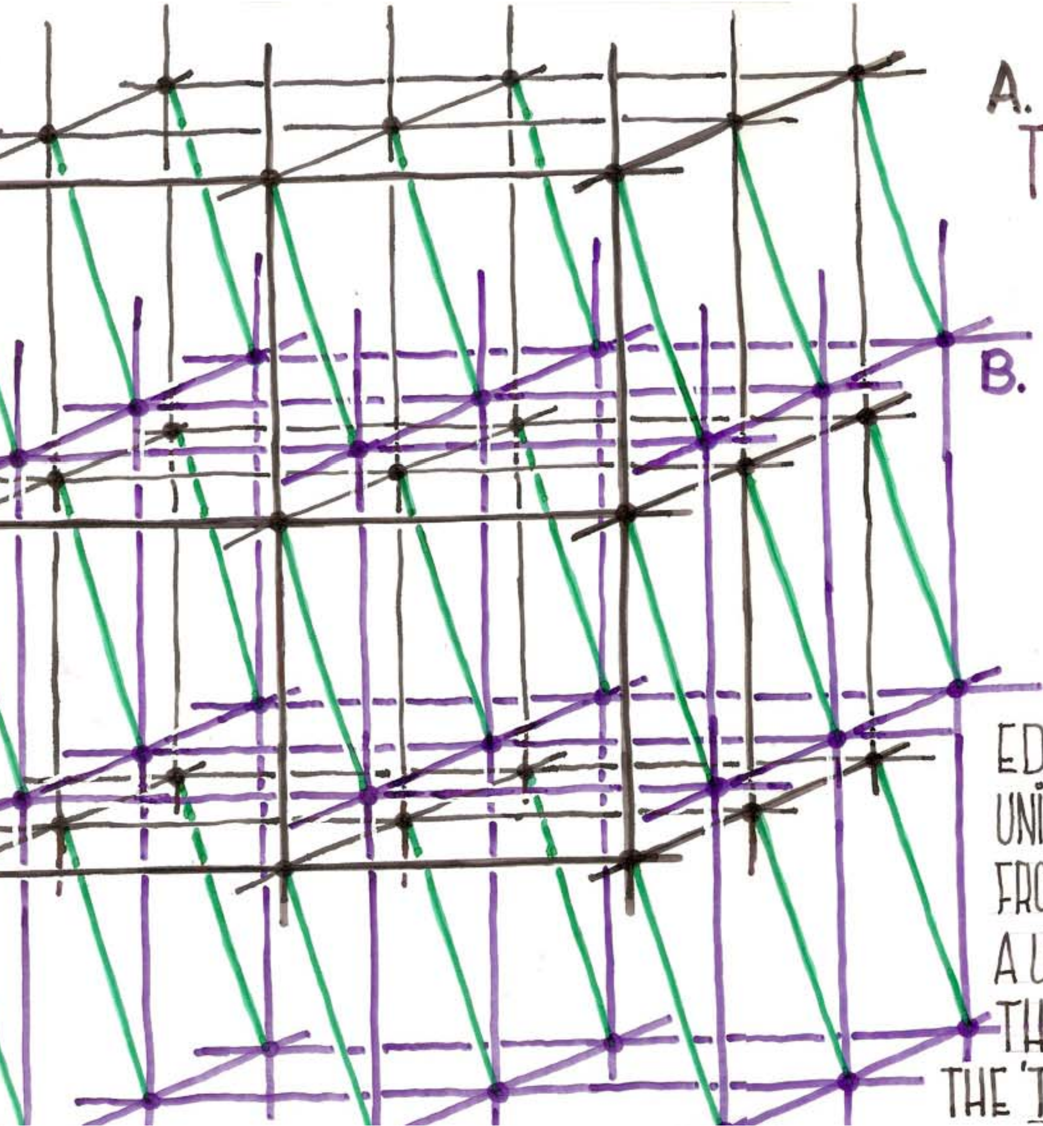
$E_{TU.} = 60$

$F_{TU.} = 30$



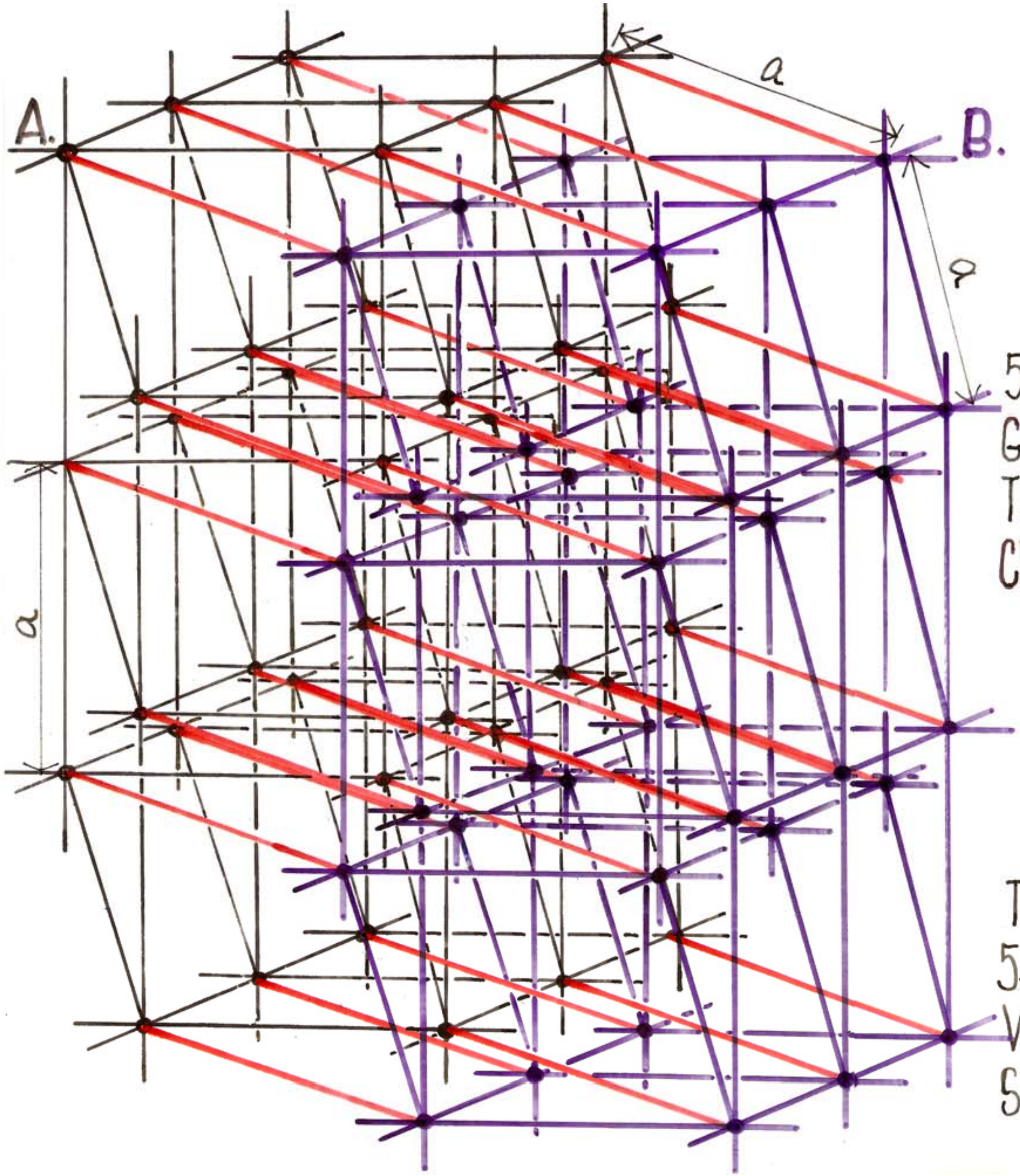
$Den._{(11\text{-}3^{15}.4^{18}_{10})} = 12.70170592\, a/a^3$

RM 11-VALENT $11\text{-}3^{15}4^{18}_{10}$ SPACE LATTICE



TRANSLATION NETWORKS

EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT (CUBIC) LATTICE FROM A-TO B-POSITION, RESULTING IN A UNIFORM SEPTAVALENT LATTICE, THE DENSITY OF WHICH IS 7.00 a/a³. THE 'TRANSLATION LATTICE' IS A 3-D

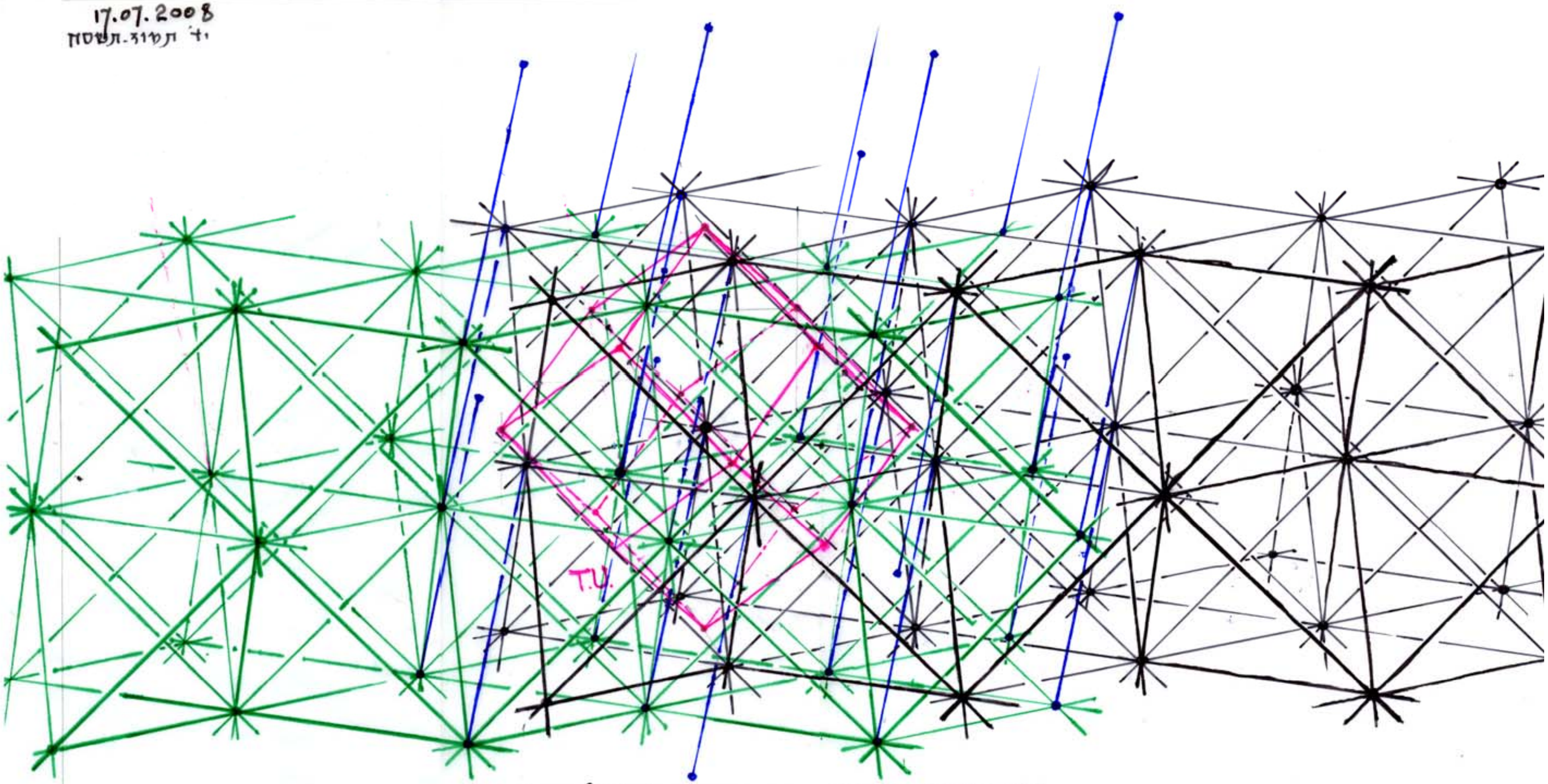


TRANSLATION NETWORKS

5-DIMENSIONAL OCTAVALENT NETWORK, GENERATED BY A DOUBLE EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT CUBIC SPACE-LATTICE.

THE 3-D REPRESENTASION OF THE 5-D NETWORK IS A UNIFORM OCTA-VALENT SPACE-LATTICE WITH A SPATIAL DENSITY OF $D_{en.}=15.00\, a/a^3$

17.07.2008
יד' תמוז-התשסח



$$\text{DENSITY}_{(13-3^{24}.4^{12}_{12})} = 18.38477631\, a/a^3$$

TWO INTER-PENETRATING UNIFORM OCTET SPACE LATTICES WITH EDGE-a, WHEN JOINED TOGETHER WITH A SET OF PARALLEL a-EDGES, GENERATE A UNIFORM 13-VALENT SPACE LATTICE.

The edge length translation could be performed **m** times, leading to a uniform **(n+m)**-valent space lattice, the spatial density of which will amount to:

$$\mathbf{Den.(n+m)} = \frac{\mathbf{Den.(n)}}{\mathbf{E_{T.U.}}} \left[2^{m} . E_{T.U.} + \frac{(1+m)m}{2} \right]$$

In fact <u>m</u> and the spatial density values can reach to infinity (!)

CATEGORIES OF UNIFORM 3D NETWORKS	Number of networks (Found so Far)
1. Centroid related networks	5+13
2. Axis related networks	33 ($\rightarrow \infty$)
3. Plane related (Double Layer) networks	65
4. Centro-Axial-Plane related networks	6 ($\rightarrow \infty$)
5. Multi - Layer Space Networks 6. Poly - Vectorial Space networks	~ 220
7. Translation networks	$\rightarrow \infty$

N

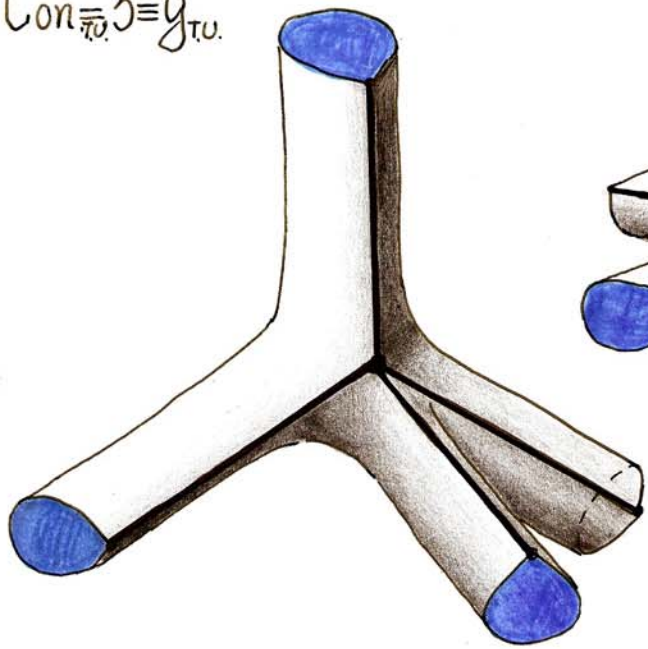
An assumption is formed that we are dealing with probably not more than few **hundreds of uniform space lattices in 3-D space and in view of the valency limiting values and symmetry constraints it seems that an exhaustive systematic search of these configurations is tenable.**

DIAMOND LATTICE PV.4-6^{12}_{3} AND THE RELATED DIAMOND UNI-HEDRON

Val. = 4

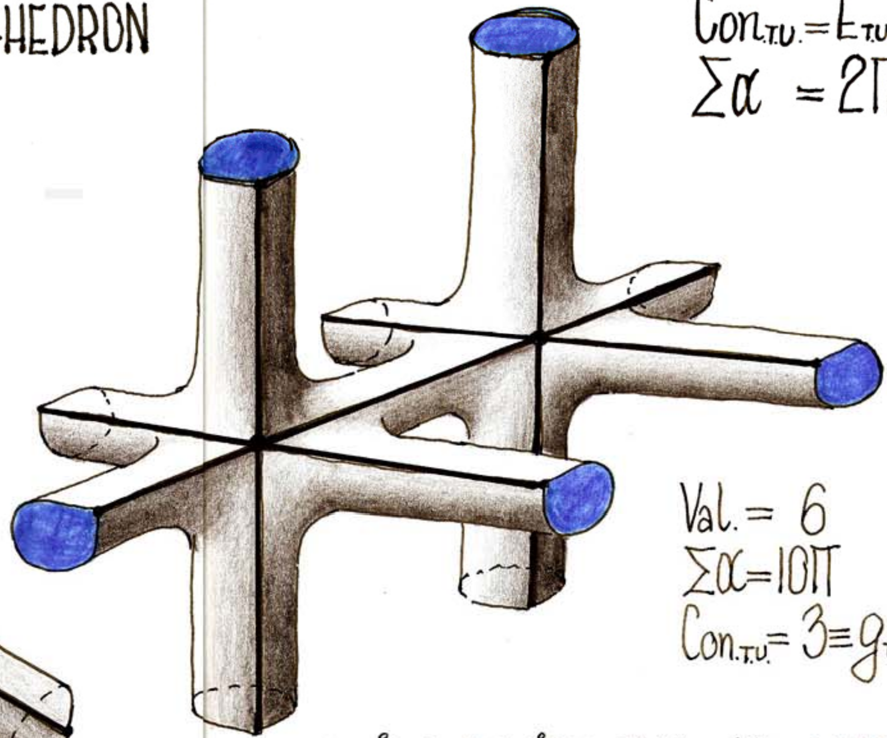
$\sum \alpha = 6\pi$

$Con_{T.U.} = 3 \equiv g_{T.U.}$



$Con_{T.U.} = E_{T.U.} - V_{T.U.} + 1 \equiv g_{T.U.}$

$\sum \alpha = 2\pi(Val. - 1)$



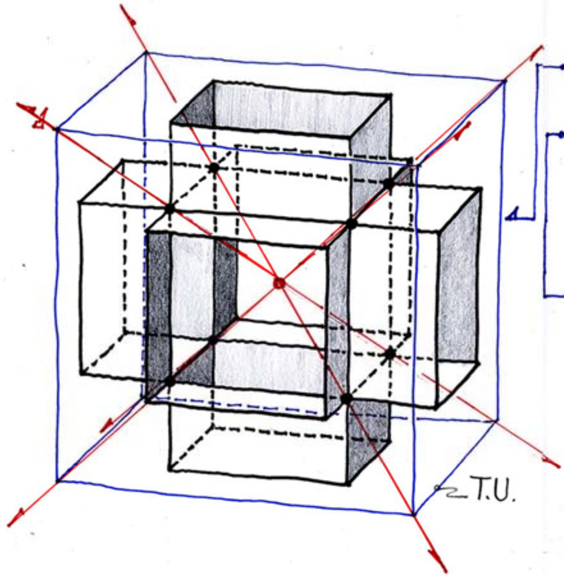
Val. = 6

$\sum \alpha = 10\pi$

$Con_{T.U.} = 3 \equiv g_{T.U.}$

CUBIC LATTICE PV.6-4^{12}_{3} AND THE RELATED CUBIC UNI-HEDRON.

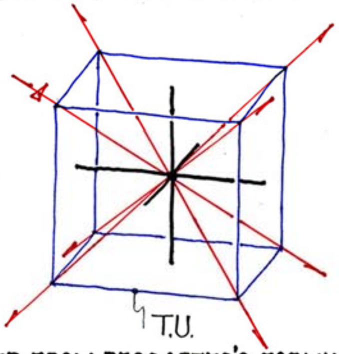
ALL 3-D SPACE NETWORKS MAY BE CONSIDERED AS UNI-HEDRAL TESSELLATIONS OF DISTINCT SPONGE SURFACES.



UNIFORM INFINITE POLYHEDRON- 4^{6}_{3}, $(6;3\pi;3)$.

UNIFORM CUBIC SPACE LATTICE PV.6- 4^{12}_{3}, $(6;10\pi;3)$.

Val.; $\Sigma\alpha$; $g \equiv C$.



DERIVED FROM DESCARTES'S FORMULA:

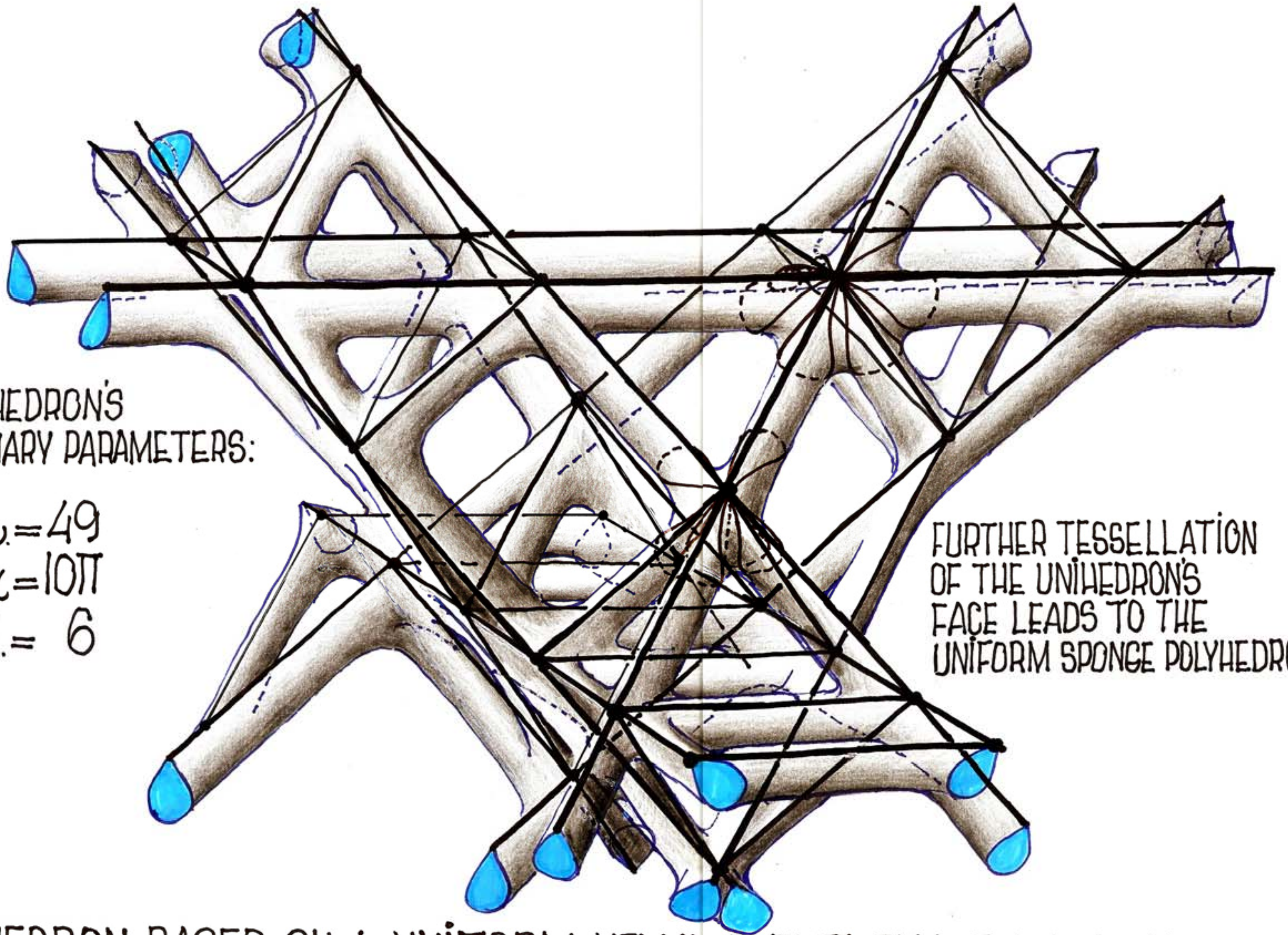
$$\Sigma\alpha = 2\pi\left[1 - \frac{2(1-g)}{V_{T.U.}}\right]; \quad \Sigma\alpha_{4^{6}_{3}} = 2\pi\left[1 - \frac{2(1-3)}{8}\right] = 3\pi$$

$$\Sigma\alpha_{PV.6-4^{12}_{3}} = 2\pi\left[1 - \frac{2(1-3)}{1}\right] = 10\pi$$

TWO LOOK-ALIKE HEXAVALENT NETWORKS DIFFER IN SYMMETRY FEATURES, TRANSLATION UNITS AND THE NUMBER OF VERTICES WITHIN

UNIHEDRON'S PRIMARY PARAMETERS:

$g_{T.U.} = 49$
$\sum\alpha = 10\pi$
$Val. = 6$



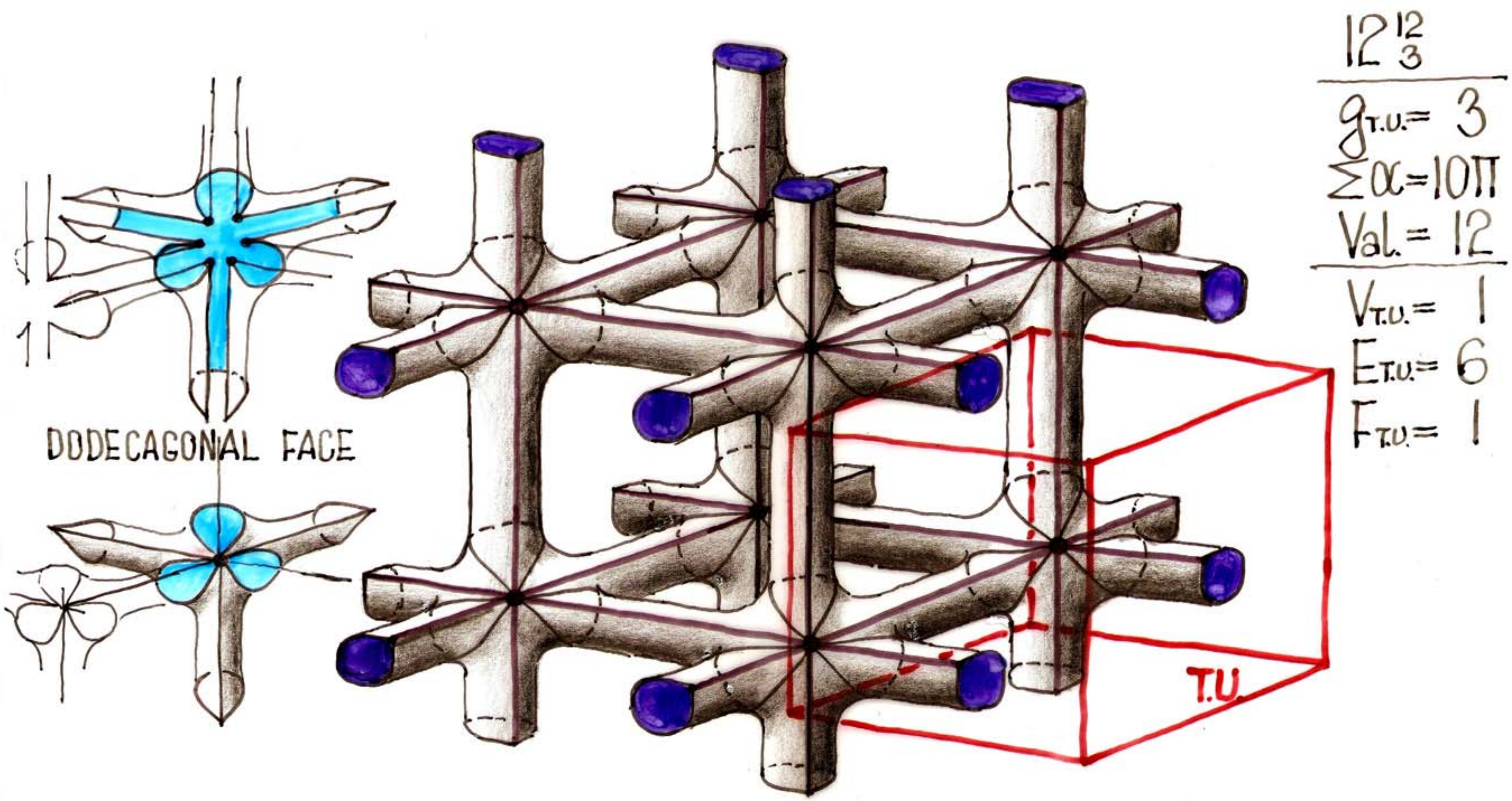
FURTHER TESSELLATION OF THE UNIHEDRON'S FACE LEADS TO THE UNIFORM SPONGE POLYHEDRON:

$(4^2.6)^6_{49}$

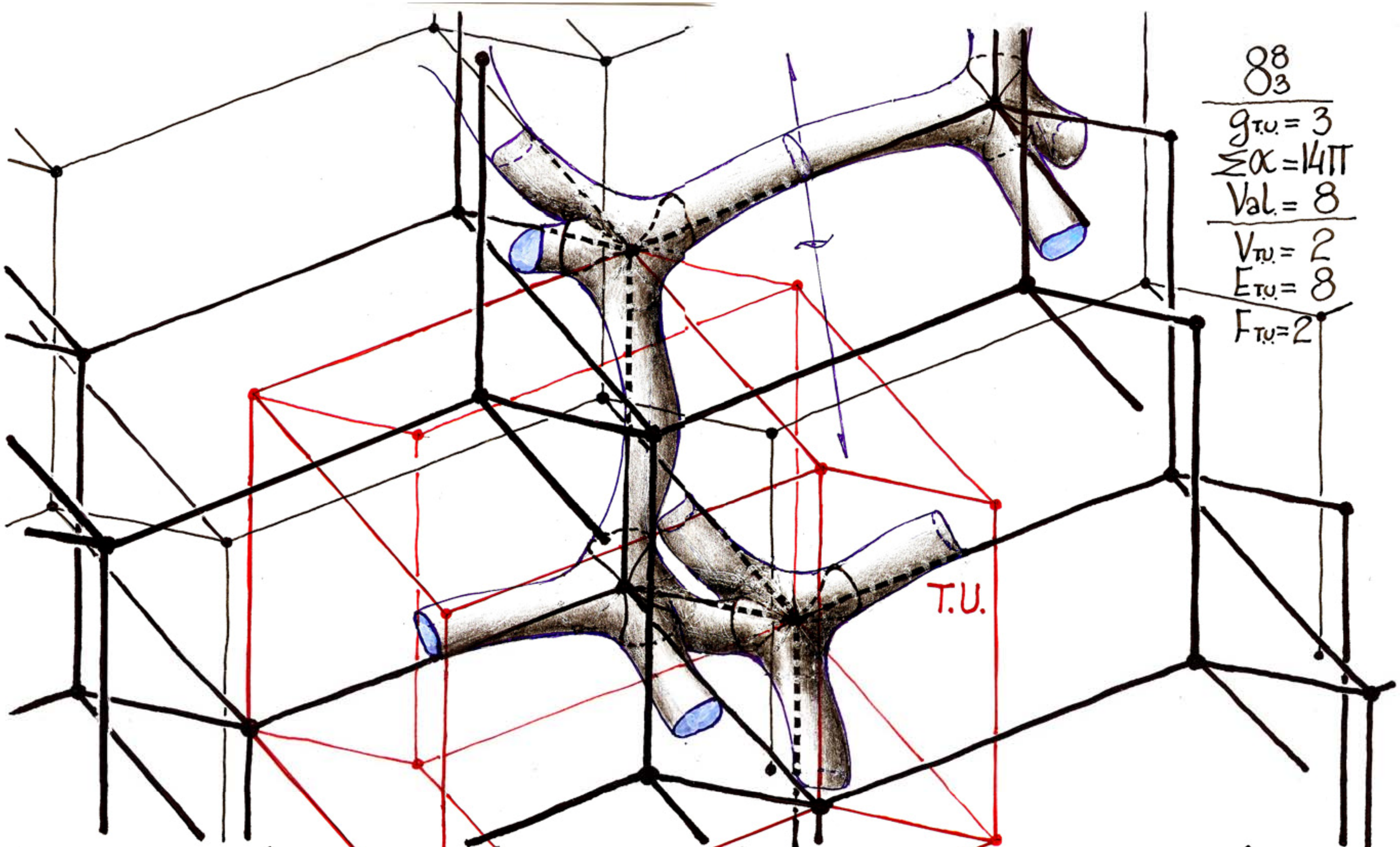
$g_{T.U.} = 49$
$\sum\alpha = 10\pi$
$Val. = 18$

$V_{T.U.} = 24$
$E_{T.U.} = 216$
$F_{T.U.} = 96$

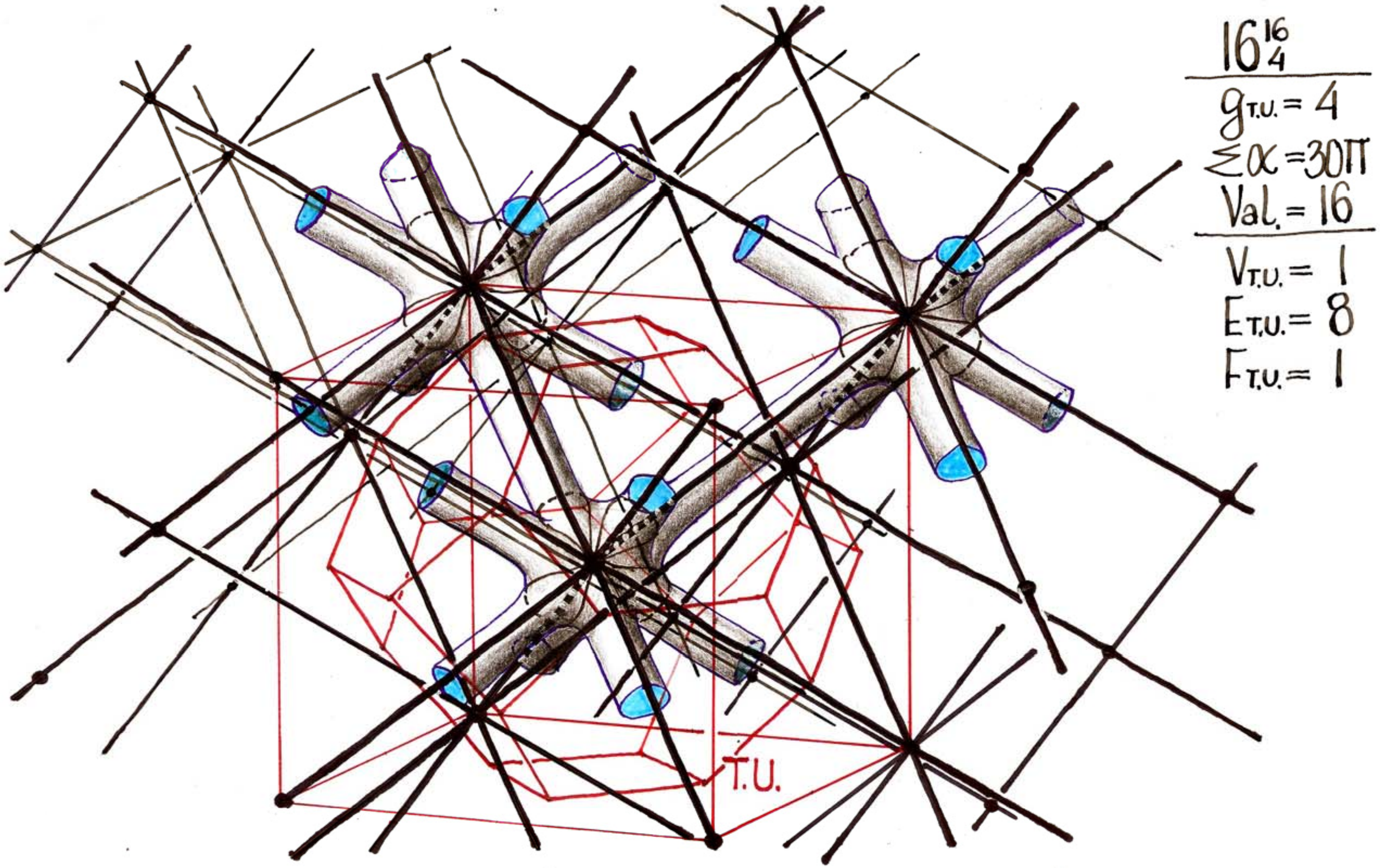
UNIHEDRON BASED ON A UNIFORM HEXAVALENT PV.6-$3^5.4^4.6^2.12^2_{49}$ SPACE LATTICE



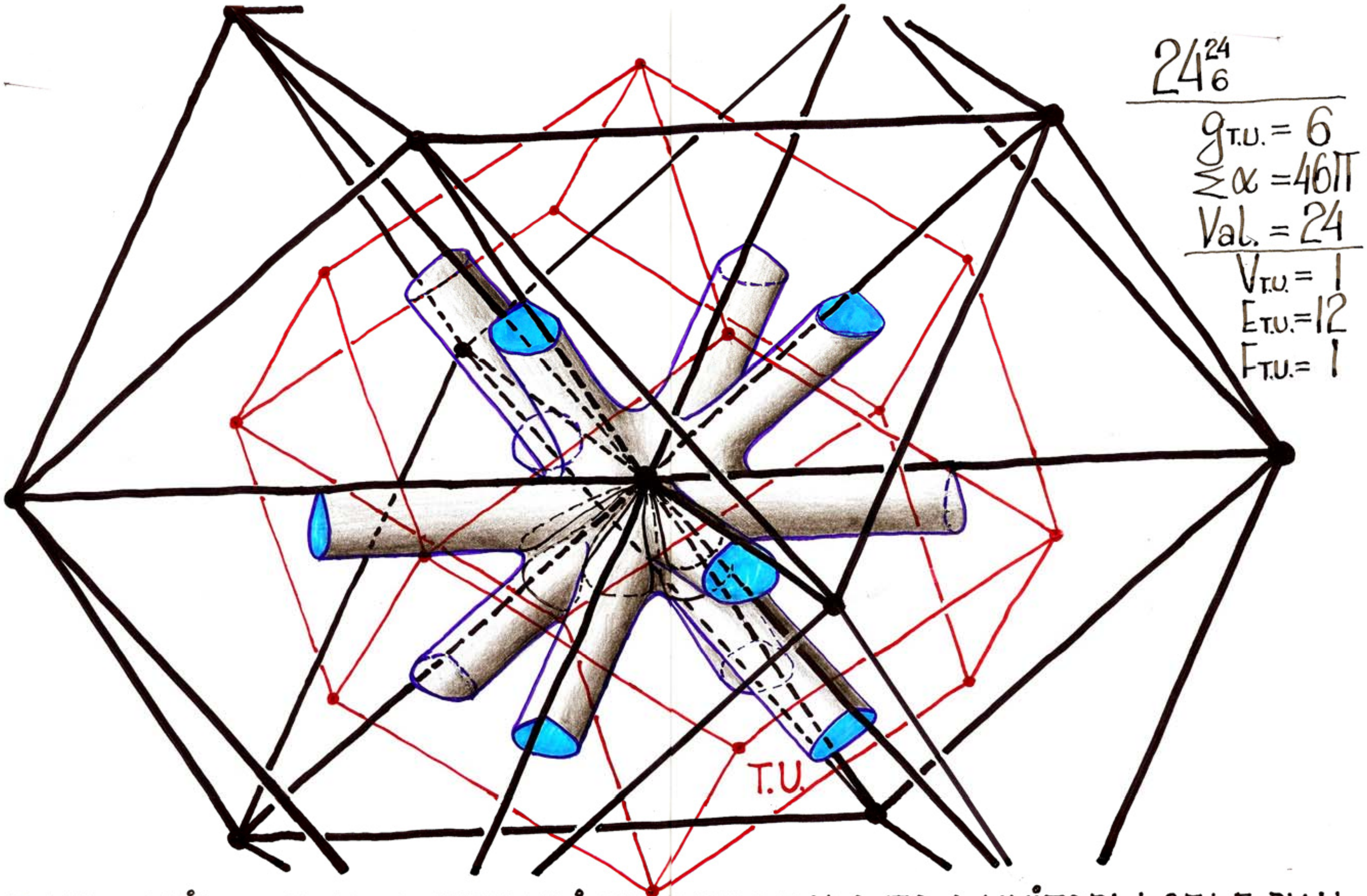
TESSELLATION OF A CUBIC UNI-HEDRON LEADS TO A UNIFORM SELF-DUAL 12^{12}_{3} SPONGE POLYHEDRON WITH JUST ONE FACE PER T.U.



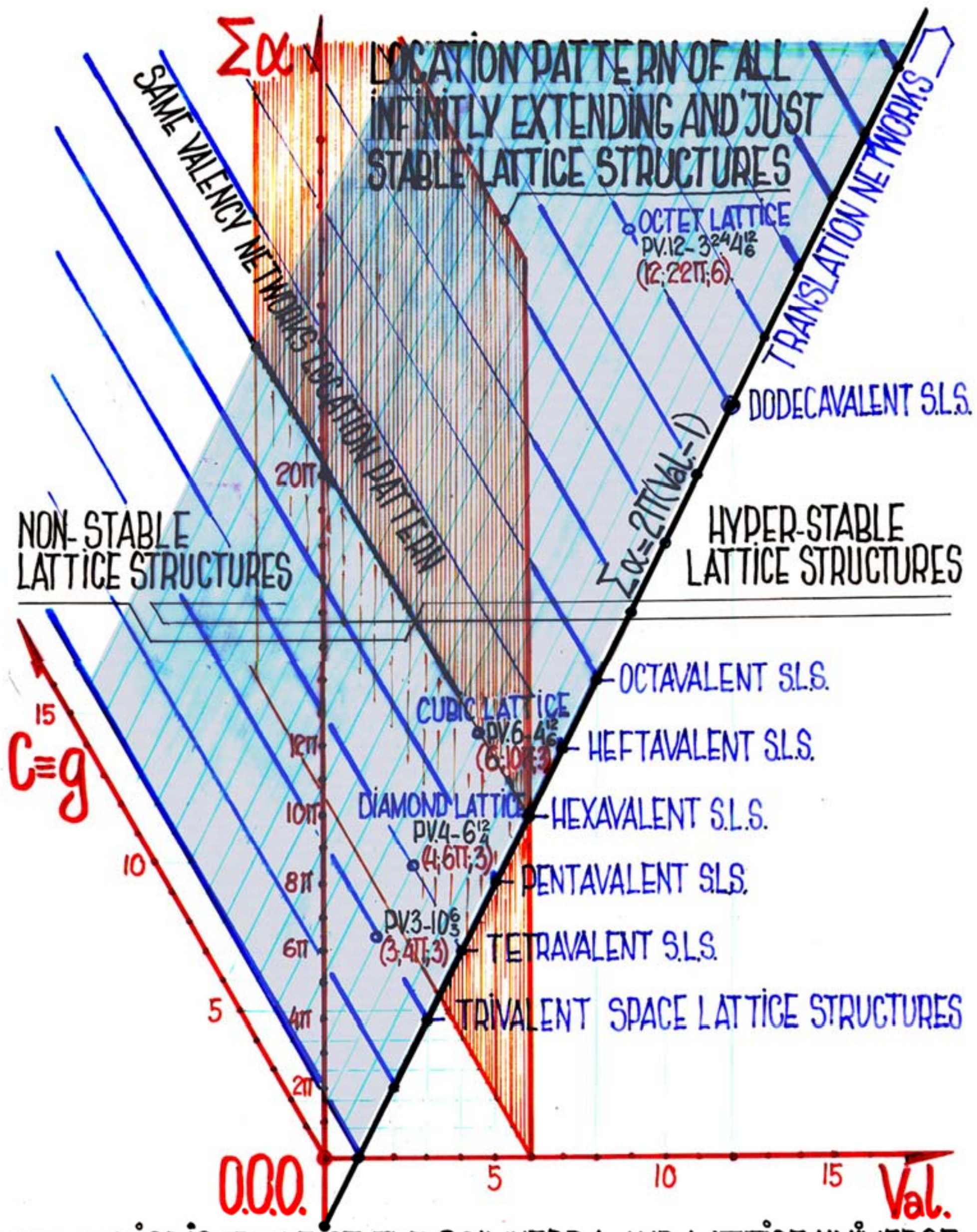
TESSELLATION OF A DIAMOND UNI-HEDRON LEADS TO A UNIFORM SELF-DUAL 8^{8}_{3} SPONGE POLYHEDRON WITH JUST TWO FACES PER T.U.



TESSELLATION OF A C.C. UNI-HEDRON LEADS TO A UNIFORM SELF-DUAL 16^{16}_{4} SPONGE POLYEDRON WITH JUST ONE FACE PER T.U.



TESSELLATION OF AN OCTET UNI-HEDRON LEADS TO A UNIFORM SELF-DUAL 24^{24}_{6} SPONGE POLYHEDRON WITH JUST ONE FACE PER T.U.



THE PERIODIC TABLE OF THE POLYHEDRA AND LATTICE UNIVERSE.
LOCATION PATTERN OF UNIFORM SPACE-LATTICE CONFIGURATIONS.

IN CONCLUSION

3D networks and the associated hyperbolical sponge surfaces seem to pose a critical aspect in all 'material sciences' and as an extension of graph-theory, dealing geometrically with any plurality that may exist, of focal entities and their inter-relations.

*After investing in the systematic research of the topic, the author claims enumerating, categorizing and graphically describing, **so far,** about 350 uniform 3D space networks and related hyperbolical sponge surface configurations.*

The effort is meant to support an evolution of new imagery which might influence scientific exploration and inspire art, architecture and innovative space structures.

F

By defining as 'morphic' those processes which display a movement toward greater 3-dimensional spatial order, symmetry or form (Whyte-1969) and **morphology as the logical preoccupation with and manipulation of those processes,** than the **research into the nature of networks and the associated sponge surfaces may be classified as the essence of morphology.**