A METHOD FOR EXHAUSTIVE ENUMERATION OF UNIFORM NETWORKS IN 3-D SPACE

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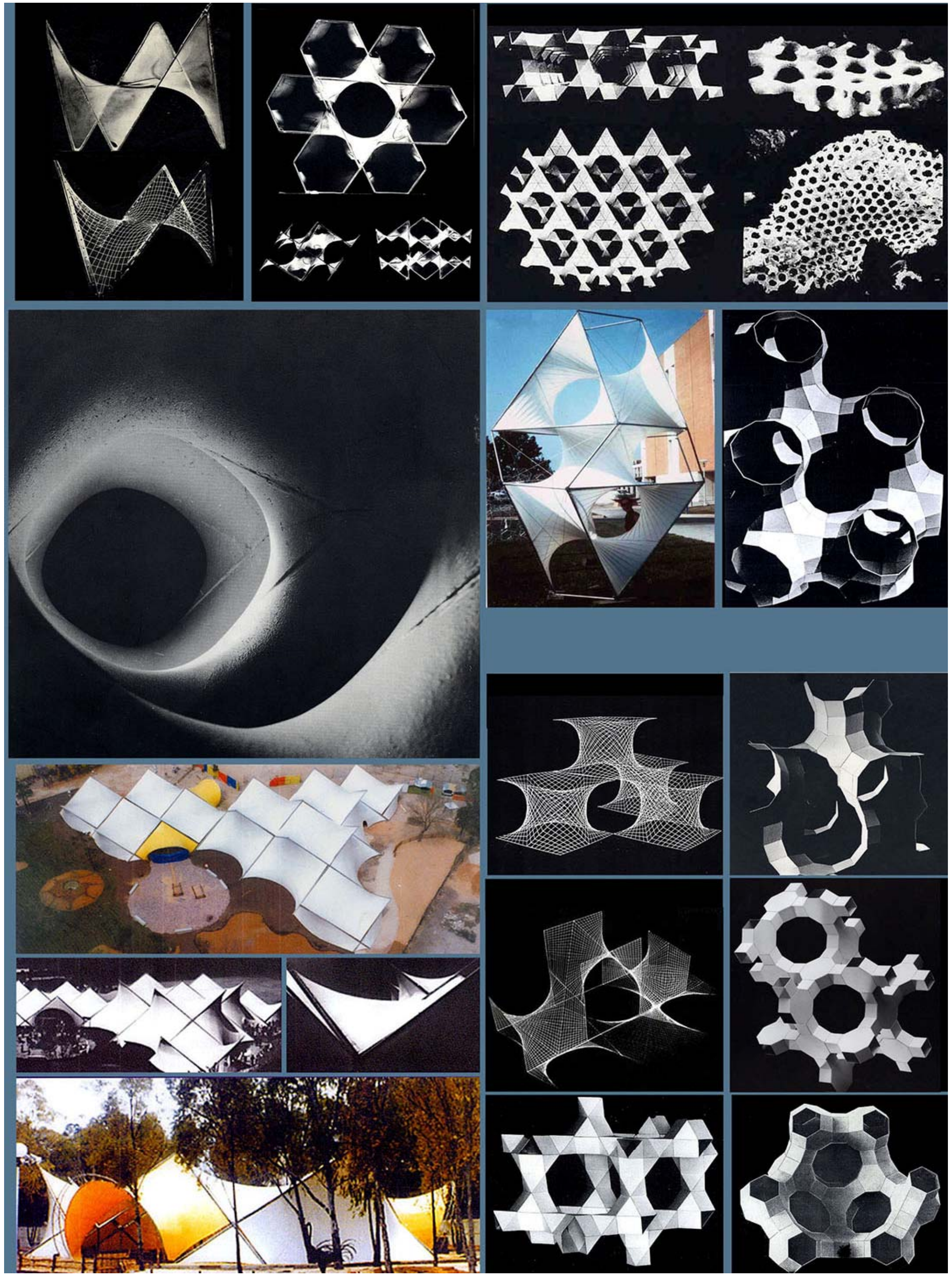
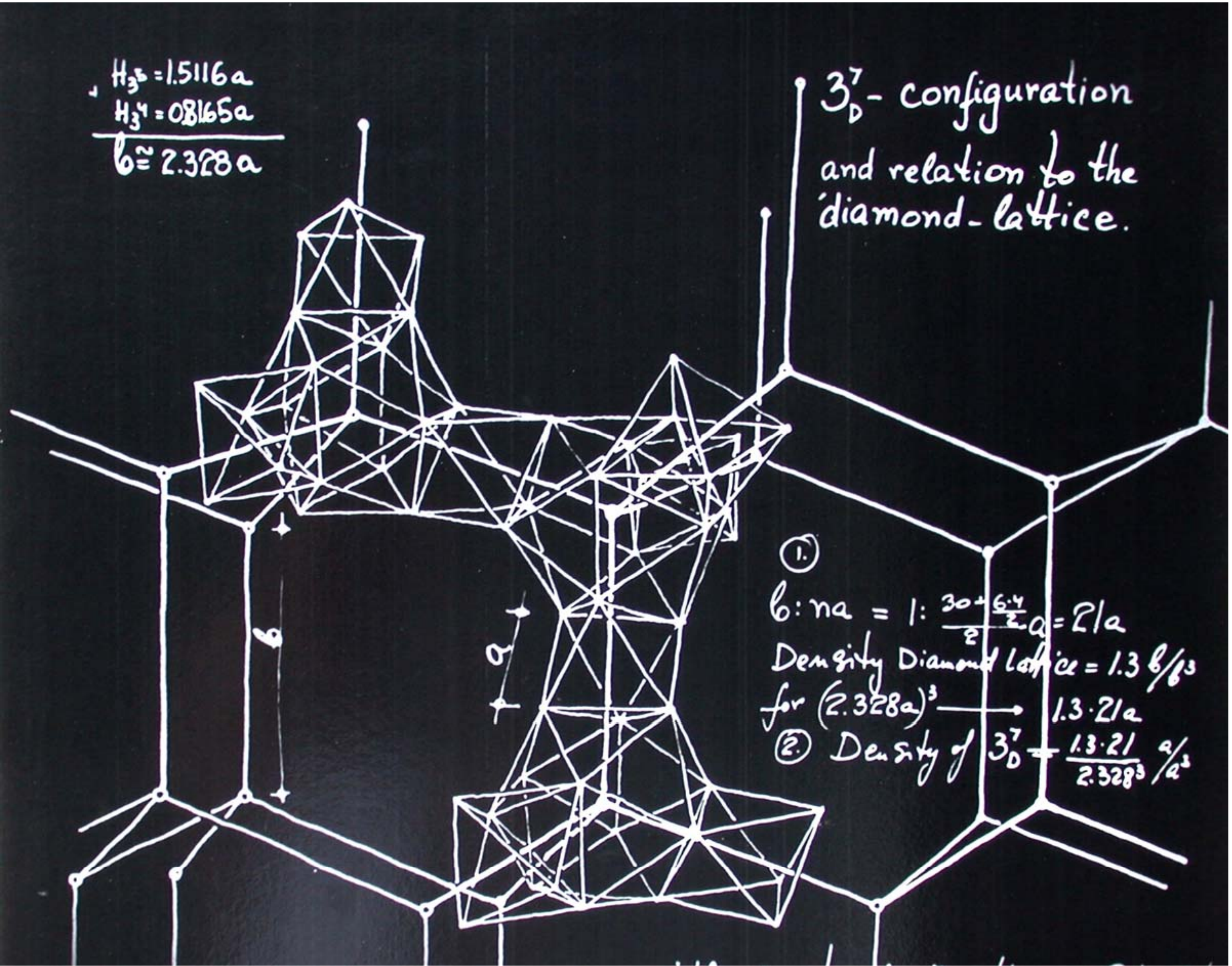
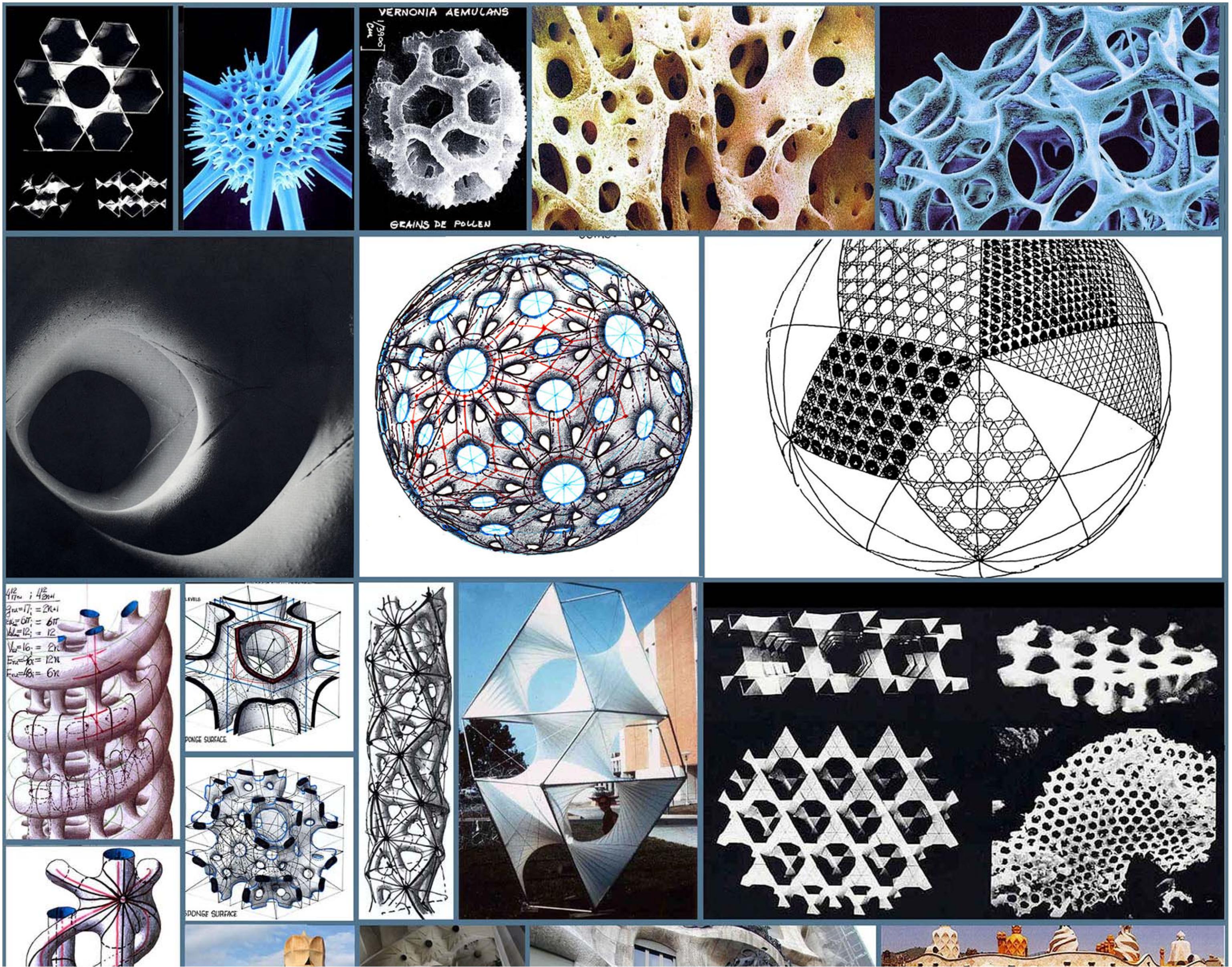
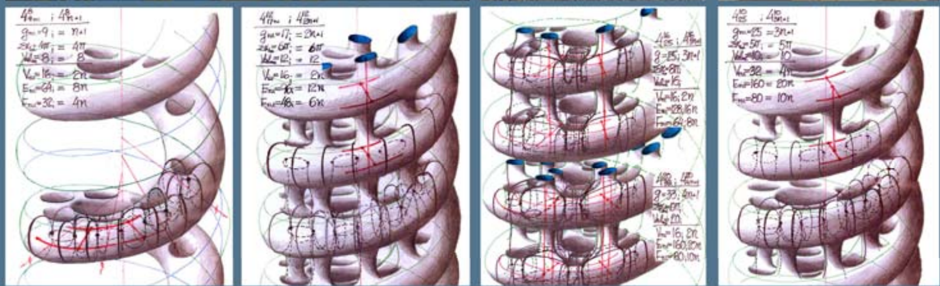


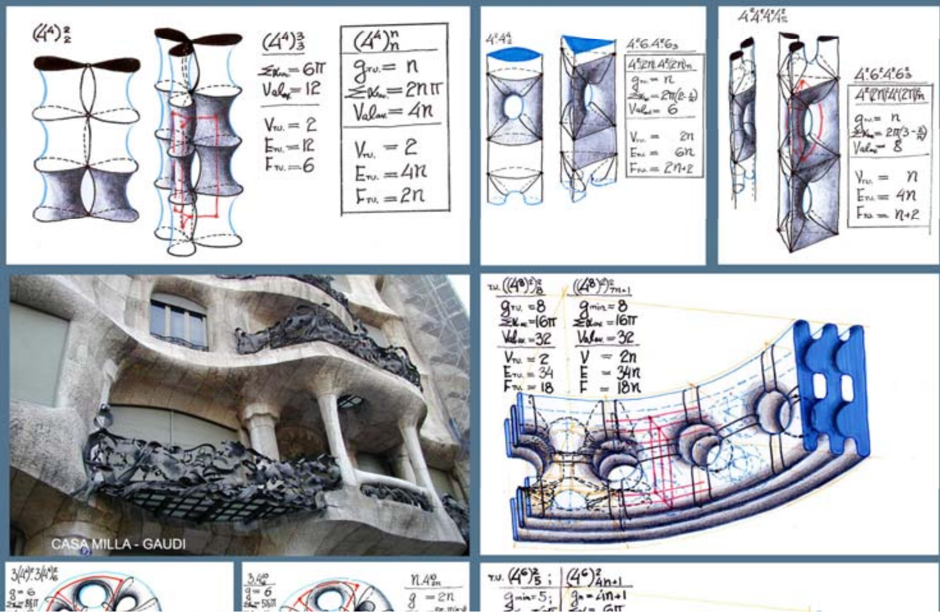


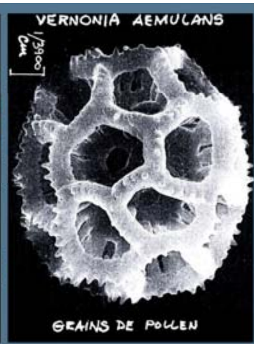
$H_3^3 = 1.5116a$
$H_3^4 = 0.8165a$
$b \approx 2.328a$
3^7_D - configuration
and relation to the
'diamond-lattice.
a
b
1.
$b : na = 1 : \frac{30+\frac{6\cdot 4}{2}}{2} a = 21a$
Density Diamond lattice = $1.3\ b/b^3$
for $(2.328a)^3 \longrightarrow 1.3 \cdot 21a$
2. Density of $3^7_D = \frac{1.3 \cdot 21}{2.328^3}\ a/a^3$

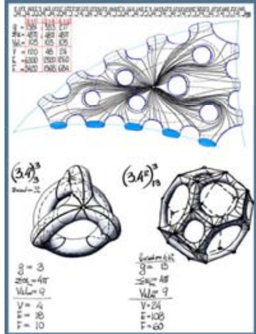

VERNONIA AEMULANS
GRAINS DE POLLEN

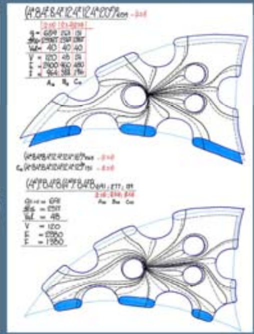


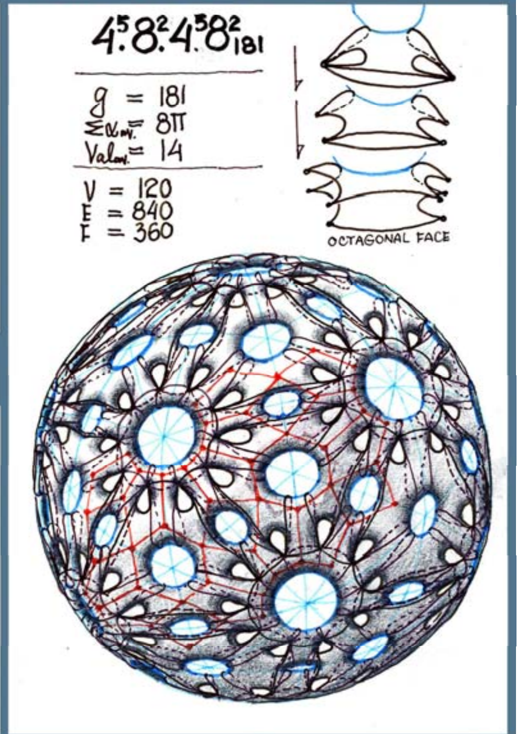
PRIMITIVE UNIFORM SPONGE POLYHEDRA M.BURT





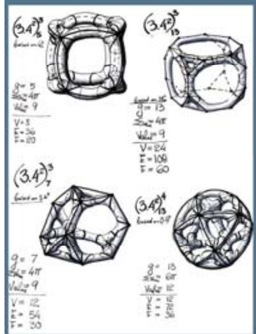


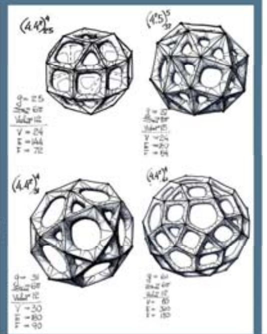


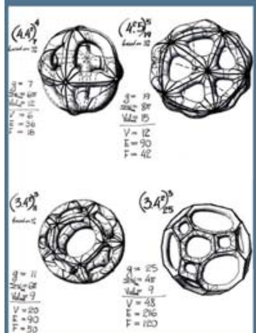


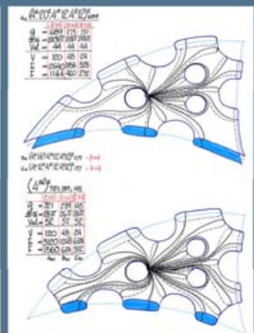
UNIFORM SPHERICAL SPONGE POLYHEDRA

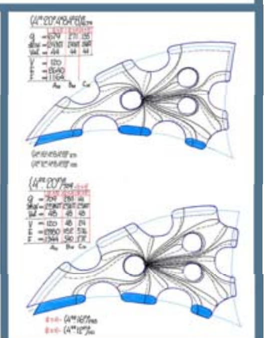
M.BURT





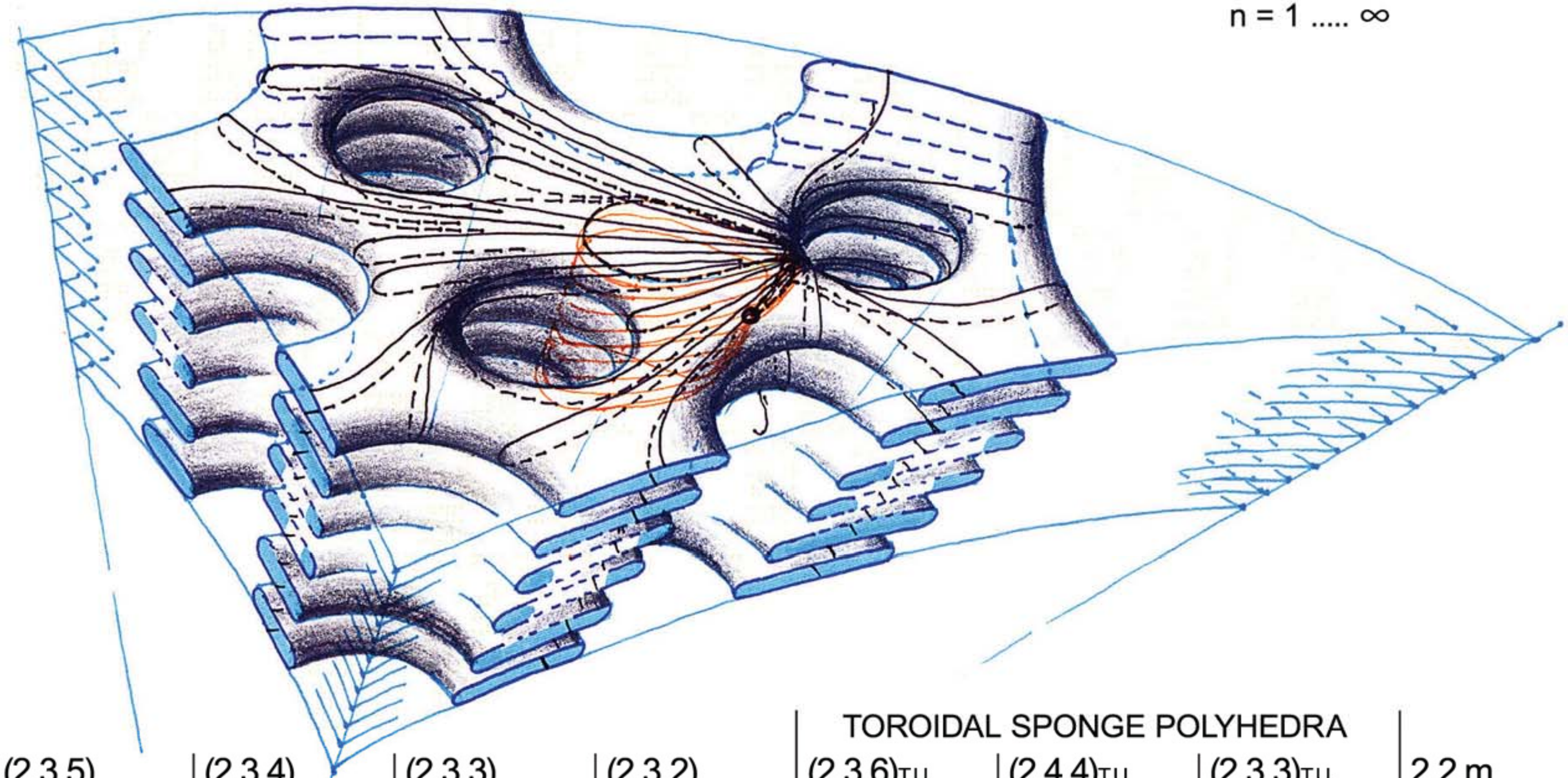






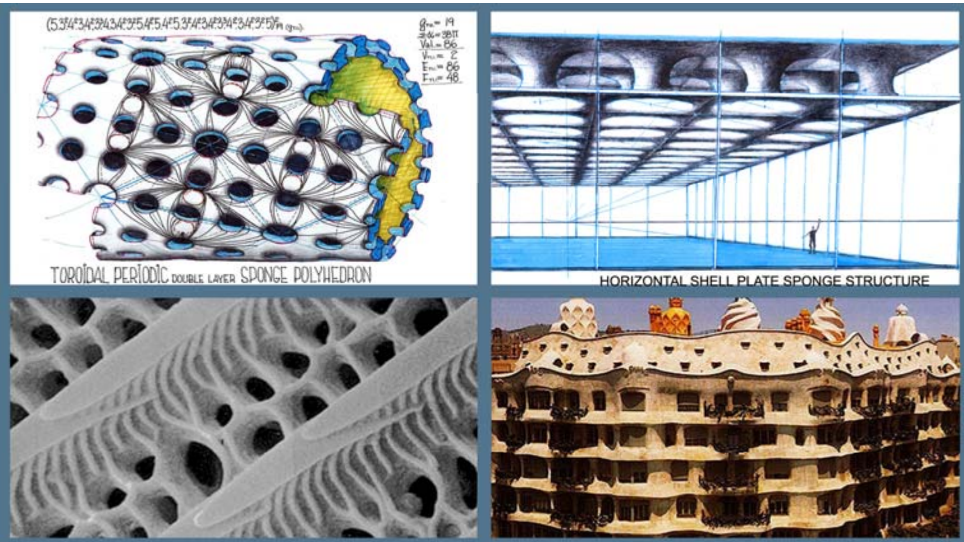
$$(3.4^{11}_{.}4^{11}_{.}3(2.4^{12}_{.}4^{12})^{n-1}_{.}2.3.4^{11}_{.}4^{11})^{2}_{1321+720(n-1)} \quad ;(2.3.5)$$

$$n = 1 \; ..... \; \infty$$

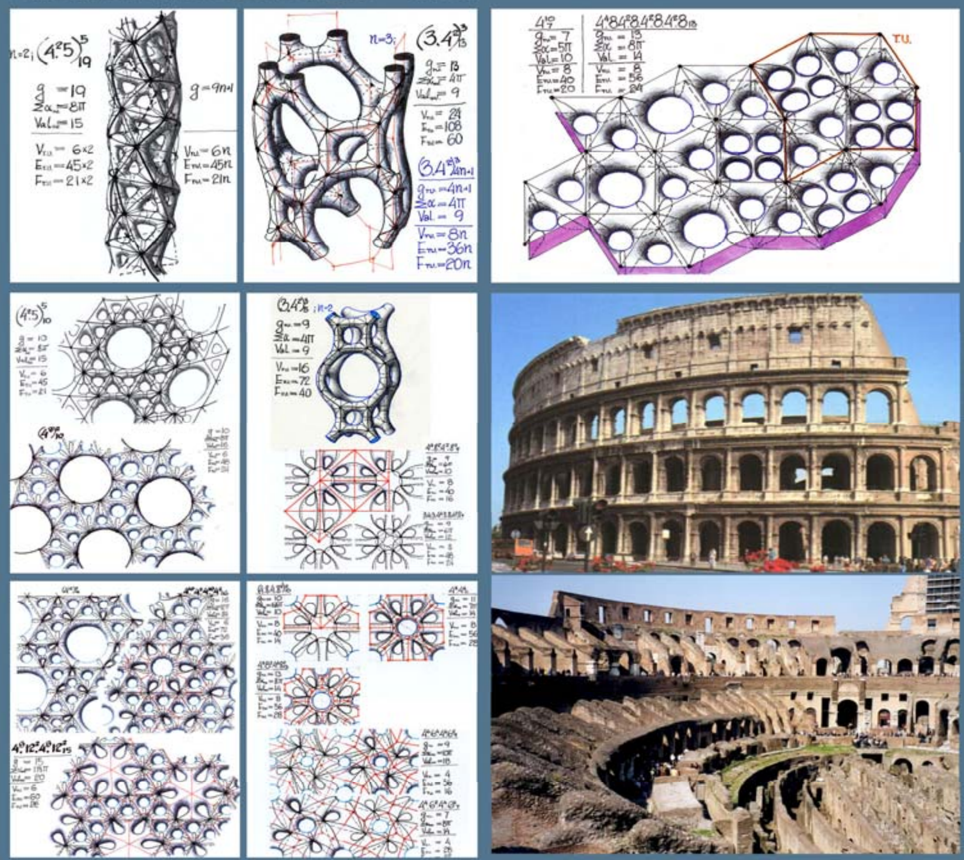


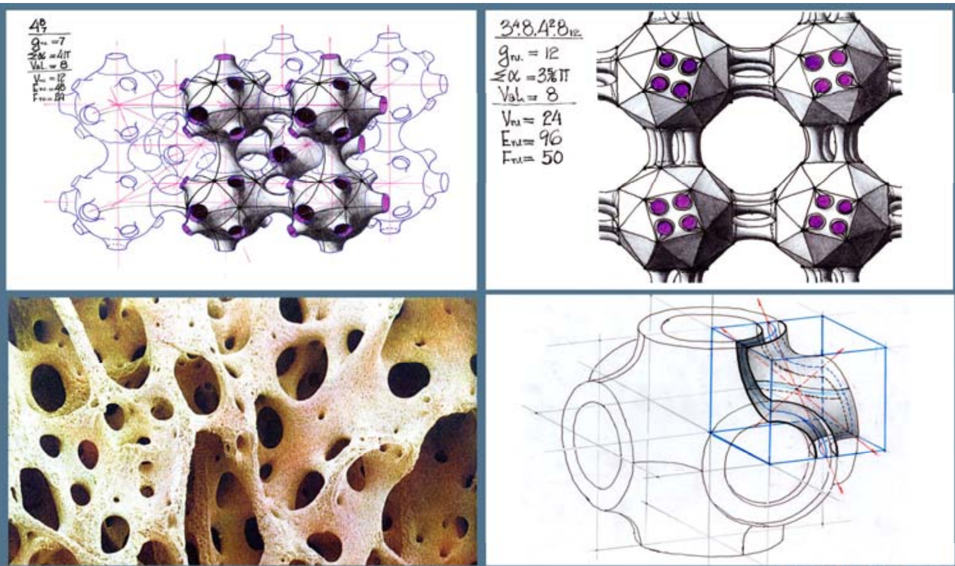
SYMM. GROUP	(2.3.5)	(2.3.4)	(2.3.3)	(2.3.2)	TOROIDAL SPONGE POLYHEDRA (2.3.6)T.U.	(2.4.4)T.U.	(2.3.3)T.U.	2.2.m
g	1321+720(n-1)	529+288(n-1)	265+144(n-1)	133+72(n-1)	133+72(n-1)	89+48(n-1)	67+36(n-1)	(22+12(n-1))m+1
$\Sigma\alpha_{av.}$	$46\pi+24\pi(n-1)$	→	→	→	→	→	→	→
$Val._{av.}$	96+50(n-1)	→	→	→	→	→	→	→
V	120	48	24	12	12	8	6	2m
E	5760+3000(n-1)	2304+1200(n-1)	1152+600(n-1)	576+300(n-1)	576+300(n-1)	384+200(n-1)	288+150(n-1)	(96+50(n-1))m
F	3000+1560(n-1)	1200+624(n-1)	600+312(n-1)	300+156(n-1)	300+156(n-1)	200+96(n-1)	150+78(n-1)	(50+26(n-1))m

Uniform Spherical multi-layer sponge polyhedron of (2,3,5) symmetry group.

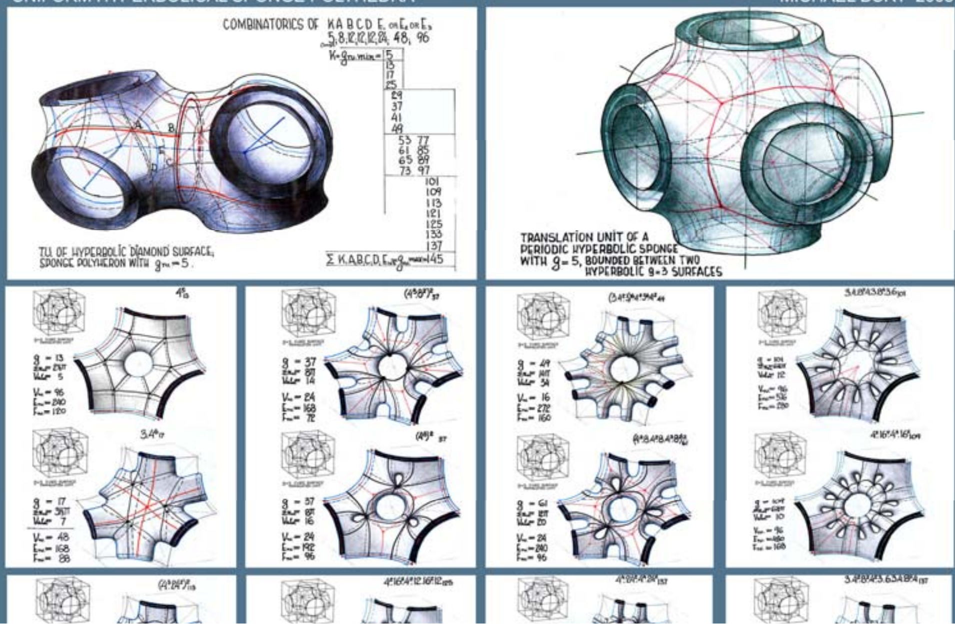


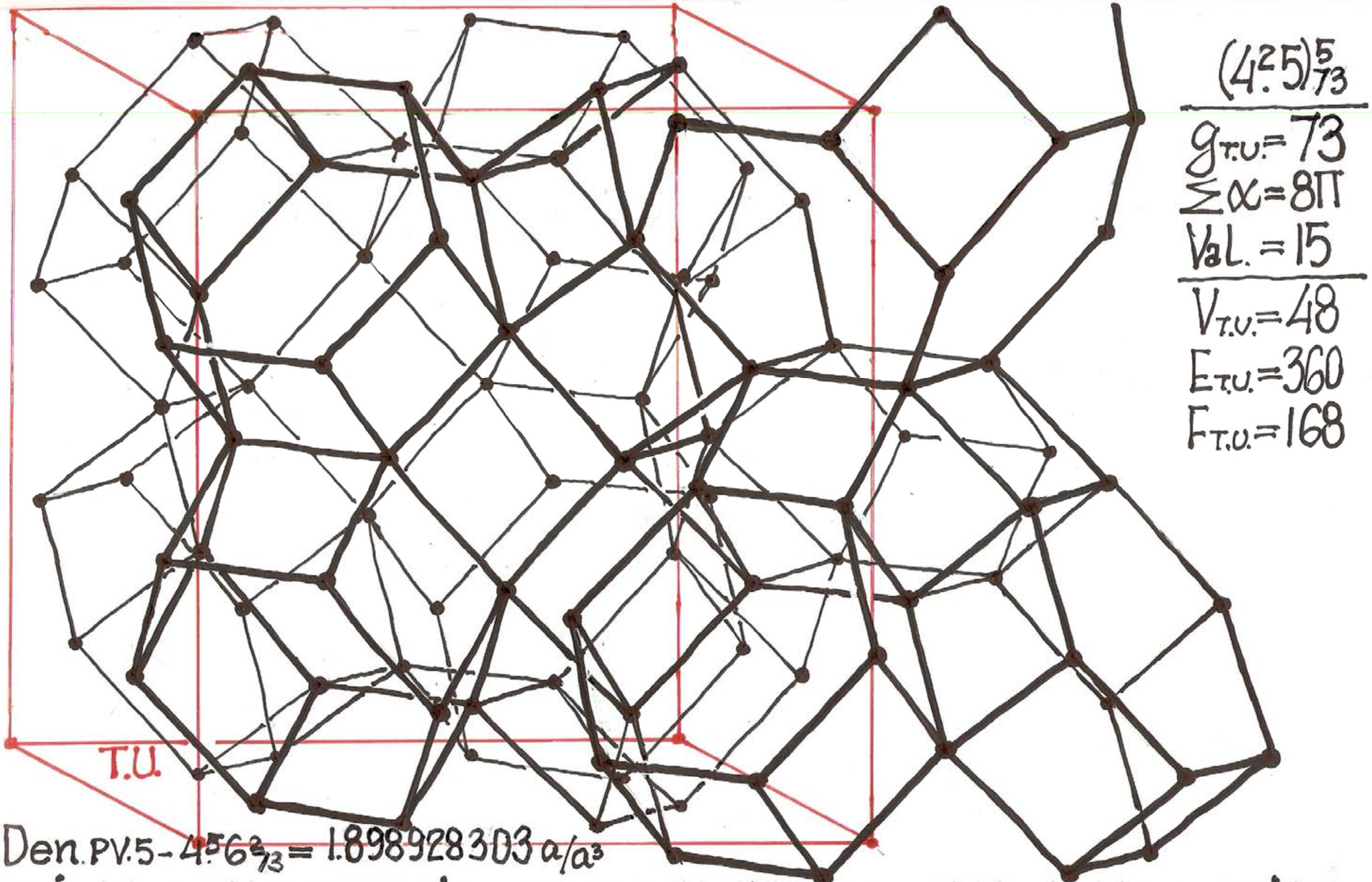
UNIFORM TOROIDAL SPONGE POLYHEDRA M.BURT





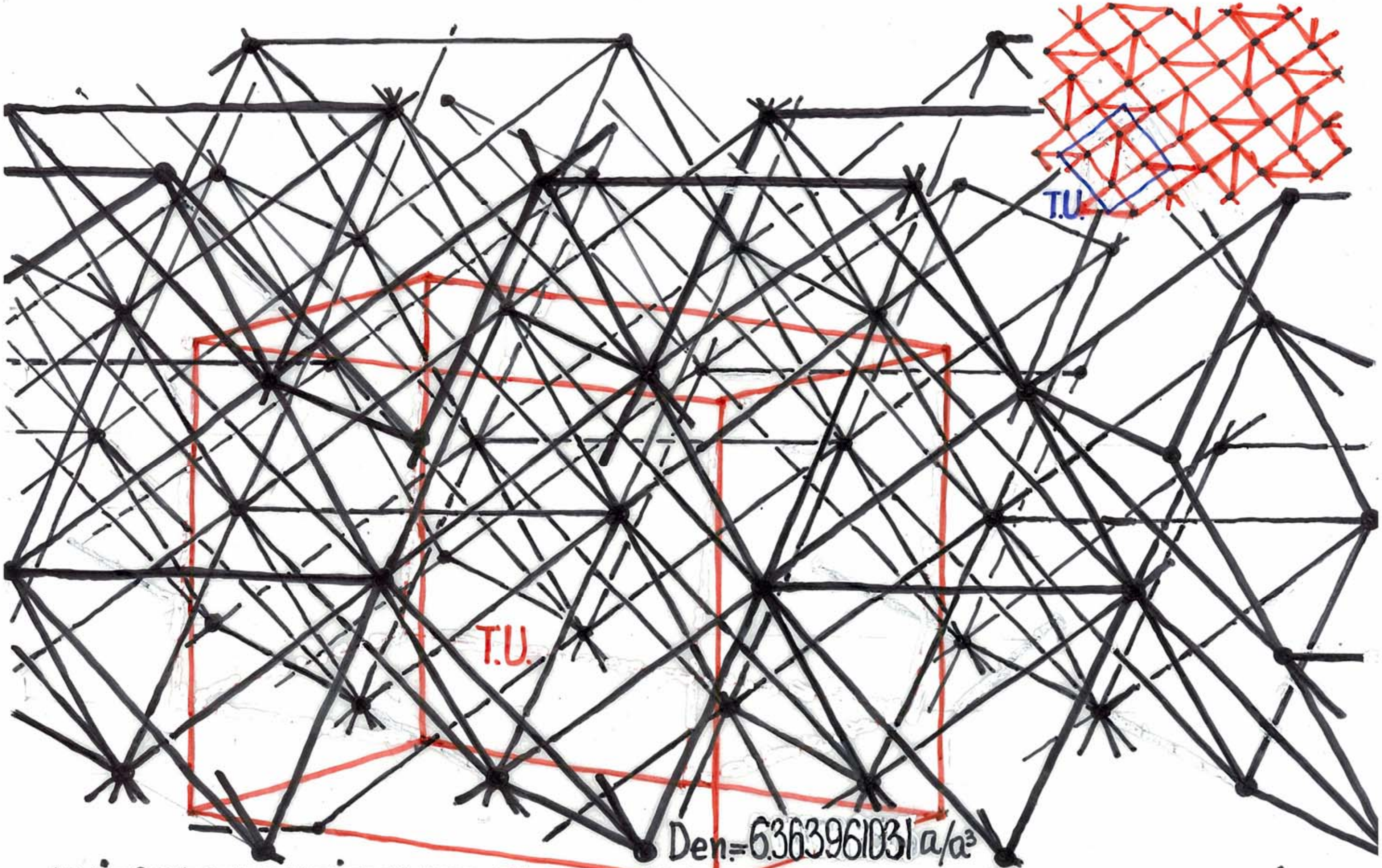
UNIFORM HYPERBOLICAL SPONGE POLYHEDRA MICHAEL BURT 2008



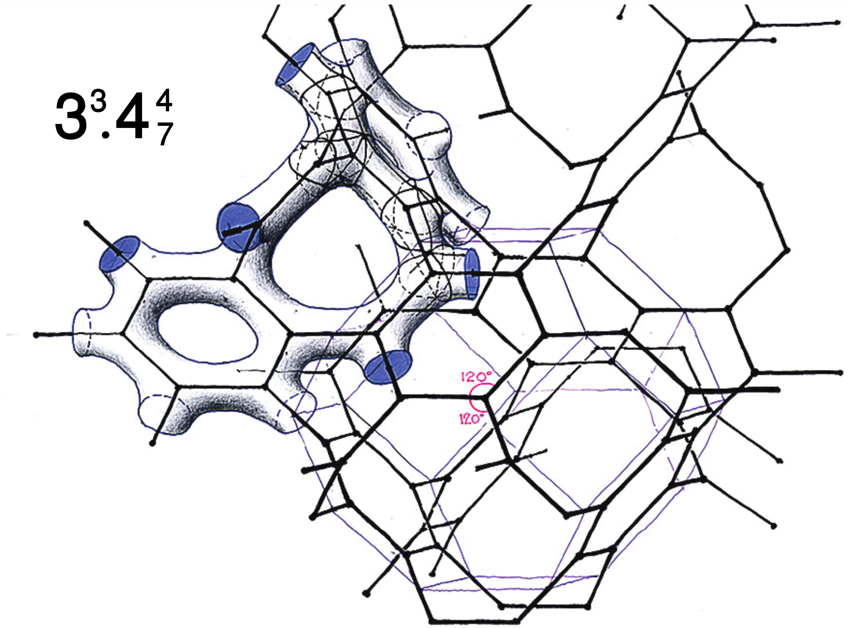


$Den._{PV.5-4^5.6^2{}_{73}} = 1.898928303\ a/a^3$

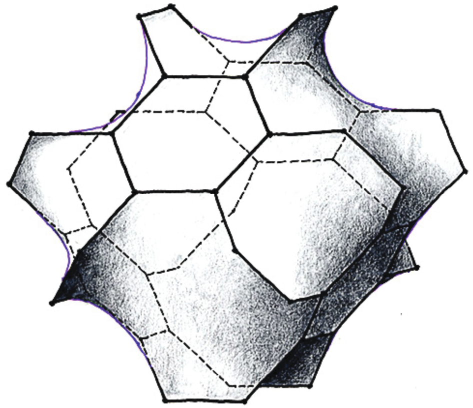
UNIFORM POLYVECTORIAL PENTAVALENT $PV.5-4^5.6^2{}_{73}$ SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.



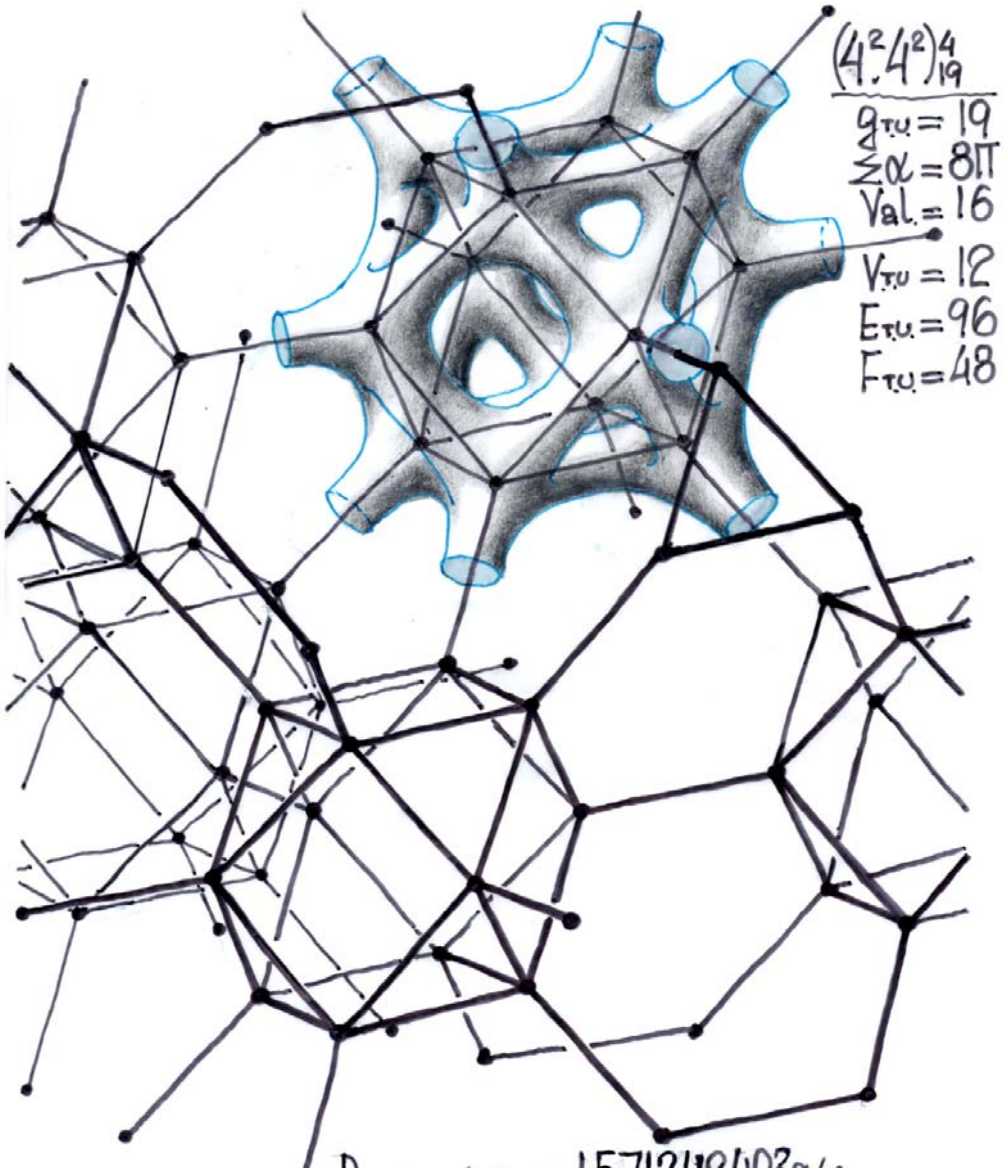
UNIFORM MULTI-LAYER ENNEAVALENT U.ML.9 - $3^{6}.4^{24}_{15}$ SPACE LATTICE.



Den.PV.3-6.8^2.12^2_7 = 0.47140452 a/a^3



SELF CLOSE-PACKING SADDLE POLYHEDRON WITH 6,8 AND 12-GONS GENERATING UNIFORM TRIVALENT PV.3-6.8^2.12^2_7 SPACE LATTICE

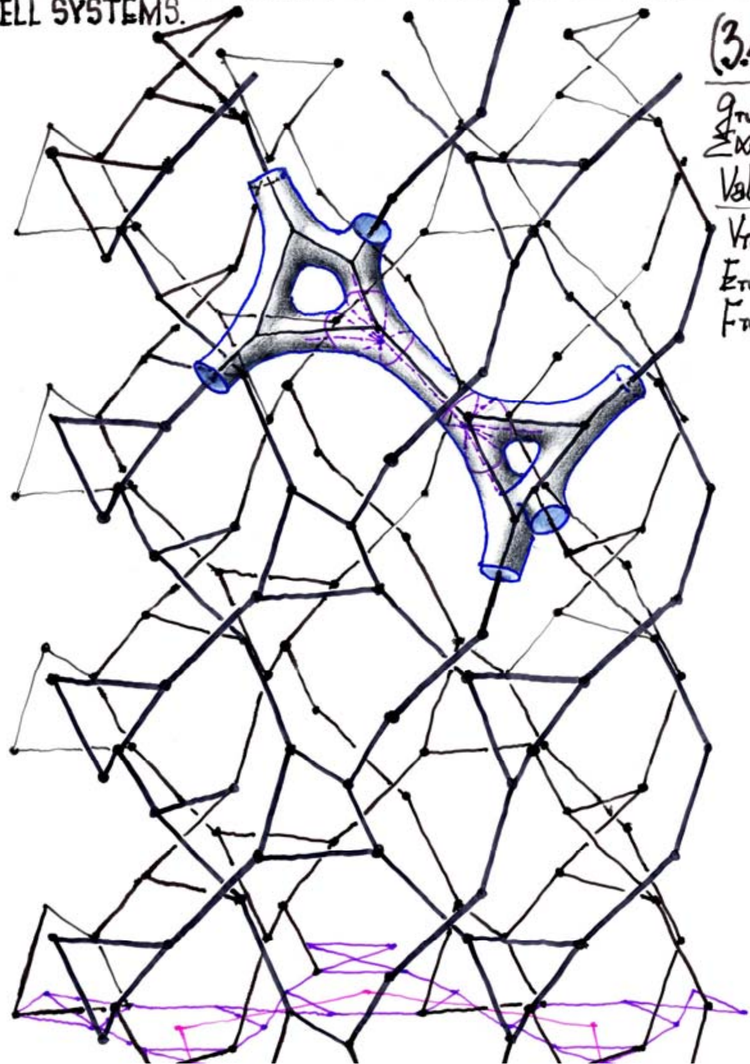


Den. $_{P5\text{-}(3.4)^2.6^6_{19}} = 1.5713484030\,a/a^3$

$P5\text{-}3^2.4^2.6^6.8^2.12^2_{19}$

UNIFORM POLY-VECTORIAL PENTAVALENT $P5\text{-}(3.4)^2.6^6_{19}$ SPACE LATTICE
AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

THE TRIVALENT SPACE LATTICE GENERATES TWO ENTANGLED CLOSE-PACKING CELL SYSTEMS.



$$(3.4^2)^3_7$$

$g_{TU.} = 7$

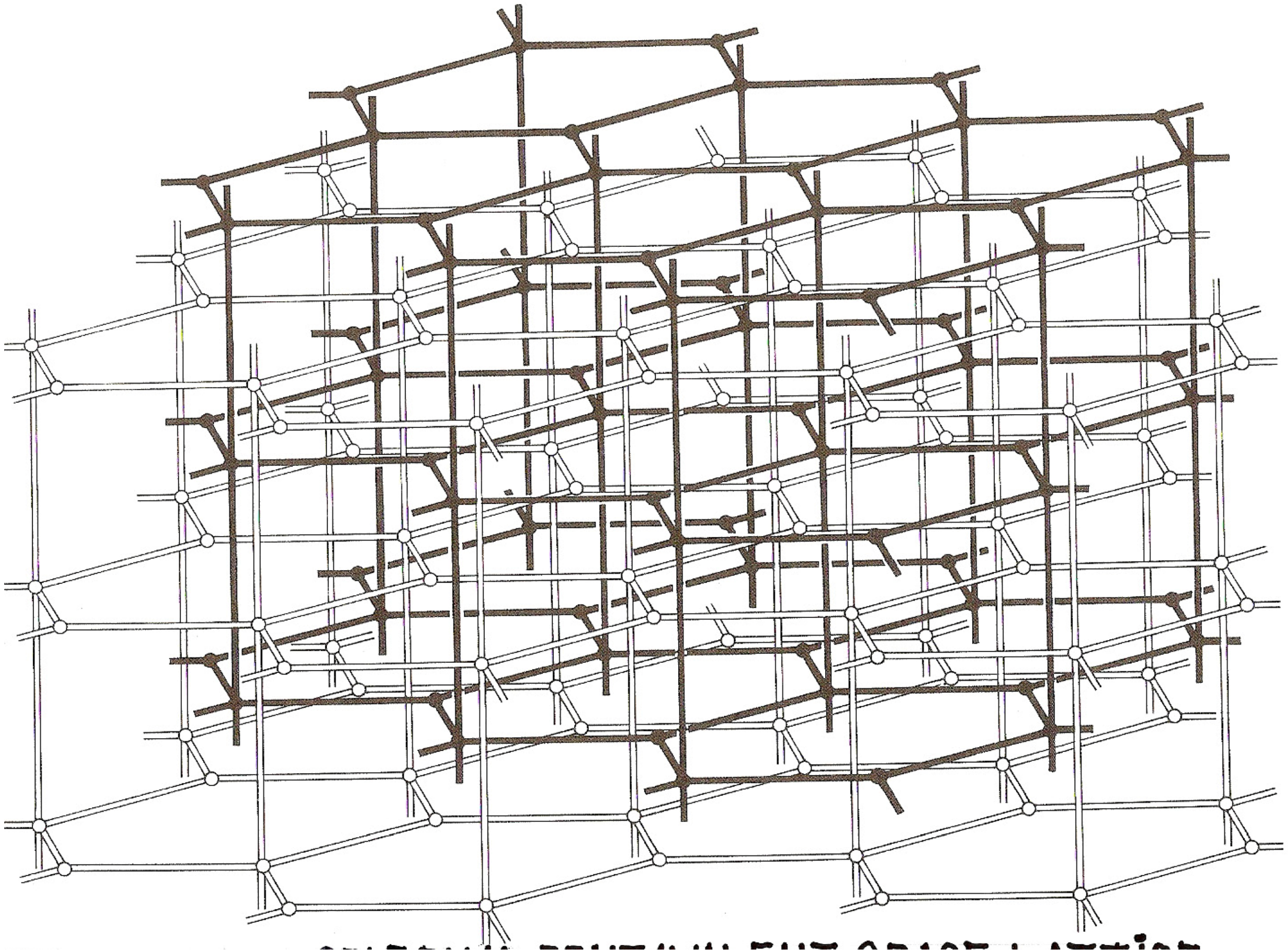
$\Sigma_{K_G} = 4\pi$

$Val. = 9$

$V_{TU.} = 12$

$E_{TU.} = 54$

$F_{TU.} = 30$



In his monumental publication on ‘Structural inorganic chemistry’ (1962) A.F. Wells makes a startling observation: “The theory of these nets does not appear to be known, and in fact no attempt to derive them systematically seems to have been made until comparatively recently” (p.101). In his later publication of ‘Three-Dimensional Nets and Polyhedra’ (1975) discussed by Grunbaum and reviewed by Coxeter in the Bulletin of the American Mathematical society (May – 1978), Wells presents all ‘systematically derived” nets, summarized in a table(p.469), the greater majority of which were already published by the author in ‘Infinite Polyhedra’.

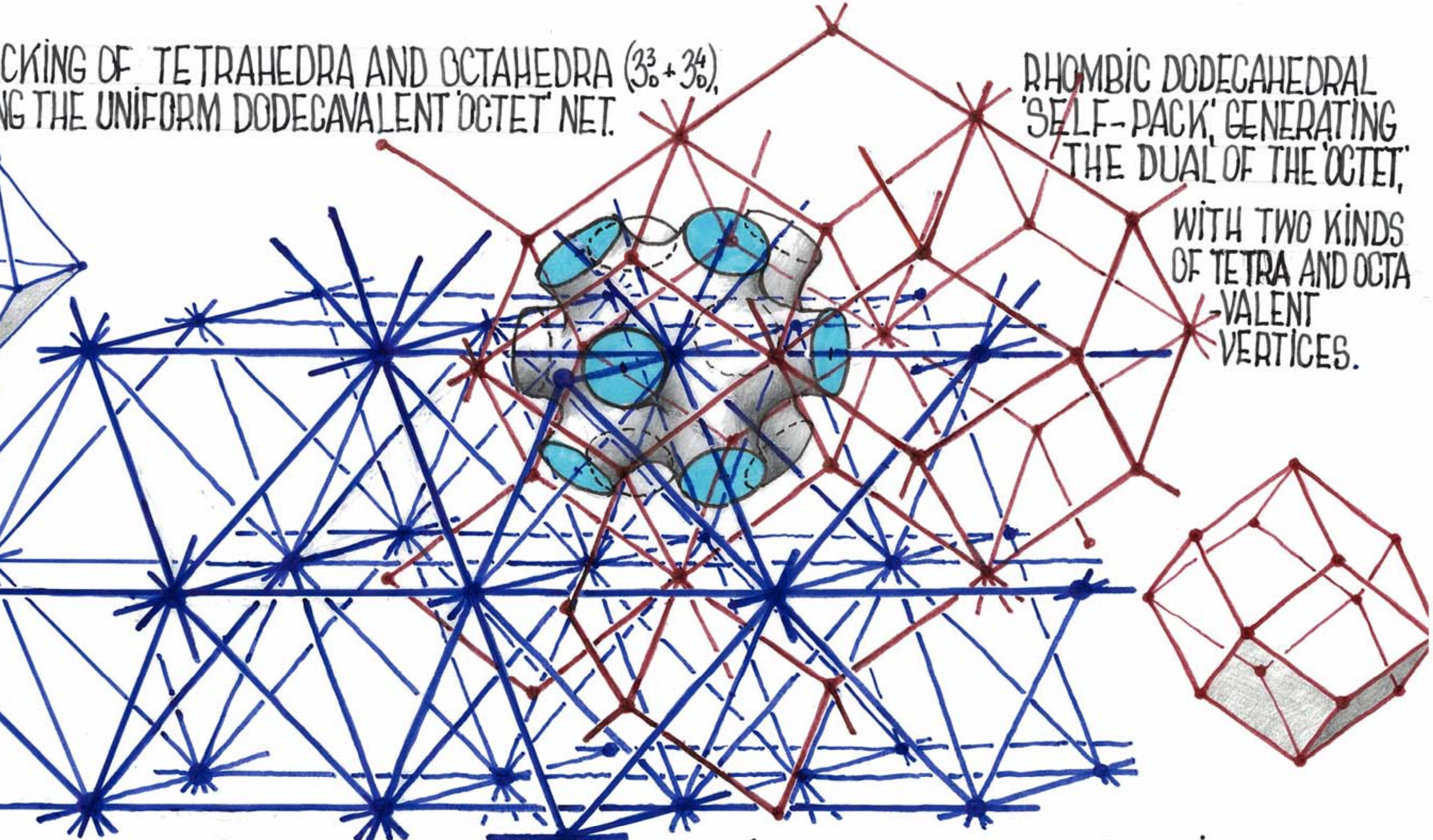
On The Nature Of The Networks Phenomenology

3D Networks are polytopal interrelating interconnected arrays of point-wise entities, which could be represented as continuous and connected configurations of vertices and segment edges, may extend to infinity and may be eventually perceived as **compact space packing's of cellular solids.** May be chaotic-accidental or ordered, to a point of being

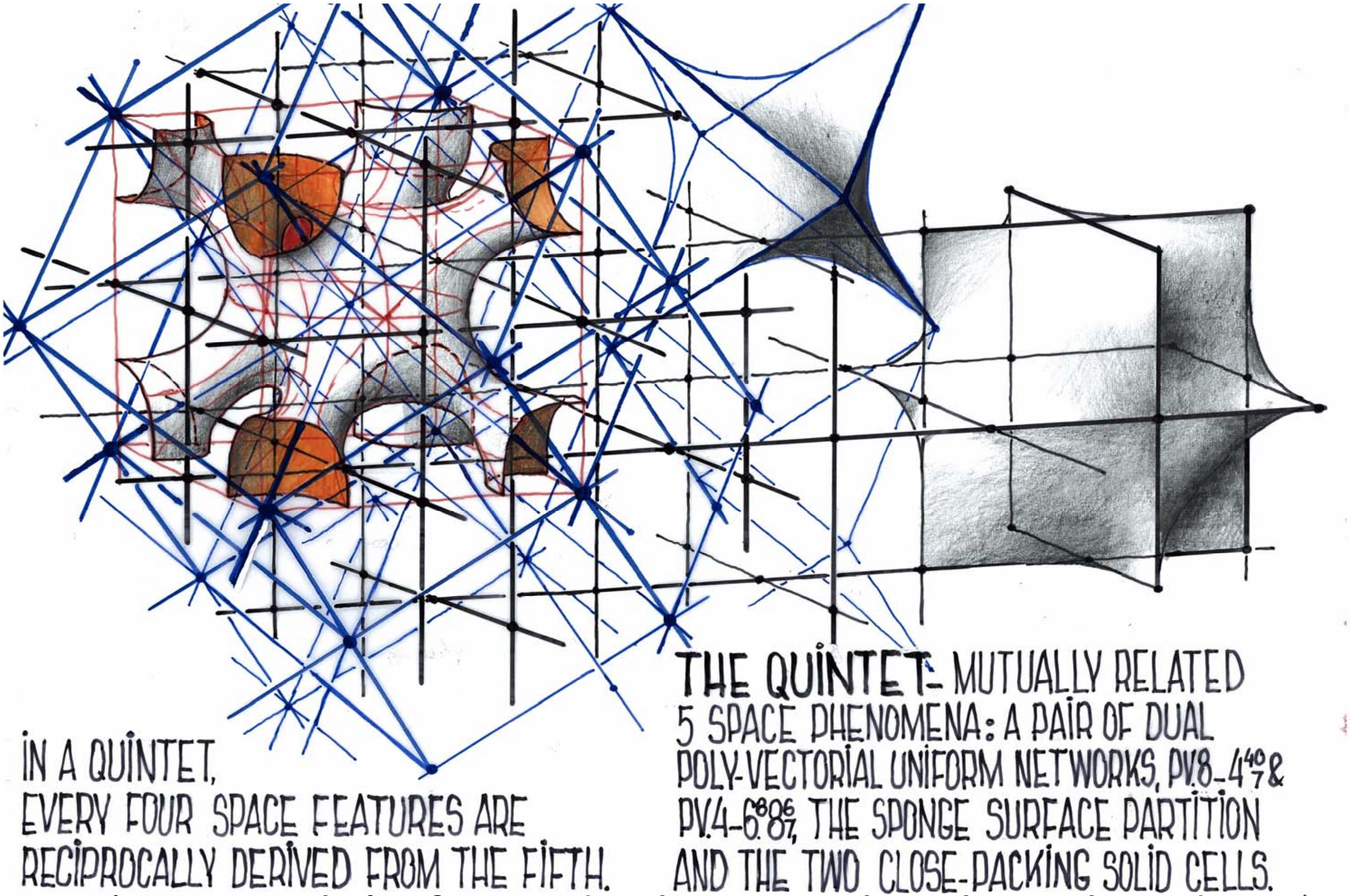
THE 'QUINTUPLE' PHENOMENON
OF 3-D SPACE

The **'quintuple'** of the **dual networks pair,** the **two related close space packings** and **the associated reciprocal sponge surface** is the most conspicuous, all pervading geometric-topological phenomenon of our 3D space, associated with its order and organization, and

CKING OF TETRAHEDRA AND OCTAHEDRA ($3^3_0 + 3^4_0$),
NG THE UNIFORM DODECAVALENT 'OCTET' NET.



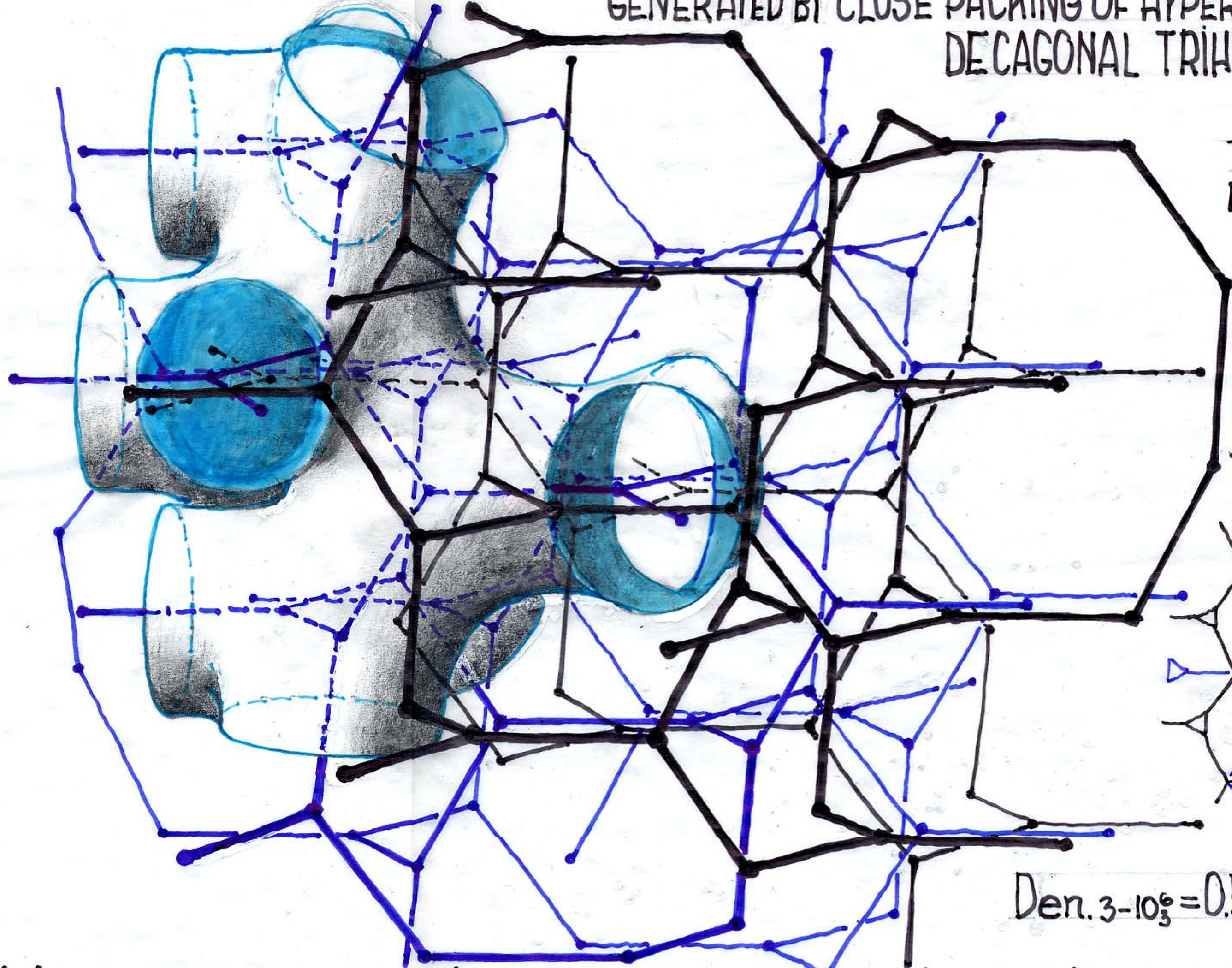
ETWORKS PAIR, THEIR RELATED CLOSE-PACKING MODES AND THE ASSOCIATED
OLICAL PARTITION SURFACE SUBDIVIDING THE SPACE BETWEEN THE TWO.



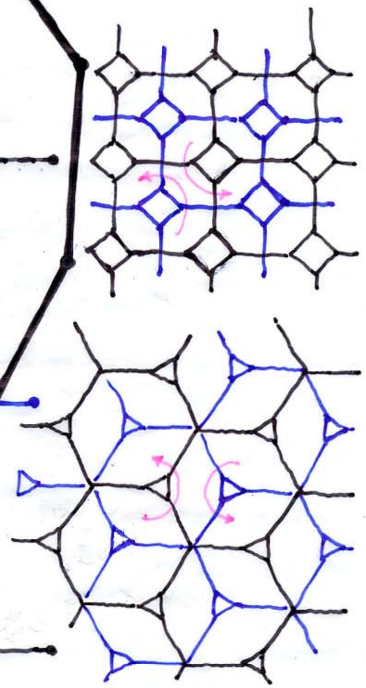
A ‘Quintuple’ of two dual networks, the related close packings and the surface in between.

The described five features of 3D-Space, namely the **dual networks pair,** the **two associated close-packing modes (**with their respective polyhedral solids and the associated **hyperbolical sponge surface,** all together represent a **‘quintuplet assembly’** which encompasses the essence of the 3D space phenomenology.

THE UNIFORM TRIVALENT (SELF-DUAL) $3\text{-}10^6_3$ LATTICE, GENERATED BY CLOSE PACKING OF HYPERBOLIC SADDLE DECAGONAL TRIHEDRA

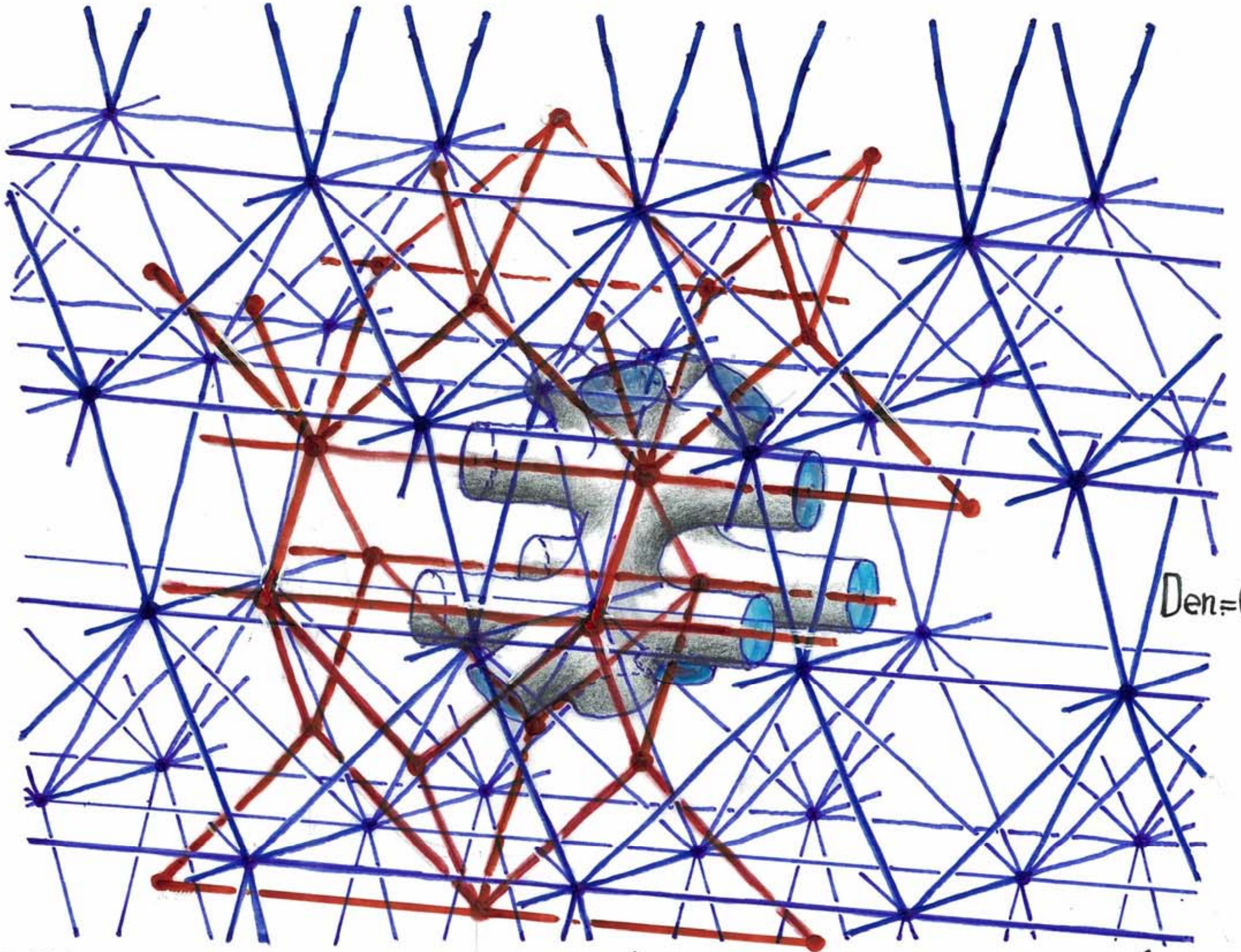


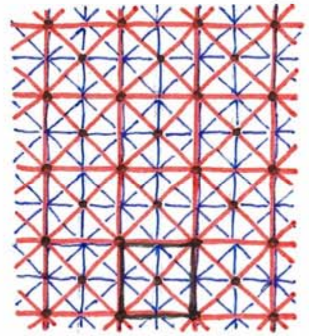
TWO TYPICAL PROJECTIONS



$\text{Den.}_{3\text{-}10^6_3} = 0.530330085\ a/a^3$

TRINITY- THE MUTUALLY RECIPROCAL TWO DUAL (IDENTICAL) UNIFORM SPACE LATTICES AND THE HYPERBOLIC SPONGE SURFACE, SUBDIVIDING THE SPACE BETWEEN THE TWO.



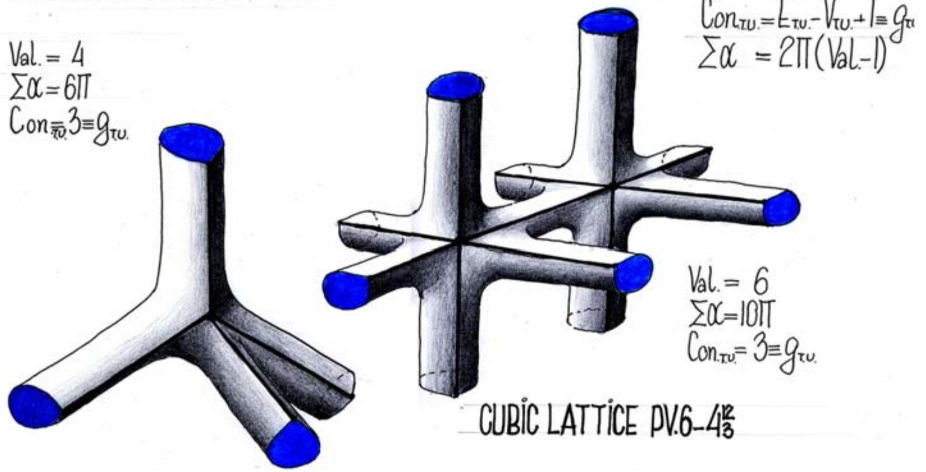


T.U.

Den.=6.356744904 a/a³

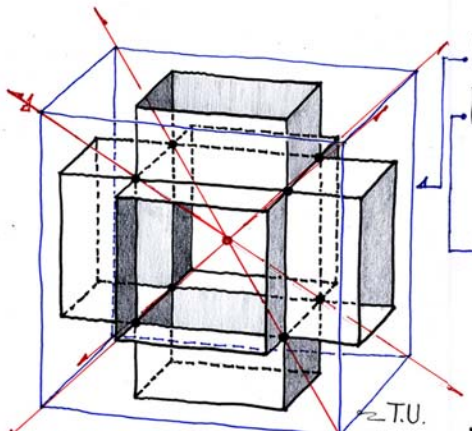
UNIFORM DECAVALENT MULTILAYER NETWORK, (Val.=10; Con.=9).
IT'S DUAL AND THE PARTITION IN BETWEEN. $U.ML.10_a\text{-}3^{15}.4^{8}_{9}$.

DIAMOND LATTICE PV.4-6^{12}_{3}



CUBIC LATTICE PV.6-4^{12}_{3}

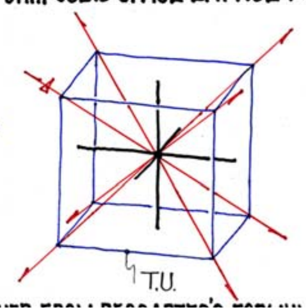
ALL 3-D SPACE NETWORKS MAY BE CONSIDERED AS POLYHEDRAL TESSELLATIONS OF DISTINCT SPONGE SURFACES.

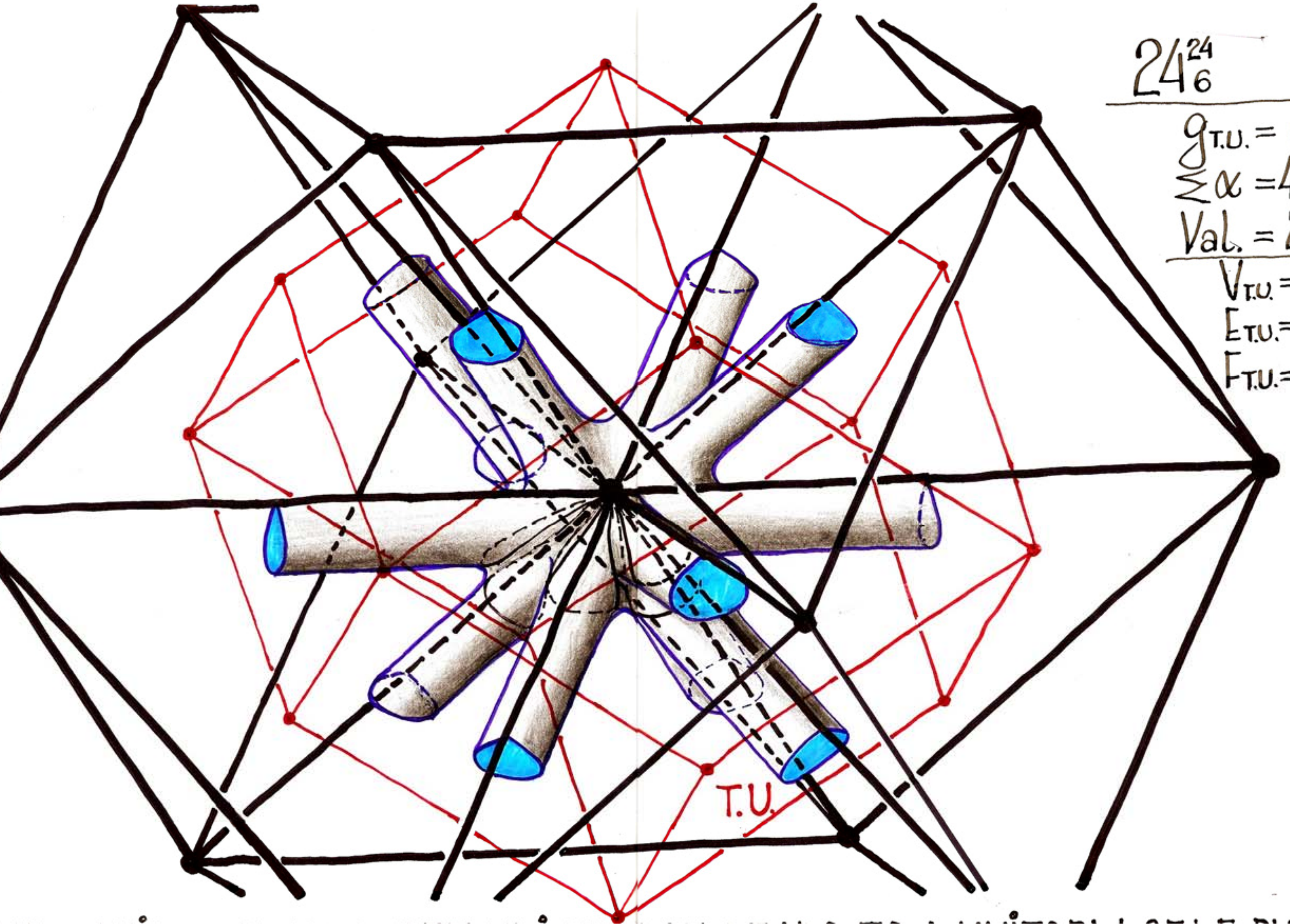


UNIFORM INFINITE POLYHEDRON-4^{6}_{3}, $(6;3\pi;3)$.

UNIFORM CUBIC SPACE LATTICE PV.6-4^{12}_{3}, $(6;10\pi;3)$.

Val.; $\sum\alpha$; $g \equiv C$.





$$24^{24}_{6}$$

$g_{T.U.} = 6$

$\sum\alpha = 46\pi$

$Val. = 24$

$V_{T.U.} = 1$

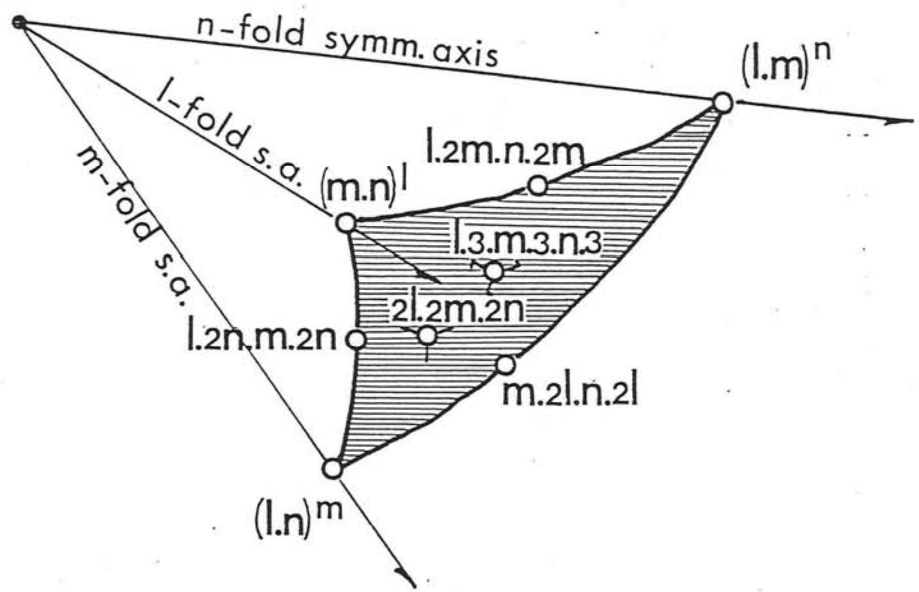
$E_{T.U.} = 12$

$F_{T.U.} = 1$

ELEMENTARY PERIODIC REGIONS – (E.P.R.)

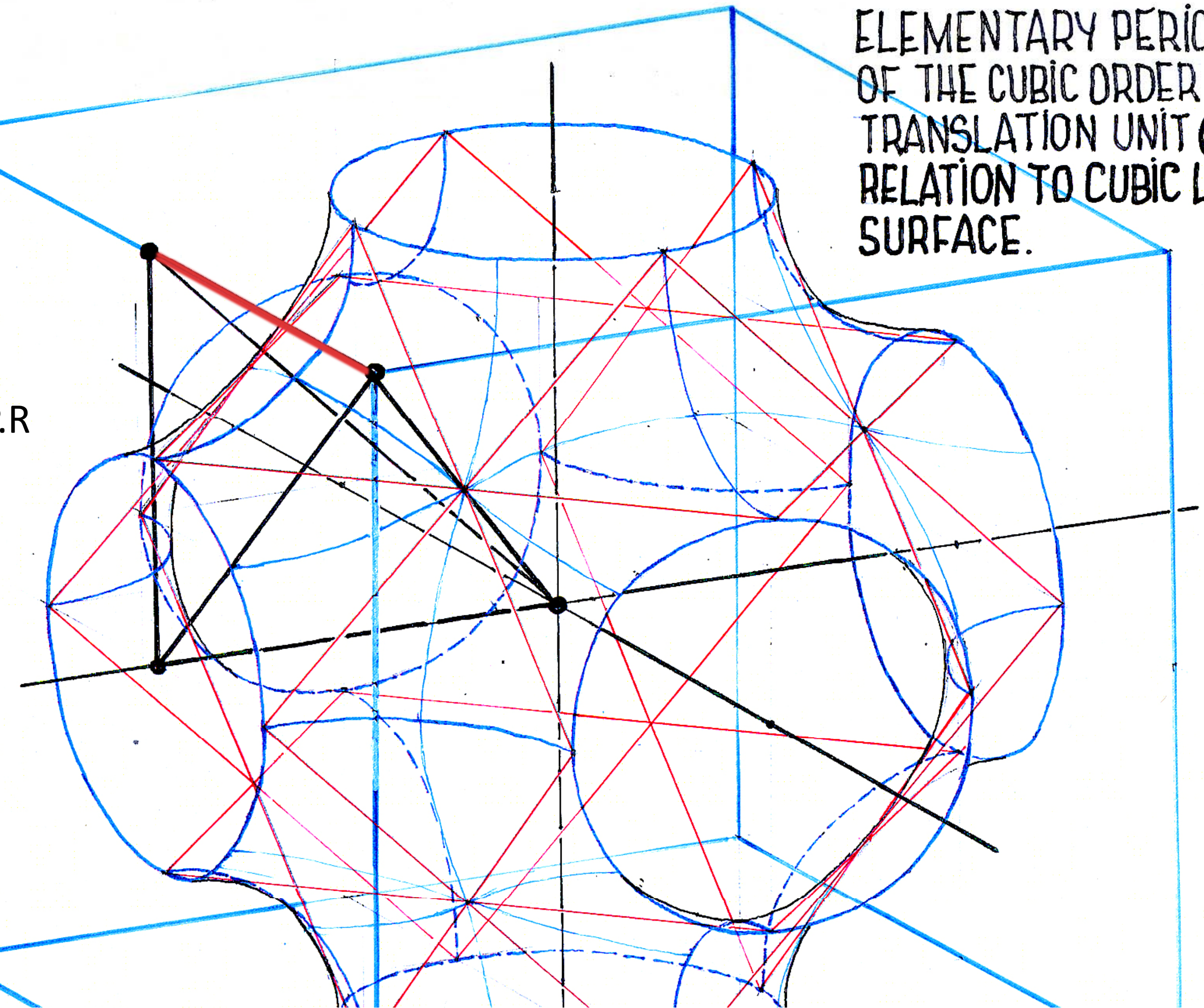
For any group of transformations, corresponding to a specific symmetry group, around a centre point, an axis, in plane or in 3D space, there exists an **Elementary Periodic Region (E.P.R),** the transforms of which cover exactly the space, without overlapings (interstices) or voids.

The E.P.R. which results from a repeating subdivision process of the periodic complex, through the application of its symmetry elements, is in fact a **volumetric repetitive unit, enveloped with symmetry axes and reflection planes**, generally in the form of a



E.P.R OF FINITE – SPHERICAL (2,3,n) SYMMETRY GROUP

ELEMENTARY PERIODIC REGION,(EPR), OF THE CUBIC ORDER WITHIN IT'S TRANSLATION UNIT (T.U.) AND IT'S RELATION TO CUBIC LATTICE AND CUBIC SURFACE.



PRIMARY PARAMETERS OF THE 3-D NETWORKS

Valency (*Val.*), **Connectivity (*Con*.)** (or genus-*g*) and ***Σα* values** represent primary parameters of the network's topology and its associated (mostly hyperbolical) sponge surface.

Connectivity (*Con*.) of a multiple vertex-edge configuration represents the maximal number of edge-disconnections which still leave the configuration un-separated into two parts.

(1). Con.=E-V+1 with E&V representing the number of edges and vertices of a connected edge-vertex configuration. In an ordered-uniform network, it's dual and the associated partition surfaces , connectivity-genus values can be applied to the **Elementary Periodic Region (E.P.R)** or to their **Translation Units (T.U.)** and thus:

(2). $\boldsymbol{Con._{T.U.} (\equiv g_{T.U.}) = E_{T.U.} - V_{T.U.} + 1}$

For all conceivable uniform networks unihedra.

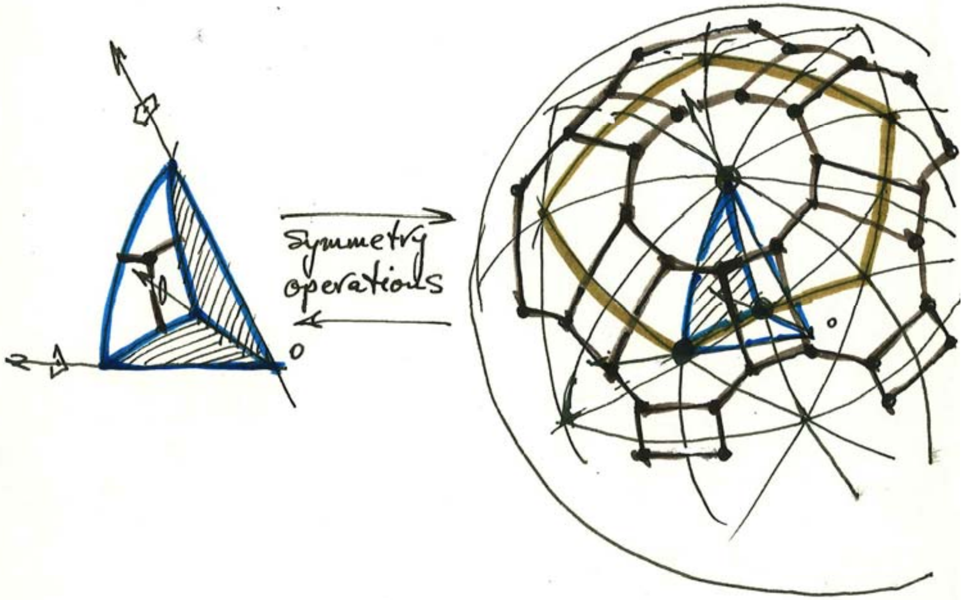
(3). $\boldsymbol{\Sigma\alpha = 2\,(Val. - 1)}$.

Generation Of Uniform Space Networks

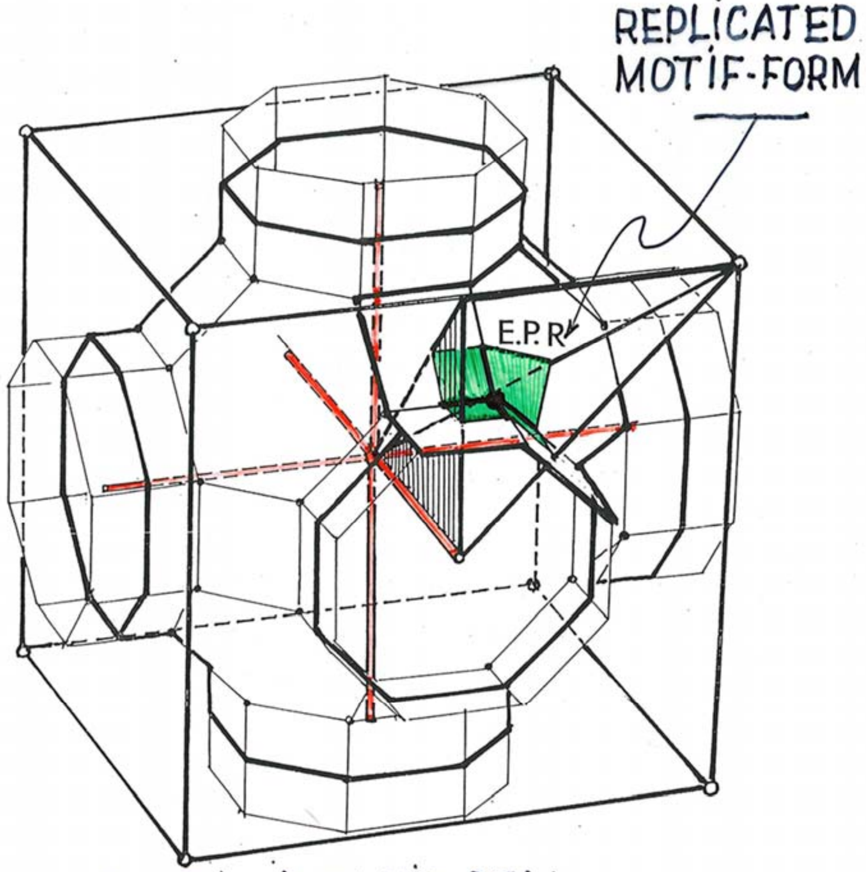
A periodic ordered space network may be formed-generated through one of the following processes:

1. **By an extended repetition of a locally and globally symmetrical association of vertex figures**. It amounts nearly to the same as planting a vertex-edge motif –form in an Elementary Periodic Region (E.P.R) of the network, characteristic of a given symmetry group, and symmetrically replicating it.
2. **By a close (compact) packing of polyhedral cells**, the vertex-edge array of which combine to form the network
3. **By a tessellation-mapping process of an unbounded periodic (2d-manifold) surface,** spherical, toroidal or hyperbolical, leading eventually to a connected 3D network.

SIMPLEX	INTERFACE MECHANISM OF DUPLICATION AND SUBDIVISION	COMPLEX
ELEMENTARY SPACE UNIT	SYMMETRY OPERATIONS WHICH CONSTITUTE A SYMMETRY GROUP	PERIODIC SPACE NETWORK (PACKING)
BASIC FIGURE MOTIF		PERIODIC CONFIGURATION



Spherical E.P.R of (2,3,5) symmetry group, with motif-form and its replication.



TRANSLATIOT UNIT CELL
CUBIC SYMMETRY SPACE GROUP

13.02.2013

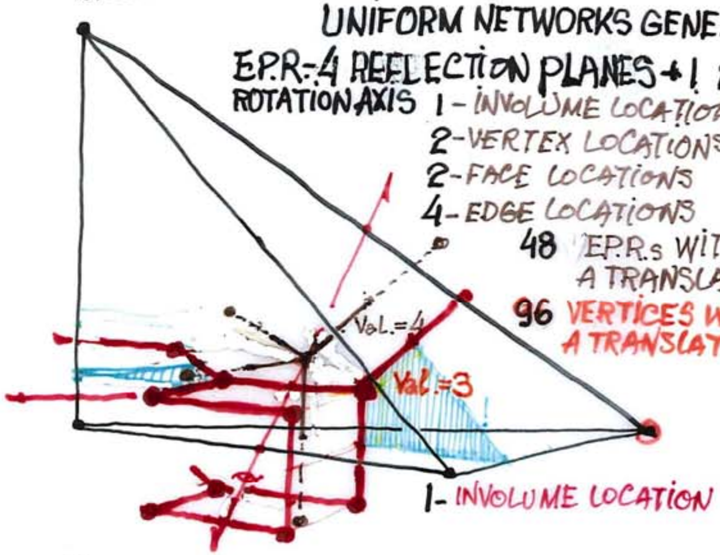
UNIFORM NETWORKS GENERATORS

E.P.R.-4 REFLECTION PLANES + 1 2-FOLD ROTATION AXIS

1 - INVOLUME LOCATION
2 - VERTEX LOCATIONS
2 - FACE LOCATIONS
4 - EDGE LOCATIONS

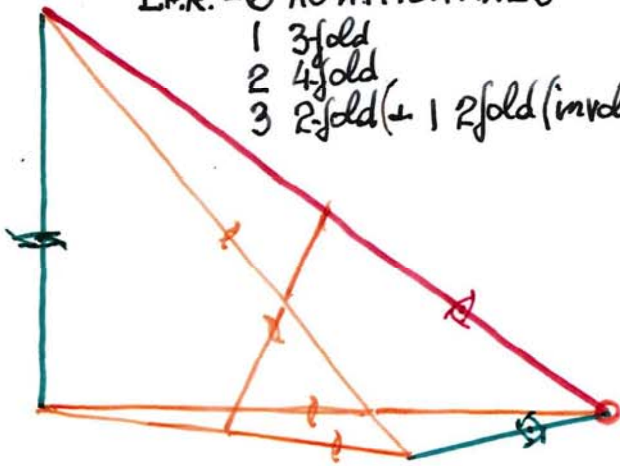
48 E.P.R.s WITHIN A TRANSLATION UNIT

96 VERTICES WITHIN A TRANSLATION UNIT

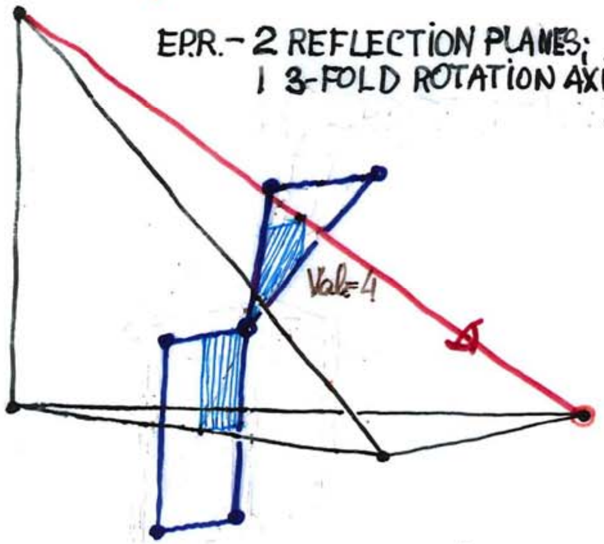


E.P.R. - 6 ROTATION AXES

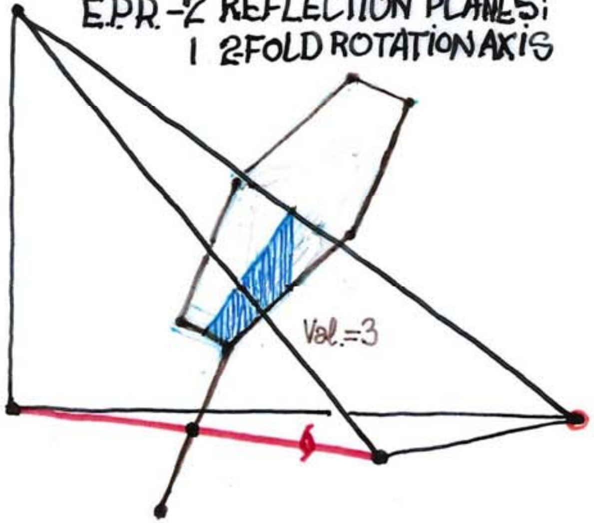
1 3-fold
2 4-fold
3 2-fold (+ 1 2fold (involume))



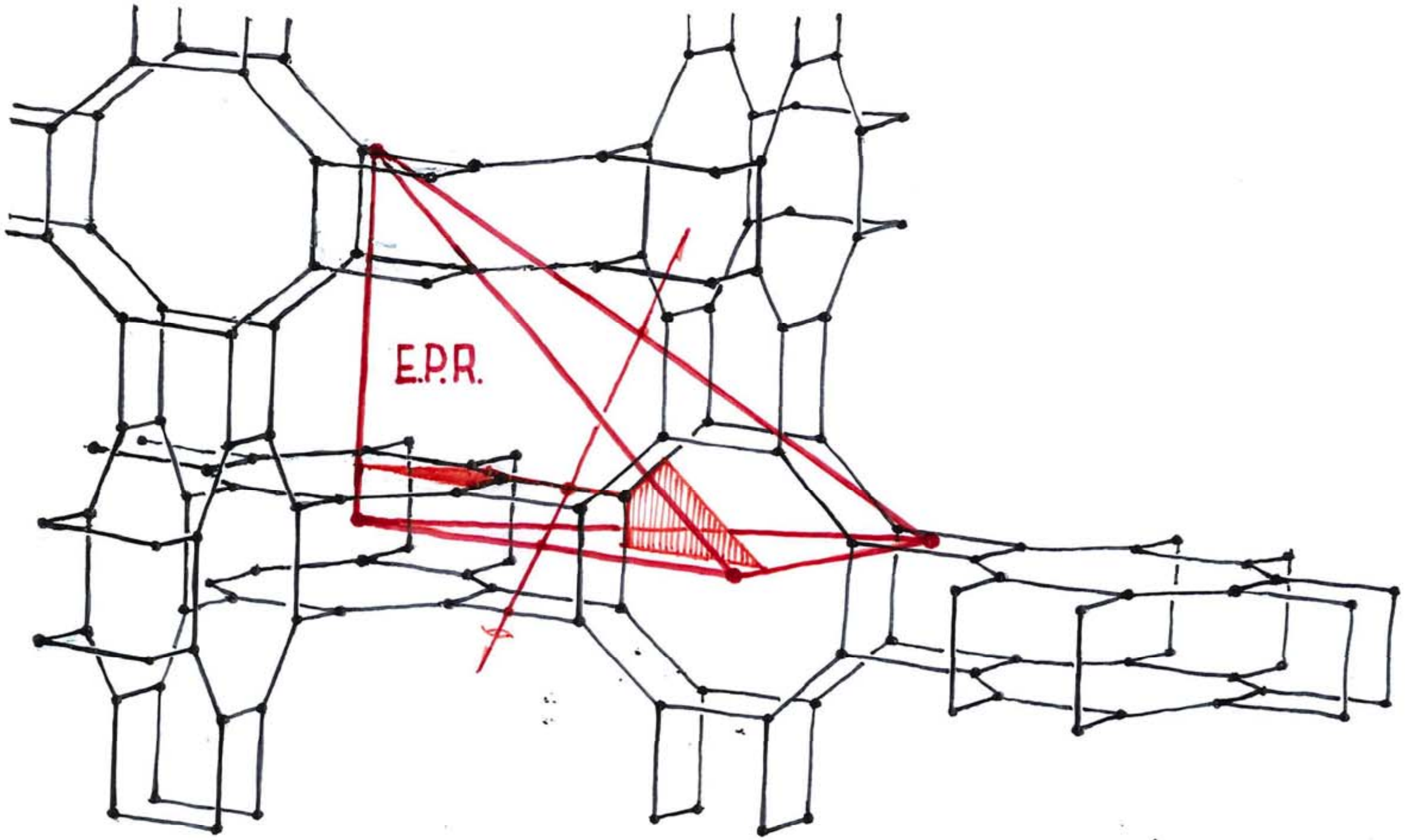
E.P.R. - 2 REFLECTION PLANES; 1 3-FOLD ROTATION AXIS



E.P.R. - 2 REFLECTION PLANES; 1 2-FOLD ROTATION AXIS



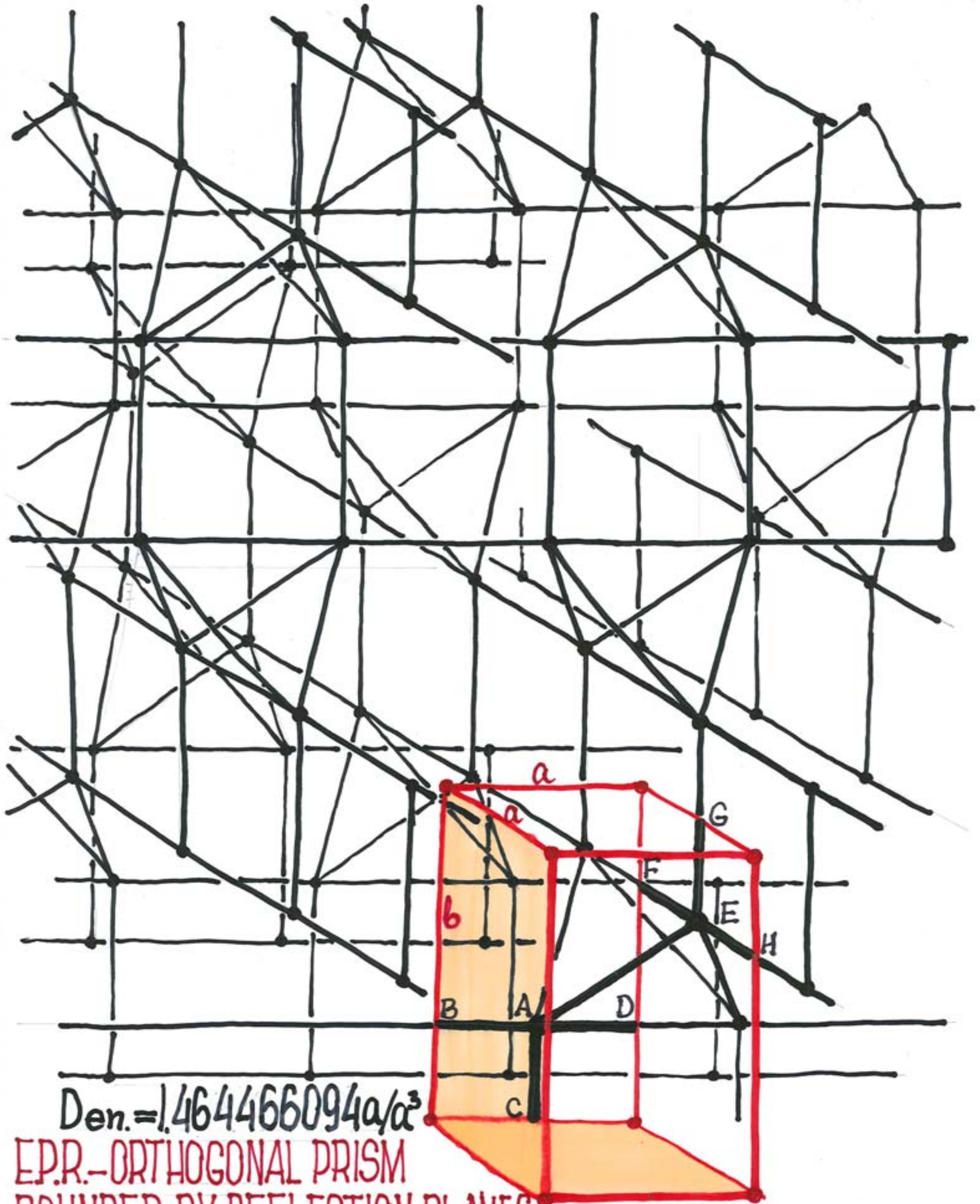
1. CUBIC E.P.R UNITS



96 VERTICES PER ONE E.P.R

$\text{Den.}_{PV\text{-}3\text{-}8^3.12^2_{49}} = 0.452272785\, a/a^3$

UNIFORM TRIVALENT POLY-VECTORIAL PV-3-$8^3.12^2_{49}$ NETWORK
IN-VOLUME E.P.R. VERTEX LOCATION AND 2-FOLD ROTATION AS GENERATORS.

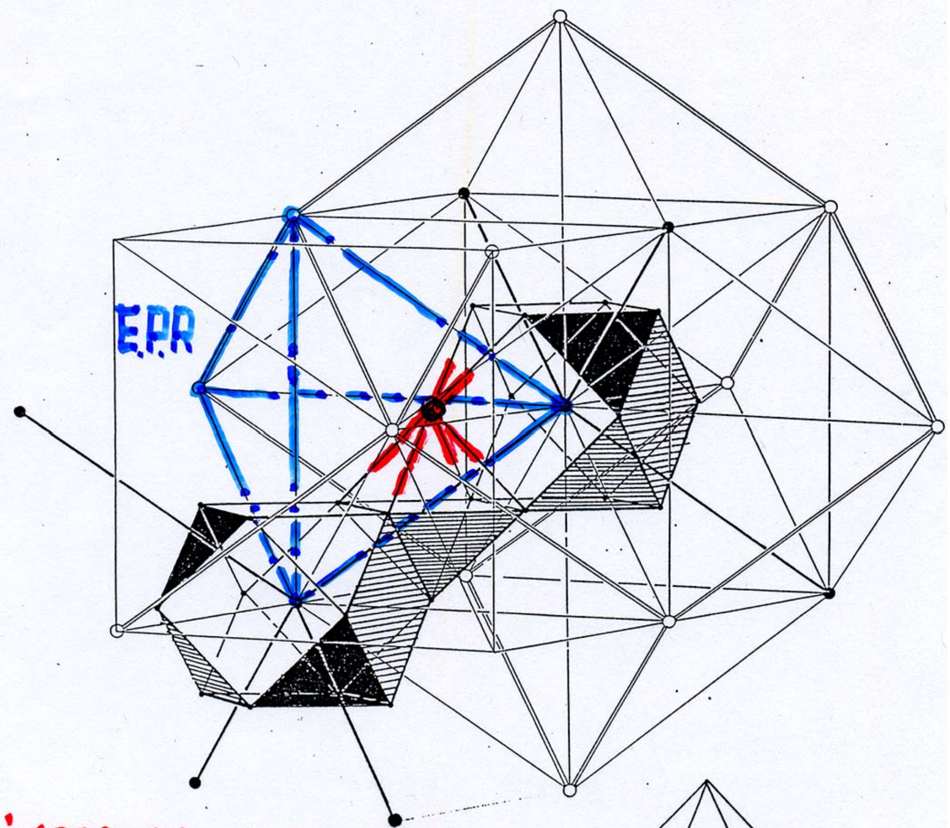


Den.$=1.464466094a/a^3$

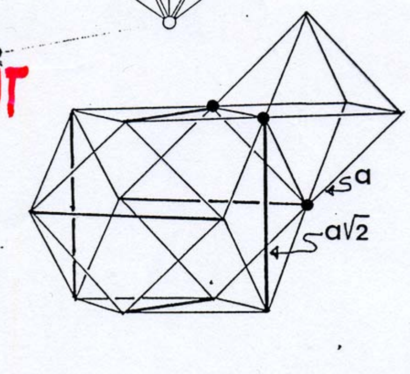
E.P.R.-ORTHOGONAL PRISM
BOUNDED BY REFLECTION PLANES

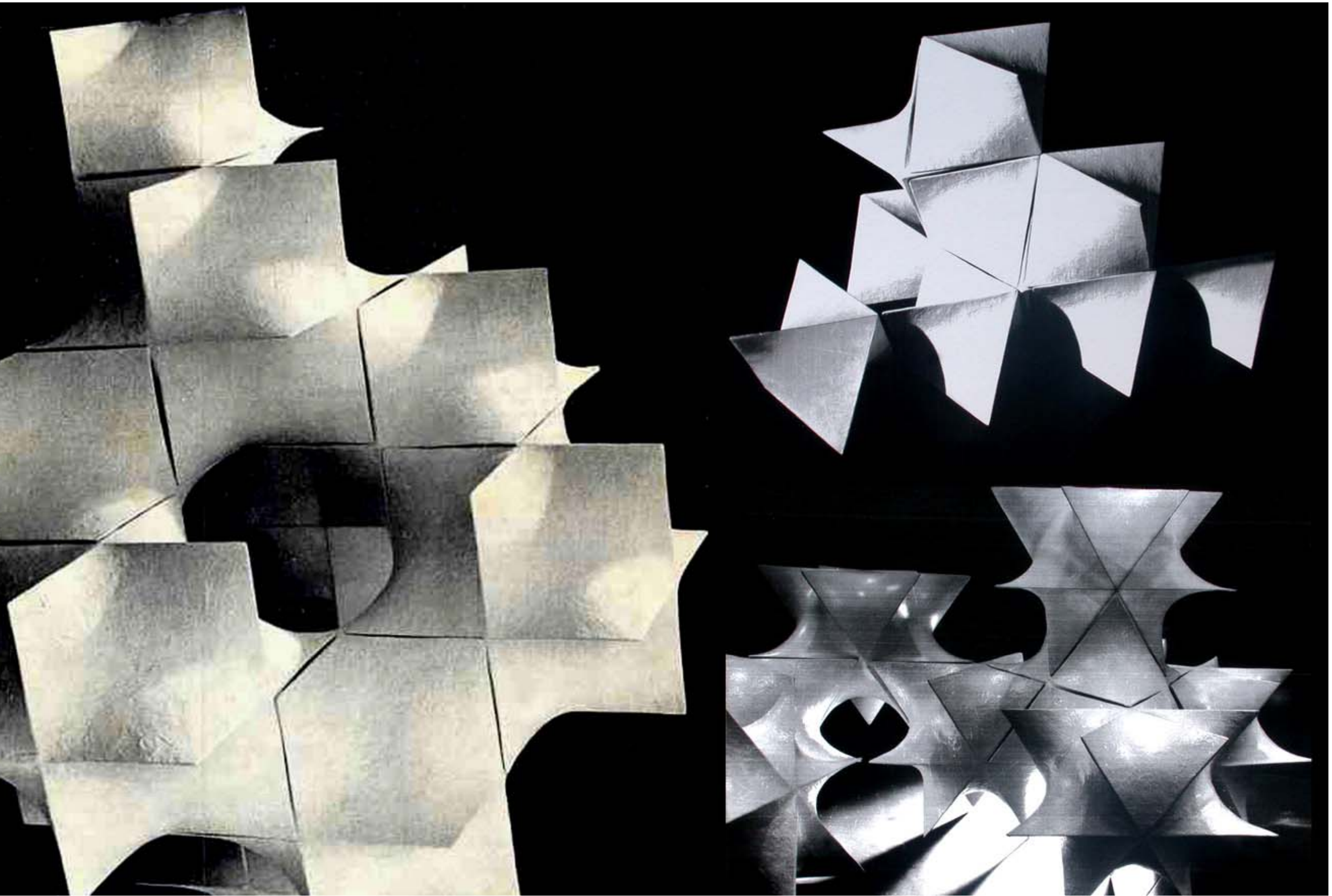
ABCD EFGH- THE MOTIF FORM

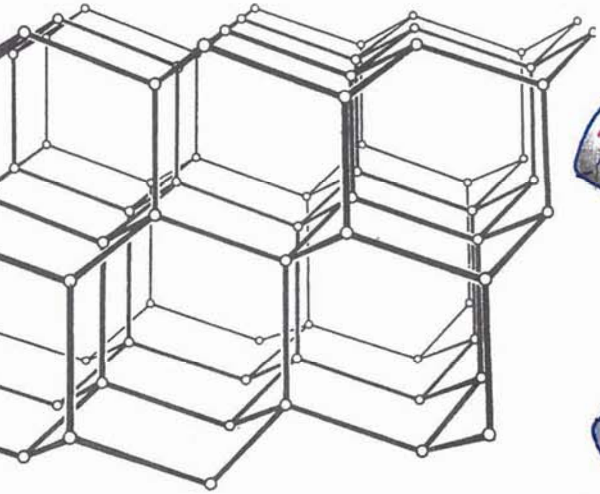
UNIFORM MULTILAYER PENTAVALENT ML4-$3^3.4^2.8^2_{13}$

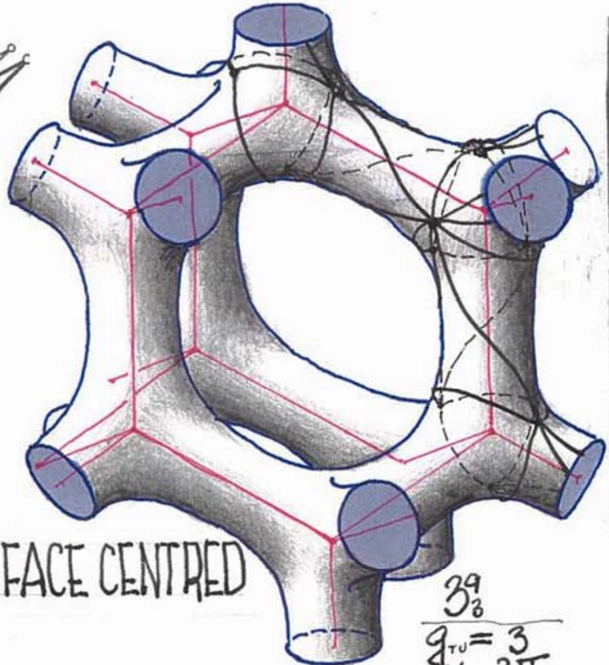


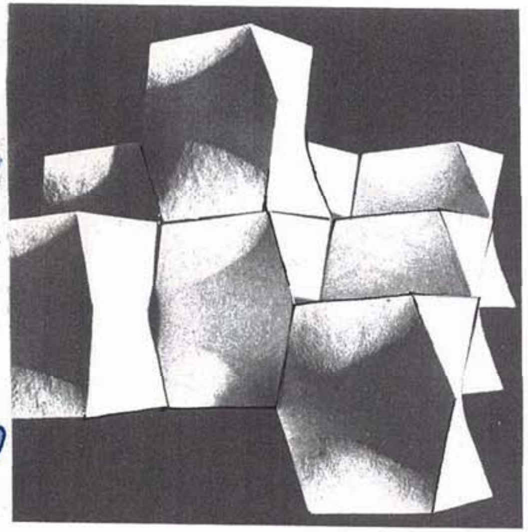
INSERTED 6-VALENT
MOTIF FORM











M TETRAVALENT DIAMOND AND FACE CENTRED
LATTICES ($4\text{-}6^{12}_{3}$ & $4\text{-}6^{8}_{4}$)

$PV.4\text{-}6^{12}_{3}$

$PV.4\text{-}6^{8}_{4}$

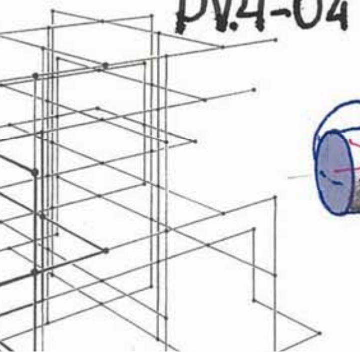
3^{9}_{3}

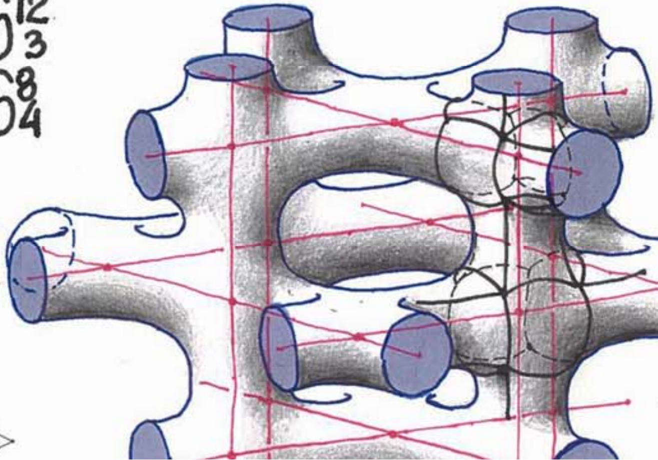
$g_{TU} = 3$
$\Sigma\kappa = 3\pi$
$Val. = 9$

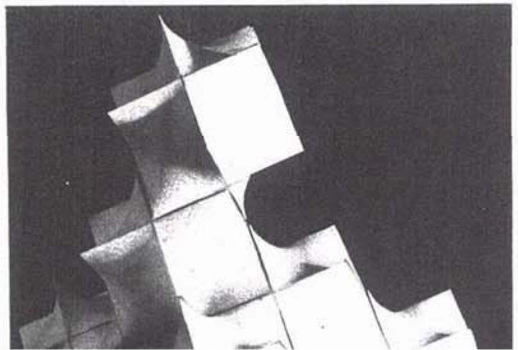
$V_{TU} = 8$
$E_{TU} = 36$
$F_{TU} = 24$

$Den_{(4\text{-}6^{12}_{3})} = 1.299038106\ a/a^3$

$Den_{(4\text{-}6^{8}_{4})} = 1.5000\ a/a^3$

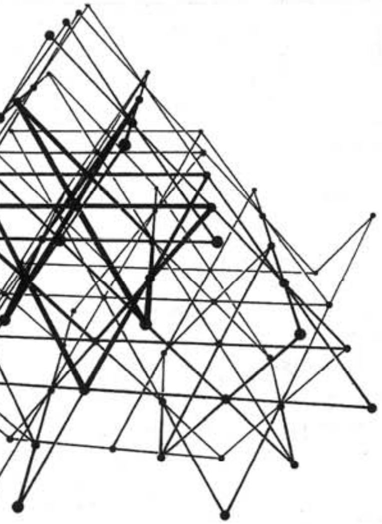




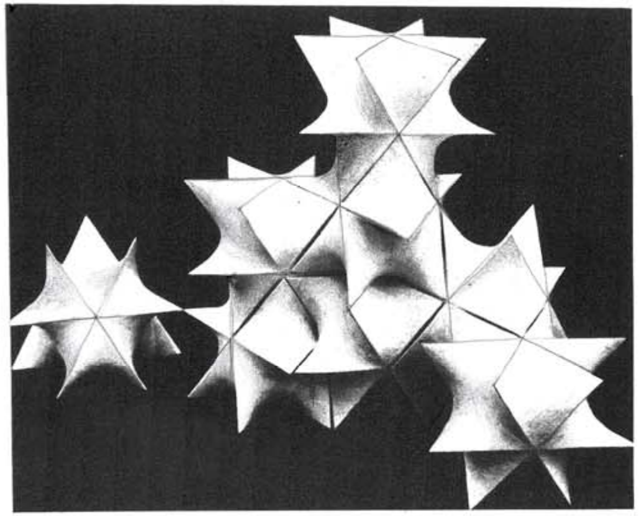


4^{5}_{4}

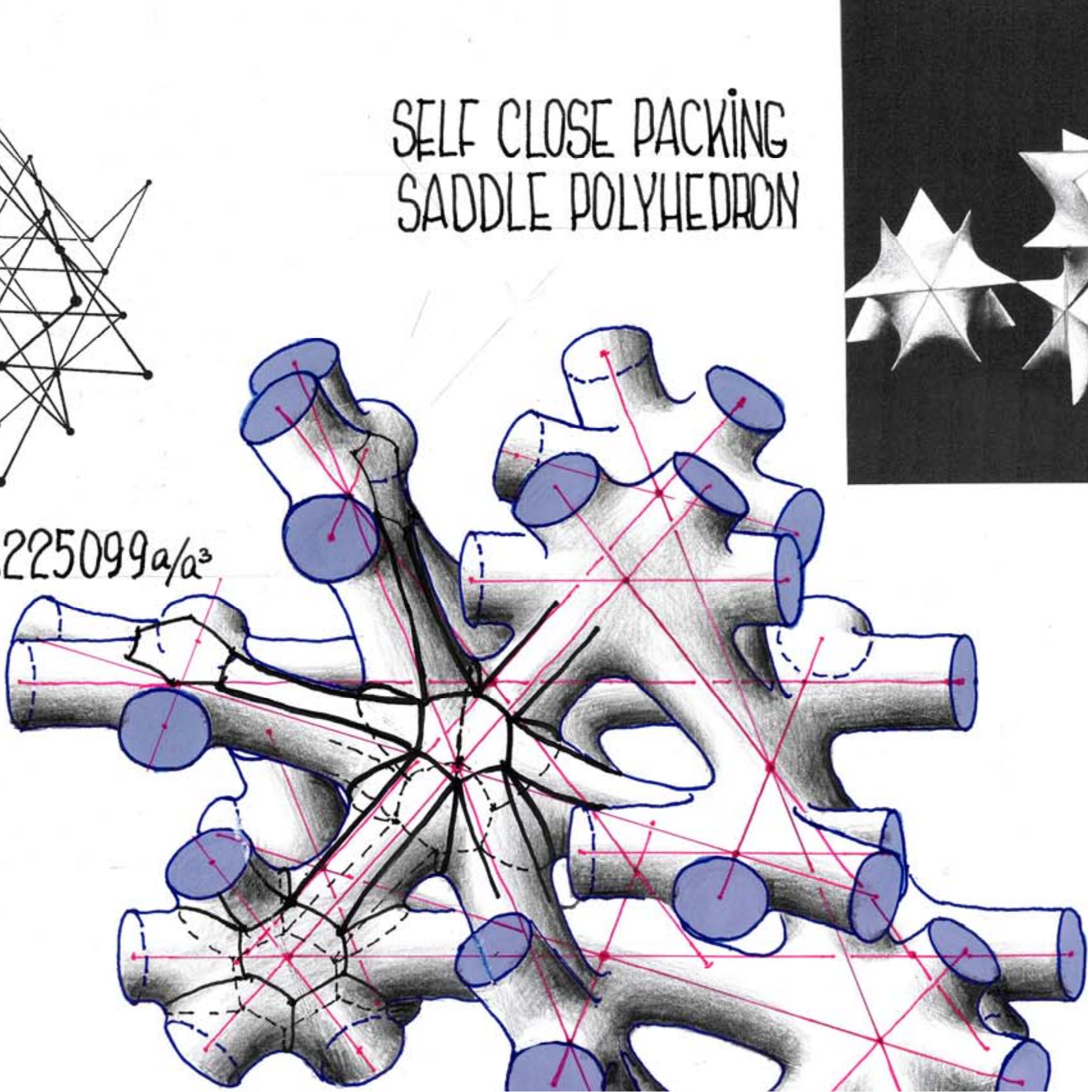
$g_{TU} = 4$



SELF CLOSE PACKING
SADDLE POLYHEDRON



$^{6}6^{6}{}_{17}) = 6.788225099\, a/a^3$



$4^4.6_{17}$

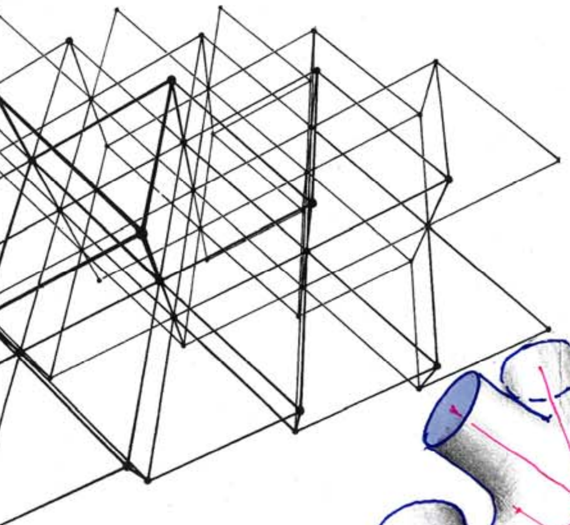
$g_{TV.} = 17$

$\sum\alpha = 2\frac{2}{3}\pi$

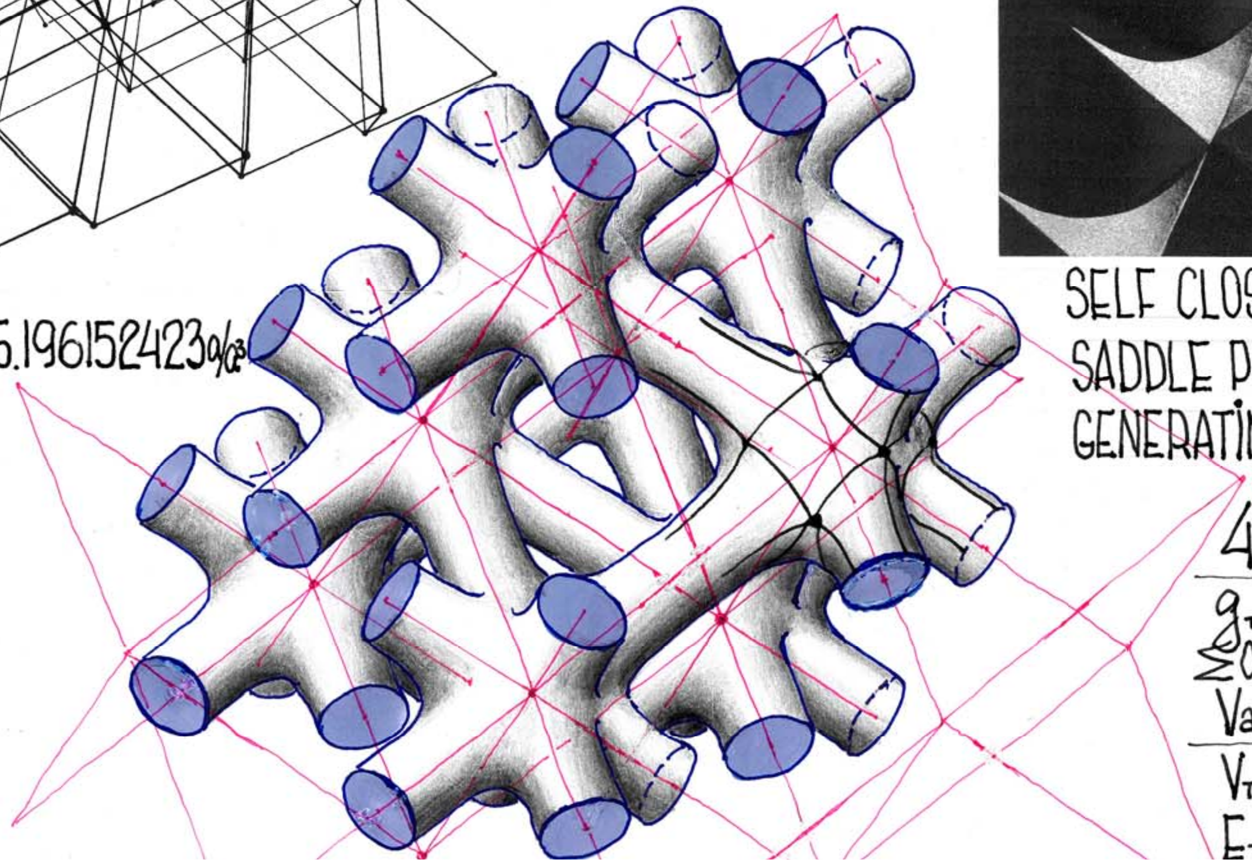
Val. = 5

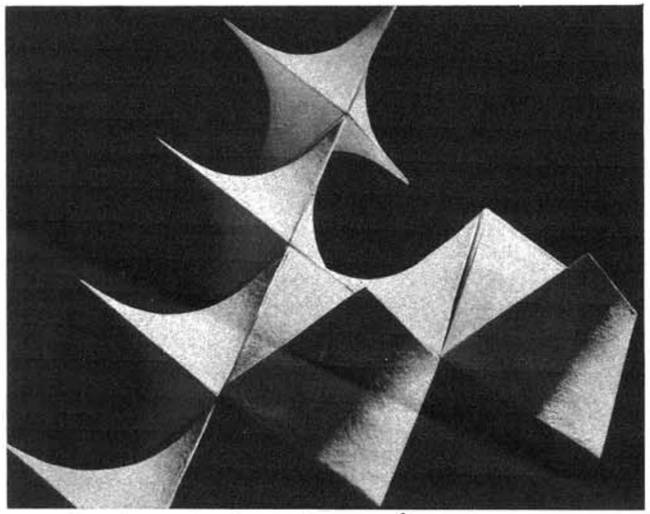
$V_{TV.} = 96$

$E_{TV.} = 240$



$$\ldots = 5.196152423\,a/c^3$$





SELF CLOSE PACKING SADDLE POLYHEDRON, GENERATING THE LATTICE.

$$4^{5}_{19}$$

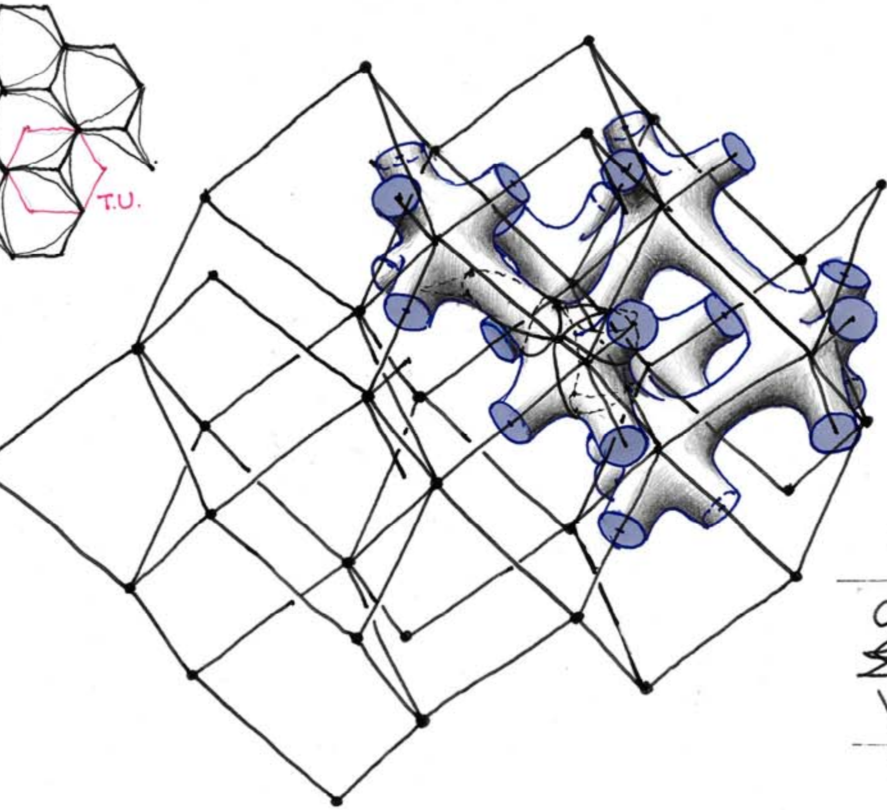
$$g_{TU.} = 19$$

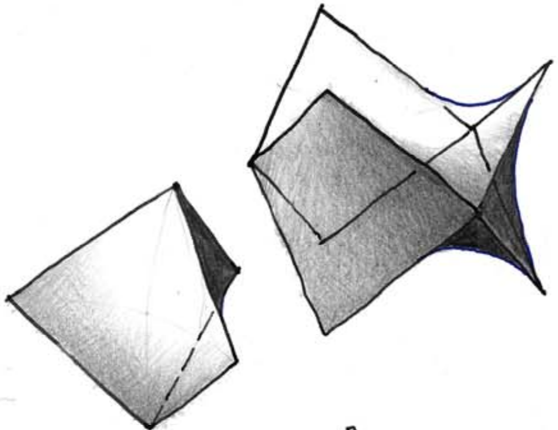
$$\sum\alpha = 2\tfrac{1}{2}\pi$$

$$\text{Val.} = 5$$

$$V_{TU.} = 144$$

$$E_{TU.} = 360$$





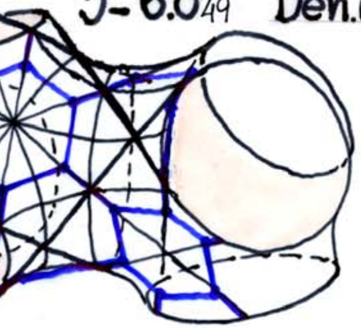
CLOSE PACKING SADDLE POLYHEDRA, GENERATING THE SPACE LATTICE.

$$3.4^{6}_{5}$$

$$g_{T.U.} = 5$$
$$\Sigma\alpha = 3\tfrac{1}{3}\pi$$
$$\text{Val.} = 7$$

$$V_{T.U.} = 12$$
$$E_{T.U.} = 42$$
$$F_{T.U.} = 22$$

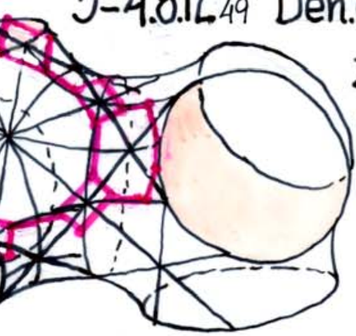
$3\text{-}6.8^2_{49}$ $\mathrm{Den.}_{(3\text{-}6.8^2_{49})} = 0.47140452\ a/a^3$



$\Sigma\alpha = 390^\circ$

$\mathrm{Con}_{T.U.} \equiv (g_{T.U.}) = 49$

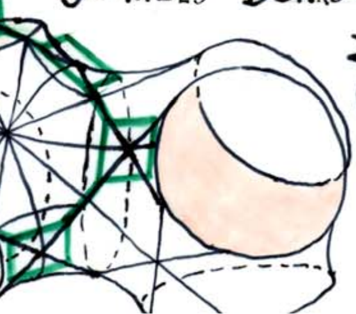
$3\text{-}4.8.12_{49}$ $\mathrm{Den.}_{(3\text{-}4.8.12_{49})} = 0.240236673\ a/a^3$



$\Sigma\alpha = 375^\circ$

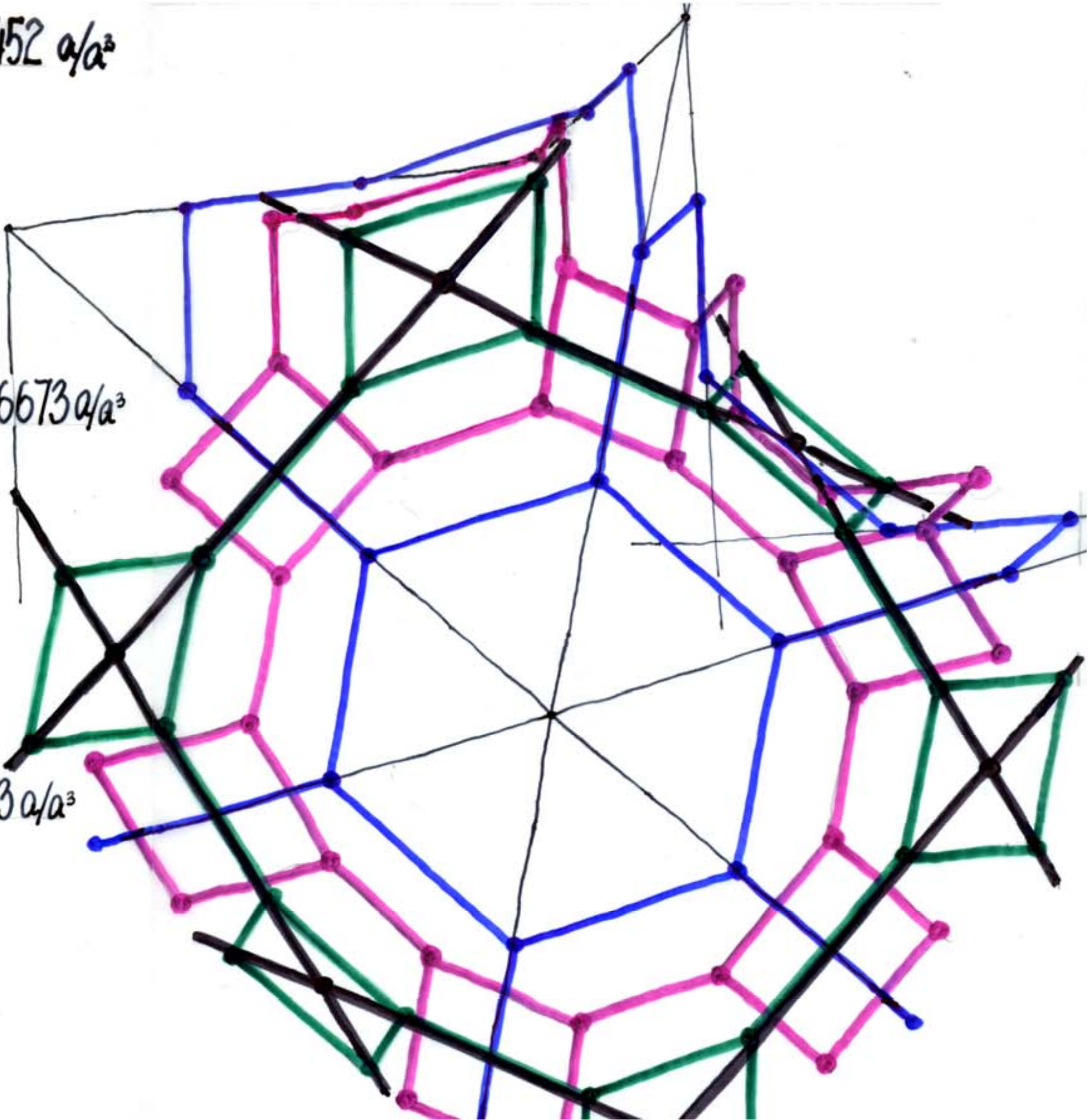
$\mathrm{Con}_{T.U.} \equiv (g_{T.U.}) = 49$

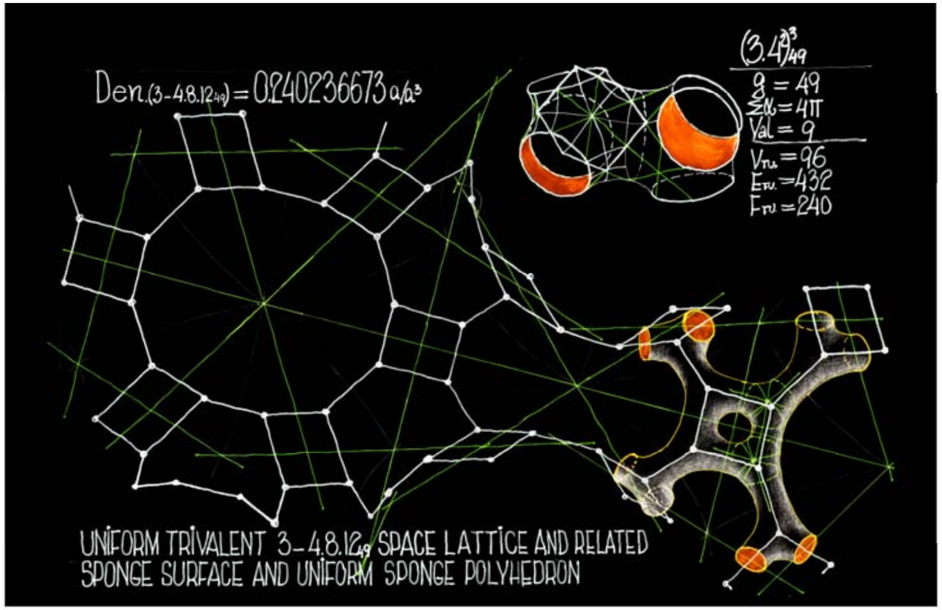
$3\text{-}4.12^2_{25}$ $\mathrm{Den.}_{(3\text{-}4.12^2_{25})} = 0.319805153\ a/a^3$



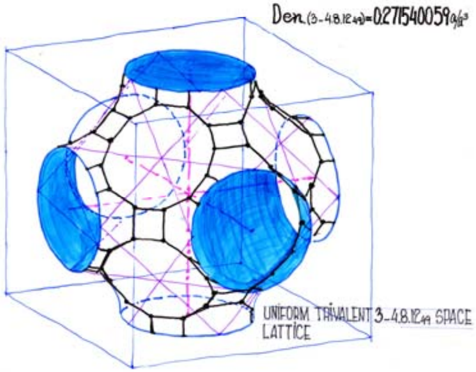
$\Sigma\alpha = 390^\circ$

$\mathrm{Con}_{T.U.} \equiv (g_{T.U.}) = 25$

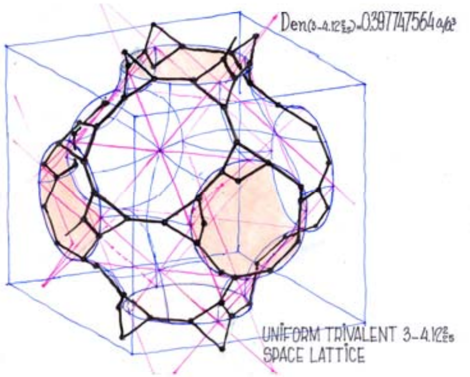




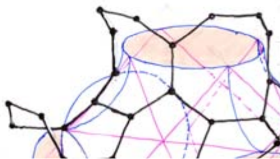
$Den._{(3-4.8.12_{49})} = 0.240236673\, a/a^3$
$(3.4^2)^3_{49}$
$g = 49$
$\Sigma\alpha = 4\pi$
$Val = 9$
$V_{TU} = 96$
$E_{TU} = 432$
$F_{TU} = 240$
UNIFORM TRIVALENT 3-4.8.12 SPACE LATTICE AND RELATED
SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON



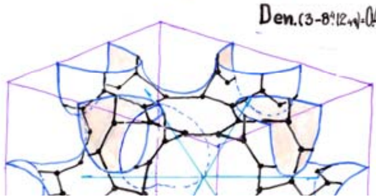
$Den._{(3-4.8.12_{49})} = 0.271540059$
UNIFORM TRIVALENT 3-4.8.12 SPACE
LATTICE



Den = 0.397747564
UNIFORM TRIVALENT 3-4.12
SPACE LATTICE



Den = 0.493078451



Den = 0.452272785

ATEGORIZATION OF 3-D NETWORKS.

nsidering all **uniform network configurations** in 3D space, we can
egorize and classify them as follows:

Centroid related networks (Centroidal), the vertices of
ch are equi-distant from a fixed point in space , (related to the imagery of
te polyhedral in 3D space).

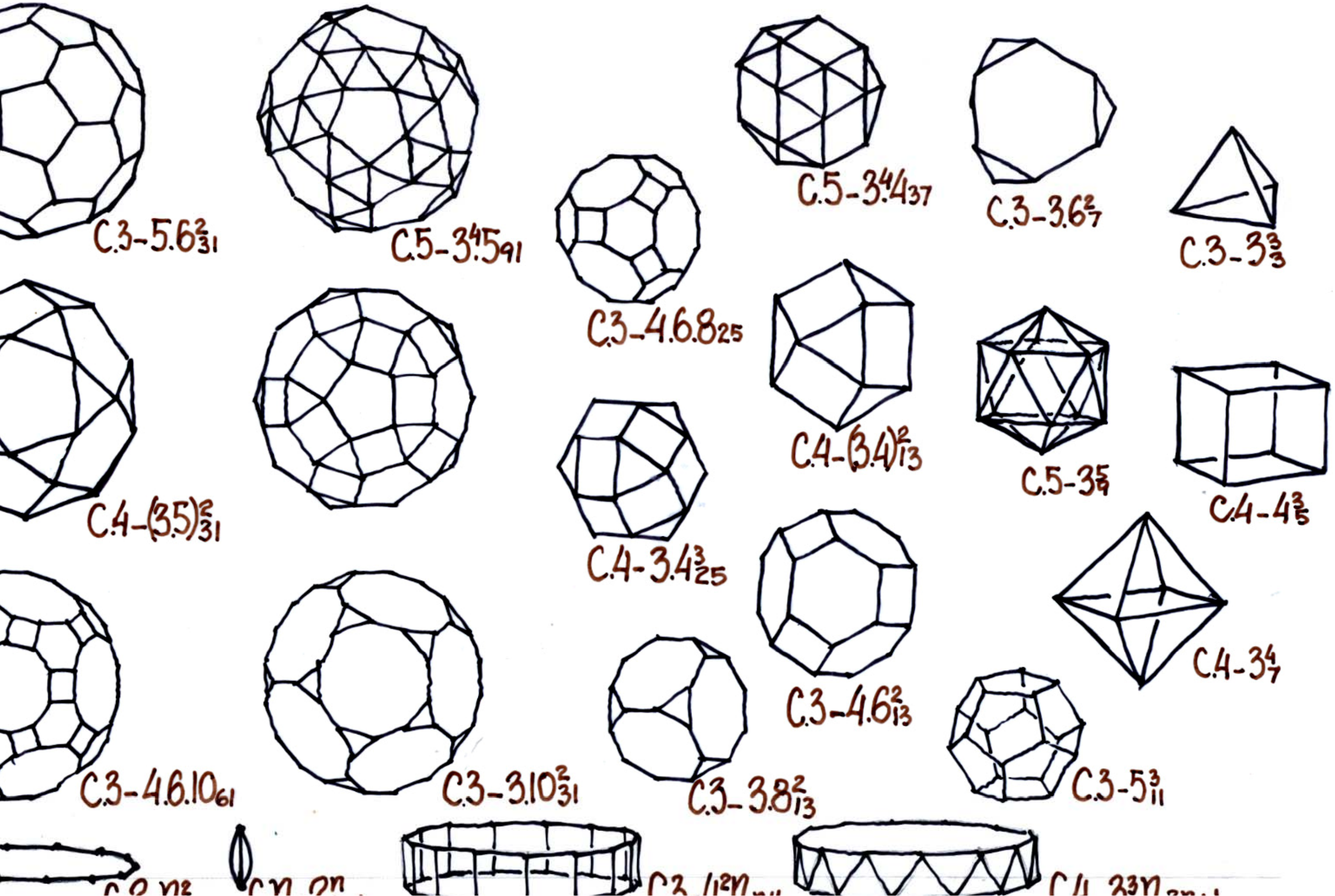
Axis related networks (Axial), the vertices of which are equi-
ant from a fixed axis in space (related some of the imagery of toroidal
yhedra).

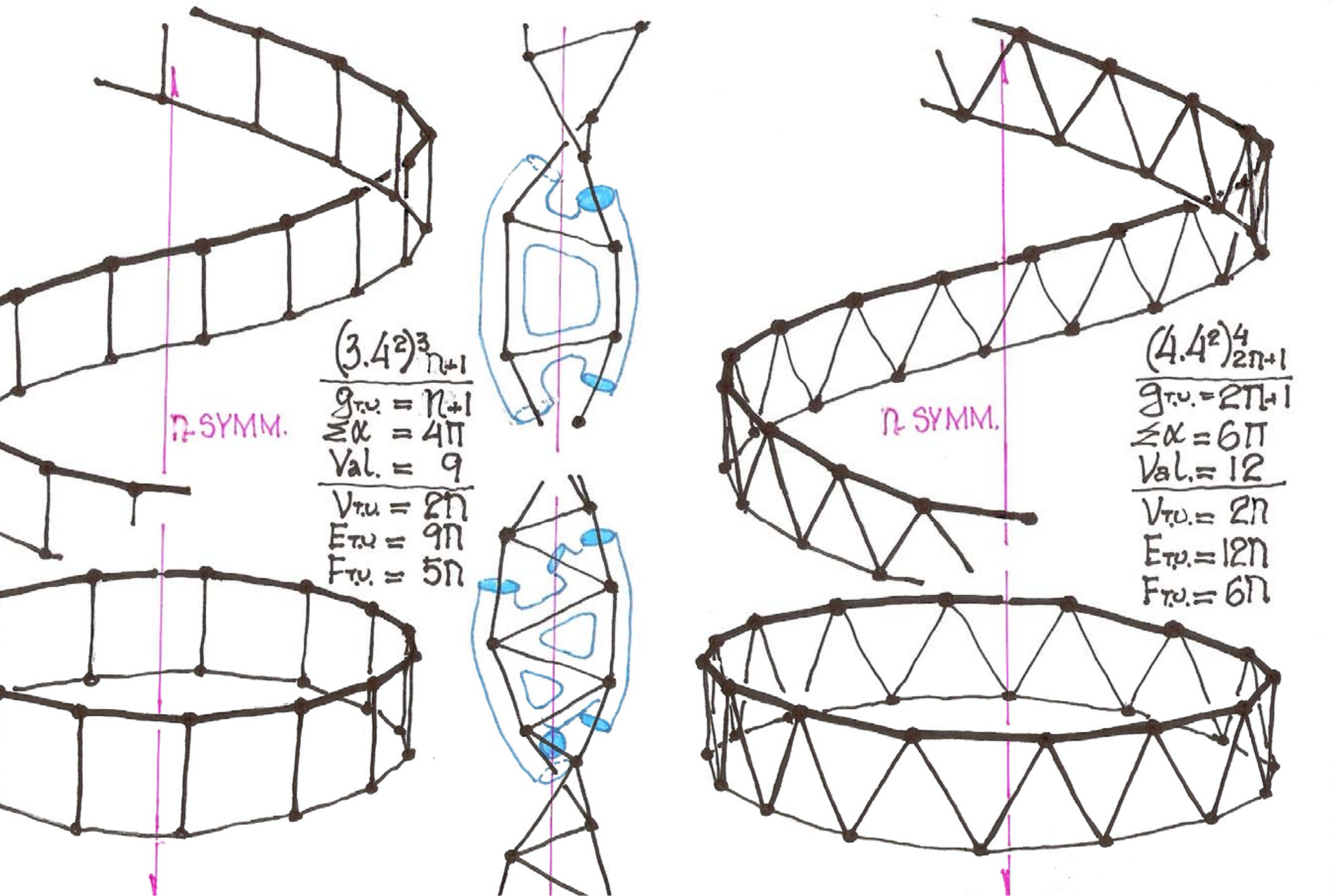
Plane related Networks, (Double Layer) the vertices of
ch are equi-distant form a fixed plane .

Centroid-Axial-Plane related Networks the vertices of
ch are equi-distant from a fixed point, axis and a plane, synchronously.

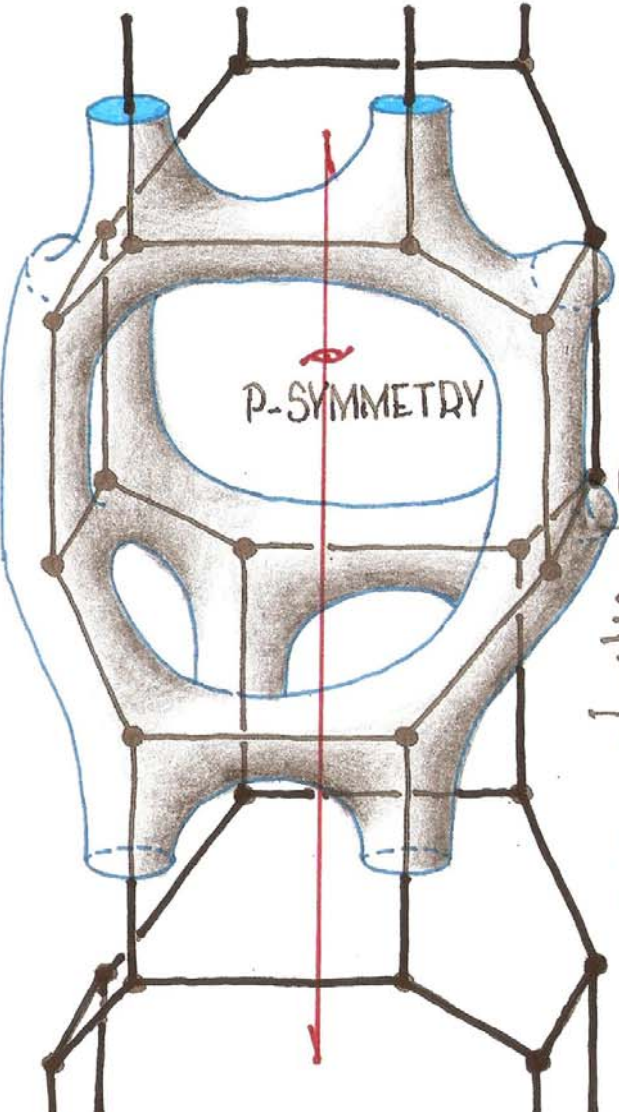
Multi-Layered Space Lattices the vertices of which are
ributed on an infinite set of parallel planes.

Poly-Vectorial Space Lattices the symmetry group of which is
not dominated by only one set of same principal axes of symmetry .


C.3-5.6$^{2}_{31}$
C.5-3^{4}.5$_{91}$
C.5-3^{4}.4$_{37}$
C.3-3.6$^{2}_{7}$
C.3-3$^{3}_{3}$
C.3-4.6.8$_{25}$
C.4-(3.5)$^{2}_{31}$
C.4-(3.4)$^{2}_{13}$
C.5-3$^{5}_{9}$
C.4-4$^{3}_{5}$
C.4-3.4$^{3}_{25}$
C.3-4.6$^{2}_{13}$
C.4-3$^{4}_{7}$
C.3-4.6.10$_{61}$
C.3-3.10$^{2}_{31}$
C.3-3.8$^{2}_{13}$
C.3-5$^{3}_{11}$



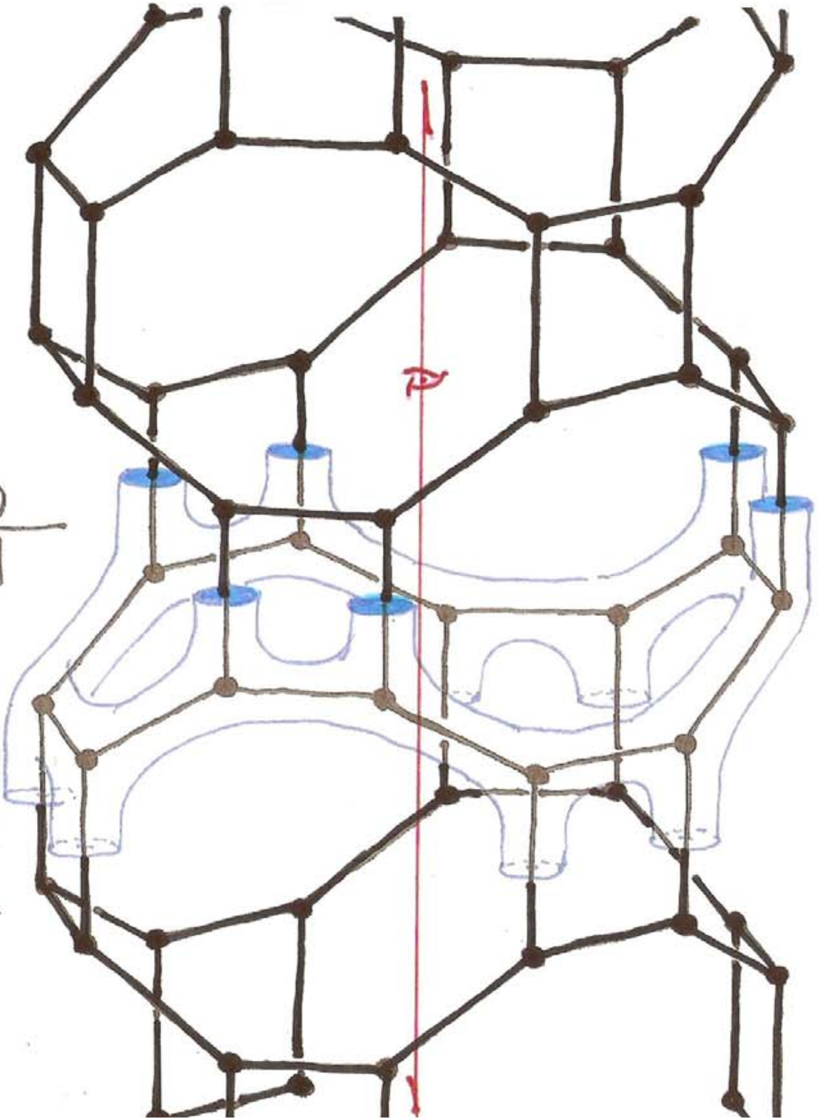
n-SYMM.
$(3.4^2)^3_{n+1}$
$g_{T.U.} = n+1$
$\Sigma\alpha = 4\pi$
Val. = 9
$V_{T.U.} = 2n$
$E_{T.U.} = 9n$
$F_{T.U.} = 5n$
n-SYMM.
$(4.4^2)^4_{2n+1}$
$g_{T.U.} = 2n+1$
$\Sigma\alpha = 6\pi$
Val. = 12
$V_{T.U.} = 2n$
$E_{T.U.} = 12n$
$F_{T.U.} = 6n$

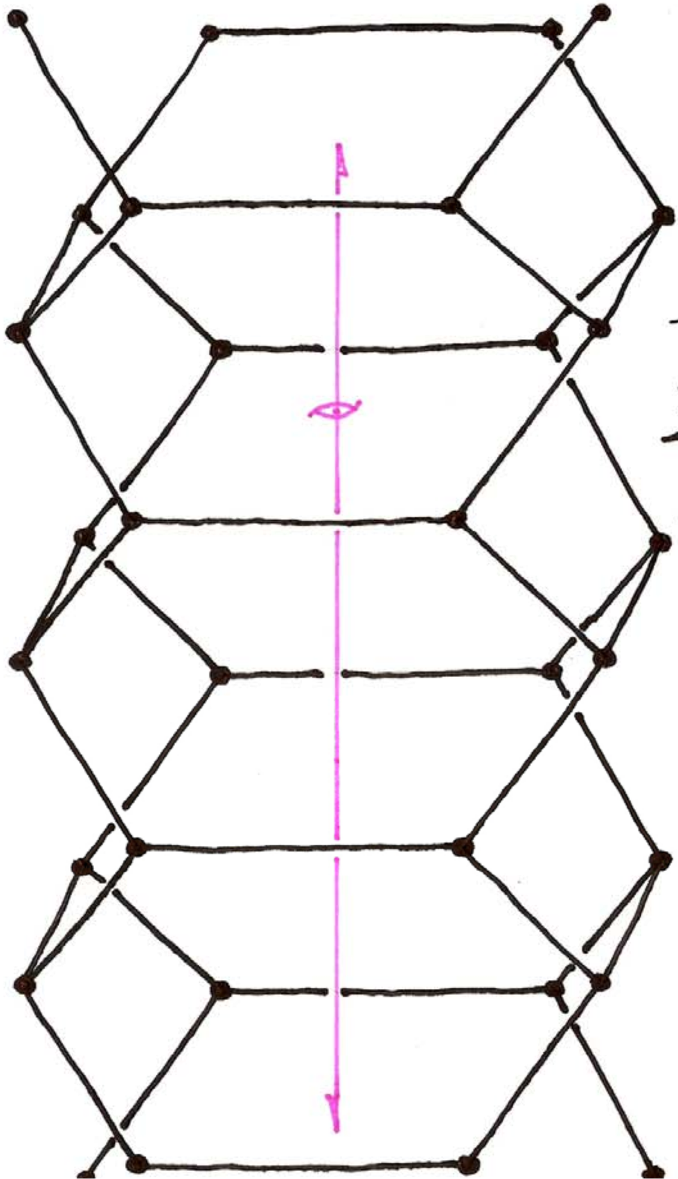


$$(3.4^2)^3_{(4n+1)}$$

$$g_{T.U.} = 4n+1$$
$$\Sigma\alpha = 4n$$
$$\text{Val.} = 9$$

$$V_{T.U.} = 8n$$
$$E_{T.U.} = 36n$$
$$F_{T.U.} = 20n$$

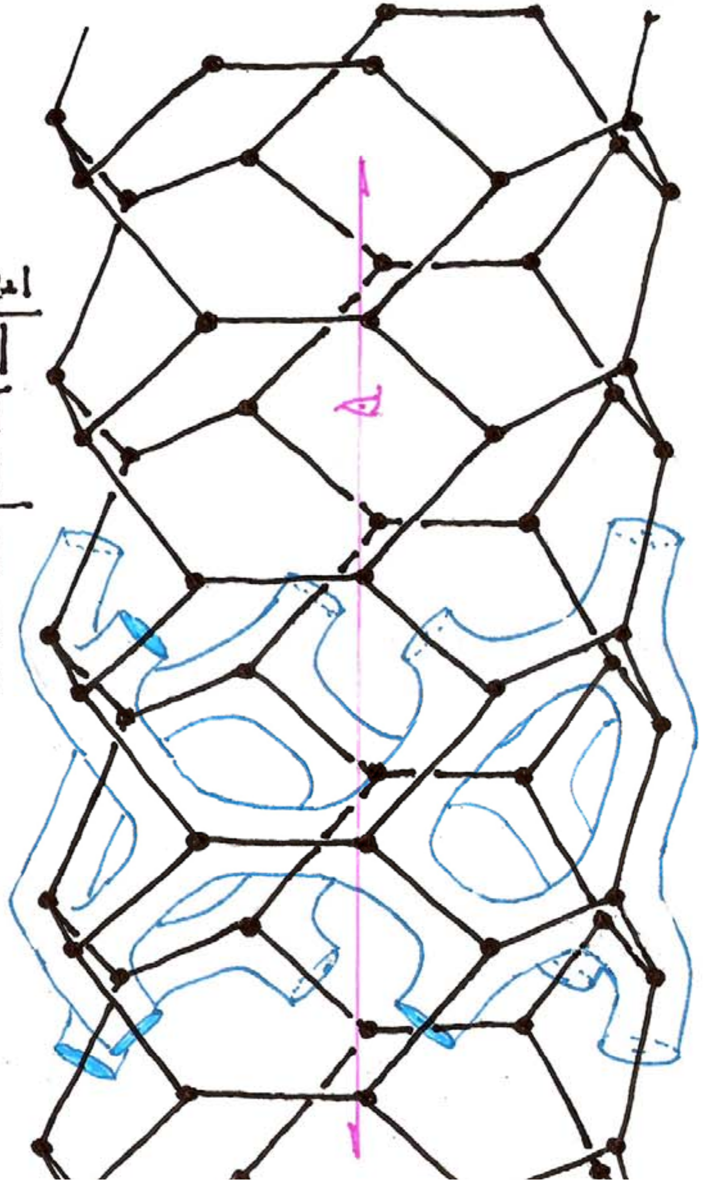




$$(3.4^2)^3_{2n+1}$$

$$g_{T.U.} = 2n+1$$
$$\Sigma\alpha = 4\pi$$
$$\text{Val.} = 9$$

$$V_{T.U.} = 4n$$
$$E_{T.U.} = 18n$$
$$F_{T.U.} = 10n$$



$$(4.4^2)^4_{13}$$

$g_{T.U.} = 13$

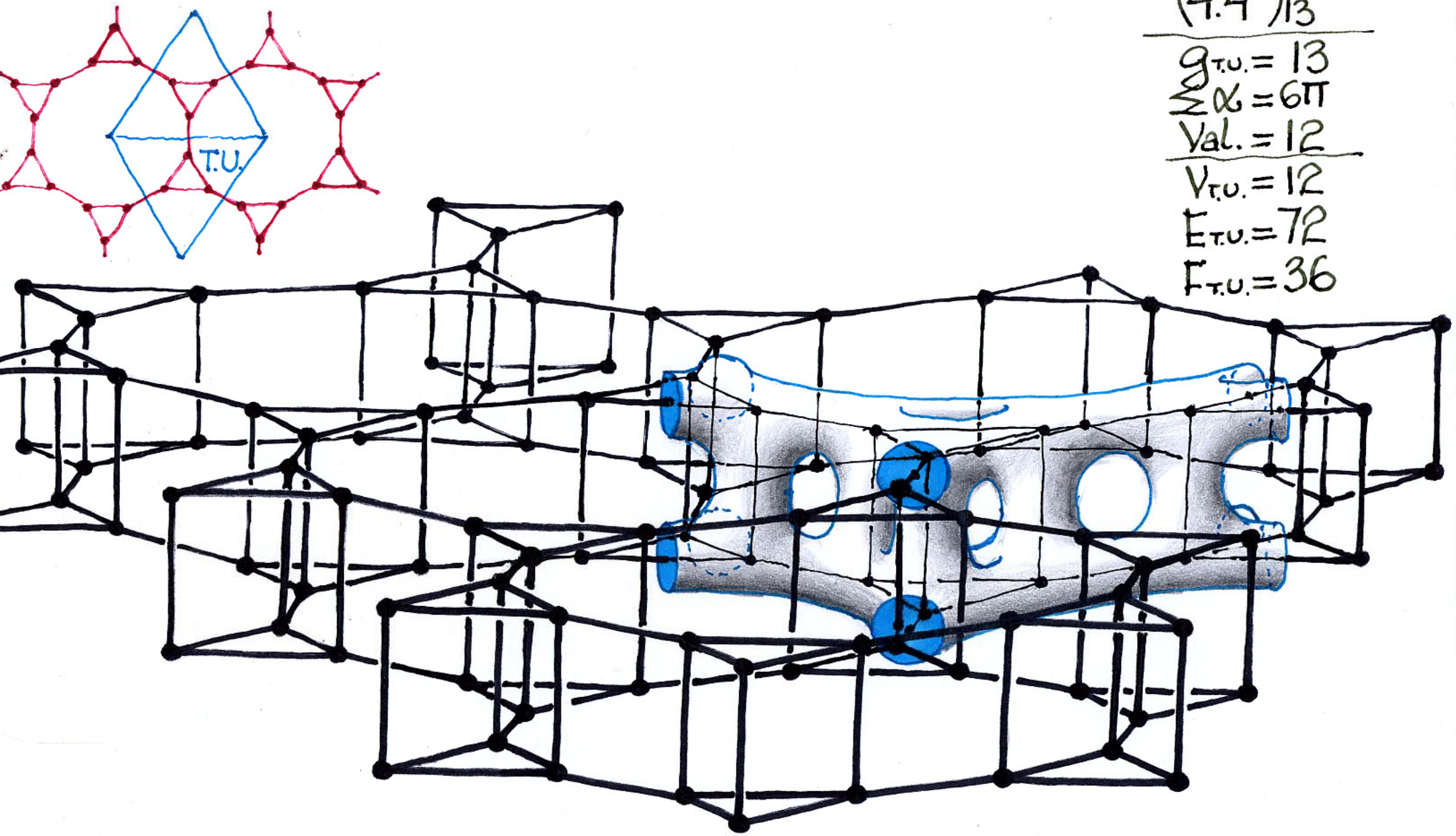
$\sum \alpha = 6\pi$

$Val. = 12$

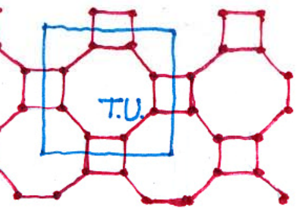
$V_{T.U.} = 12$

$E_{T.U.} = 72$

$F_{T.U.} = 36$



IFORM DOUBLE-LAYER TETRAVALENT $D.4\text{-}3.4^3.12^2_{13}$ SPACE LATTICE



$$(4.4^2)^4_{17}$$

$g_{T.U.} = 17$

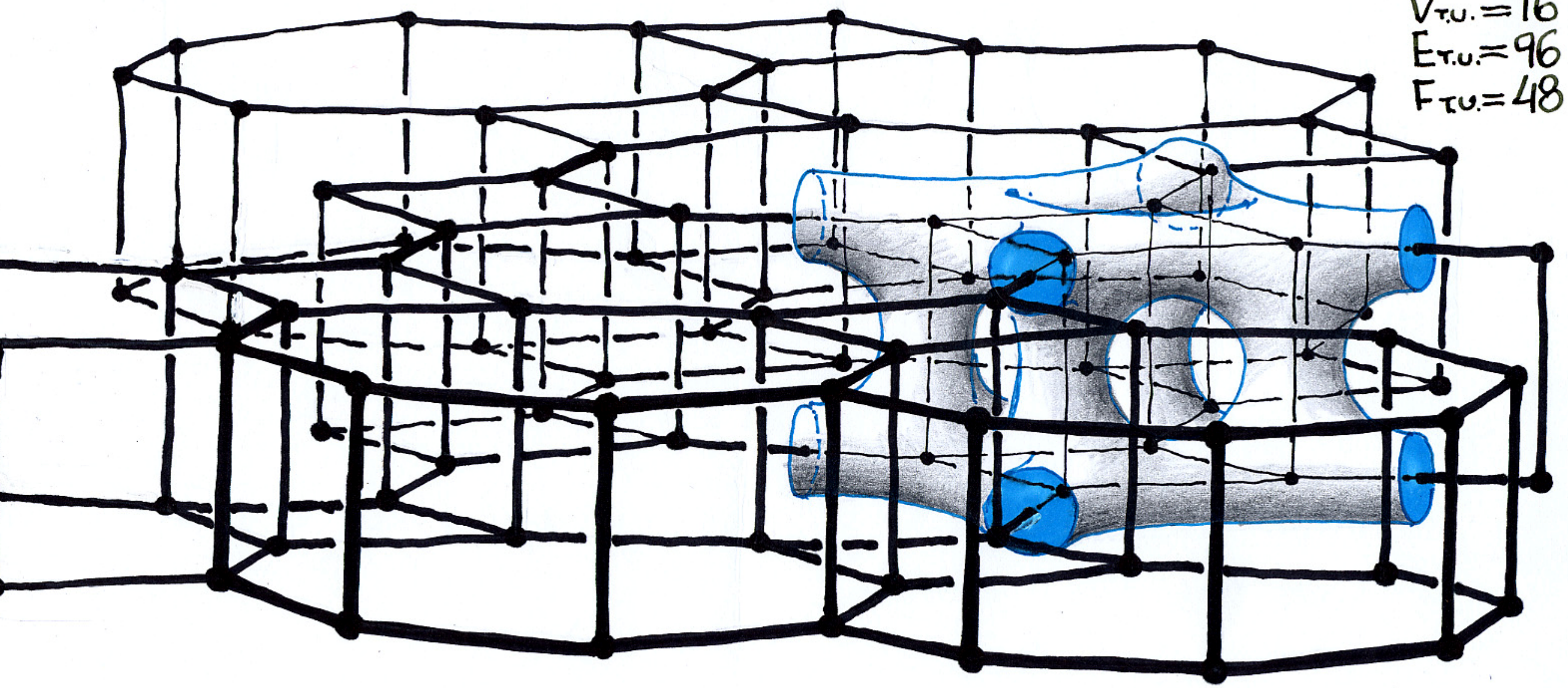
$\sum\alpha = 6\pi$

$Val. = 12$

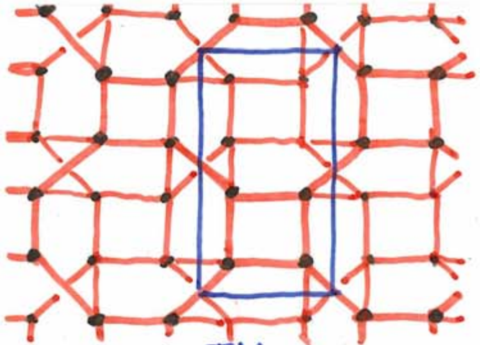
$V_{T.U.} = 16$

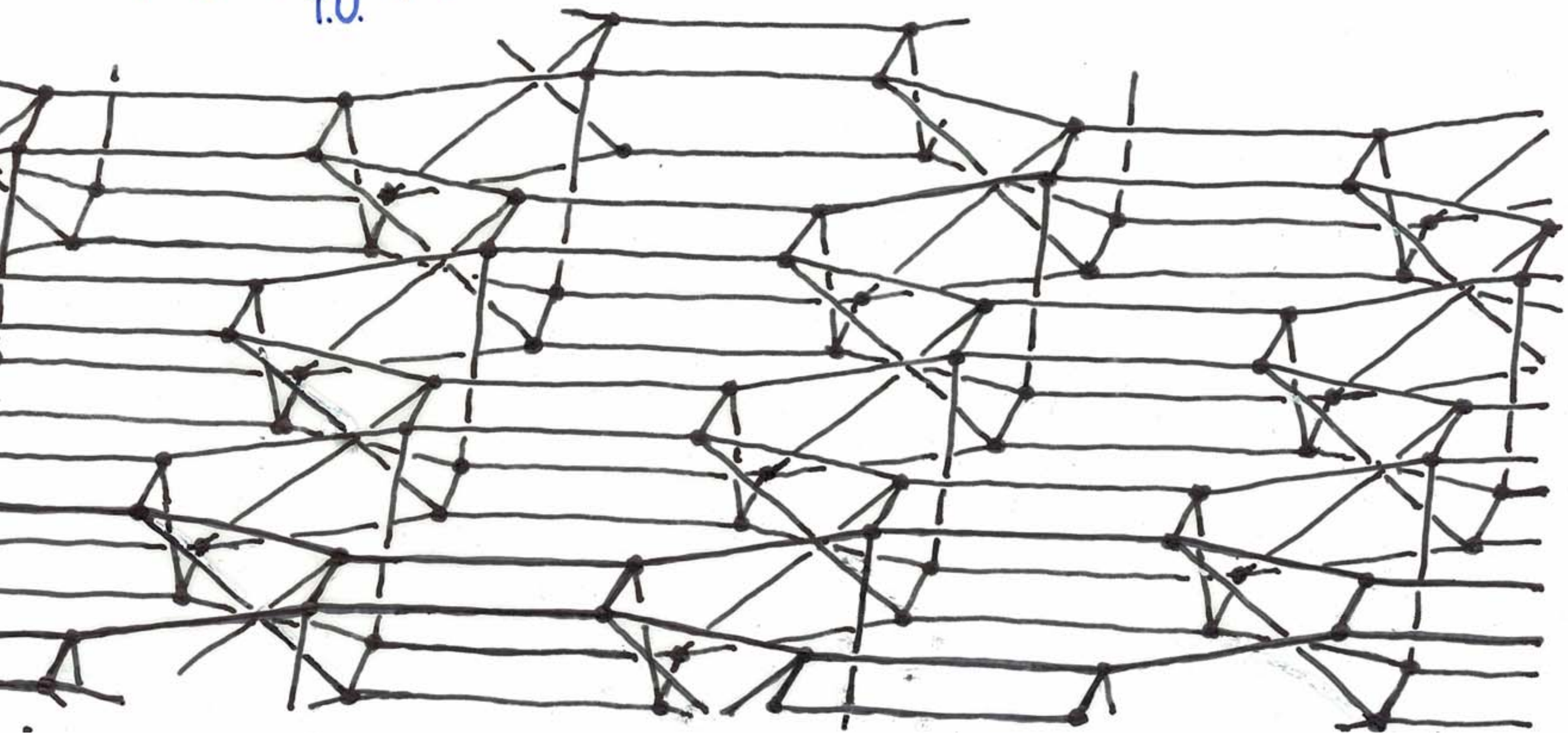
$E_{T.U.} = 96$

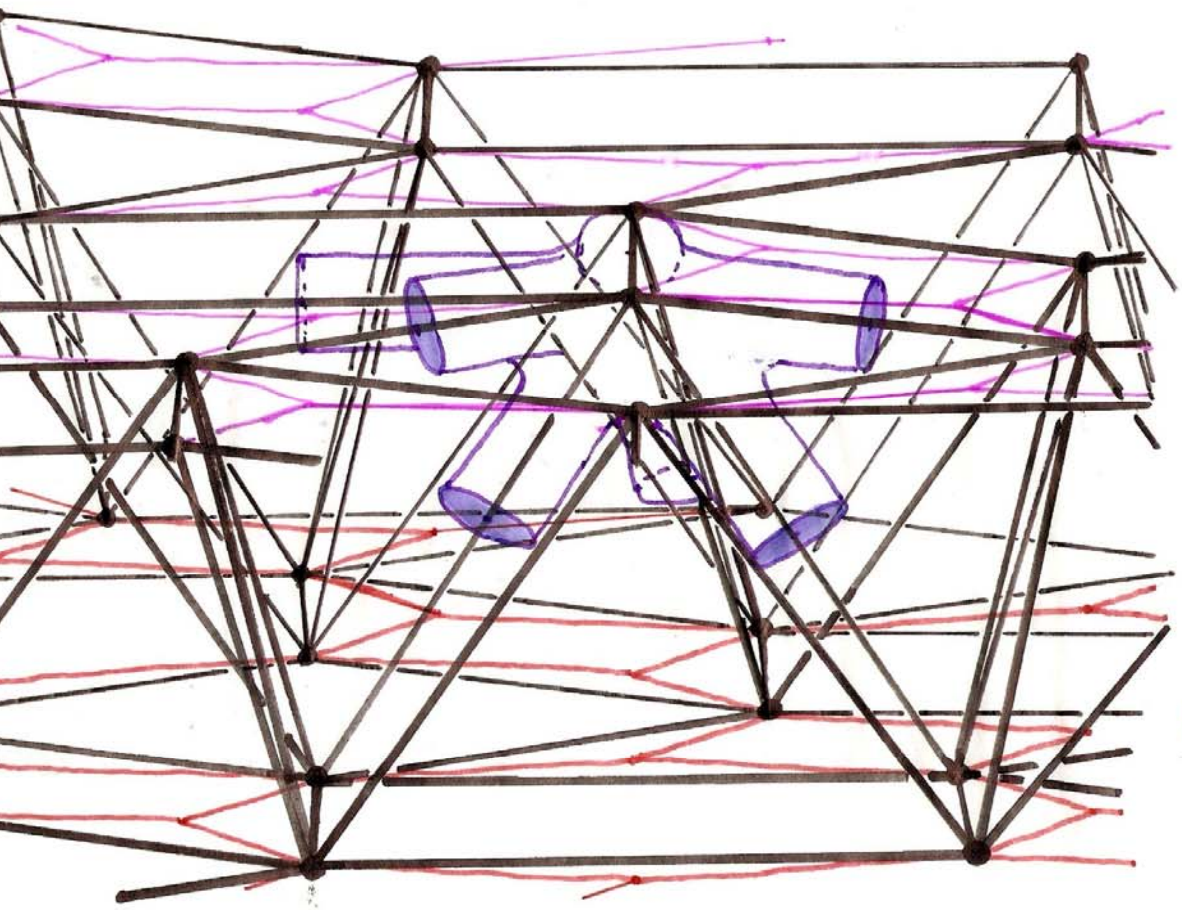
$F_{T.U.} = 48$



NIFORM DOUBLE-LAYER TETRAVALENT D.4-$4^3.8^2_{17}$ SPACE LATTICE


T.U.

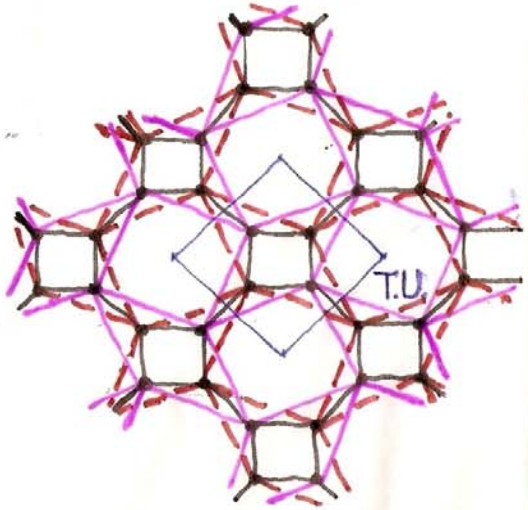




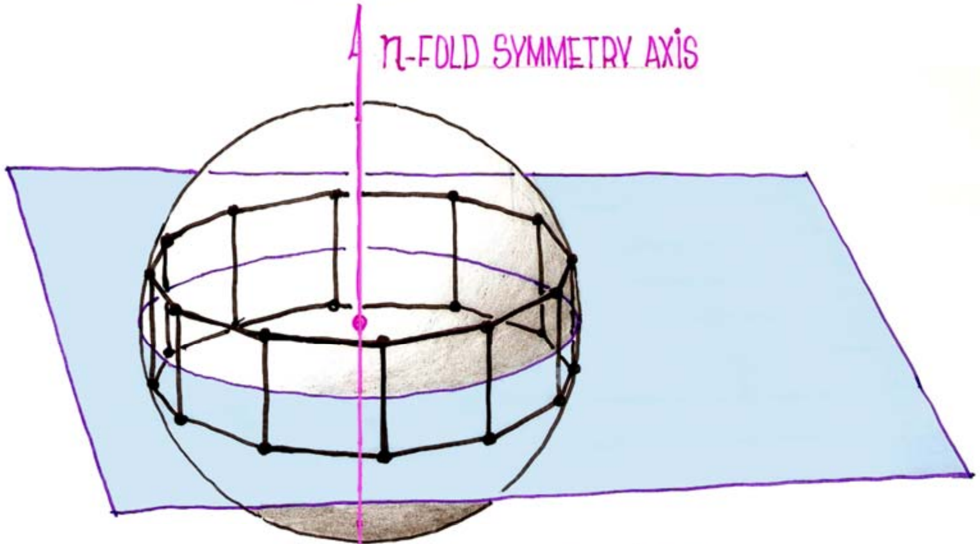
$$(4^2.7)^{7}_{11}$$

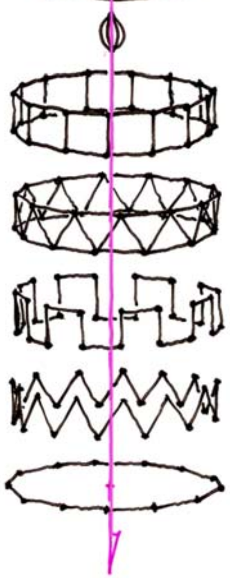
$$4\quad g_{T.U.}=11$$
$$\Sigma\alpha=12\pi$$
$$Val.=21$$

$$V_{T.U.}=4$$
$$E_{T.U.}=42$$
$$F_{T.U.}=18$$



ORM DOUBLE LAYER HEPTAVALENT D7 24/5 SPACE LATTİCE





$CAD.n\text{-}2^{n}_{n-1}$ 1. POLYDIGONS

$CAD3\text{-}4^{2}.n_{n+1}$ 2.

$CAD4\text{-}3^{3}.n_{2n+1}$ 3.

$CAD2\text{-}4n_{1}$ 4.

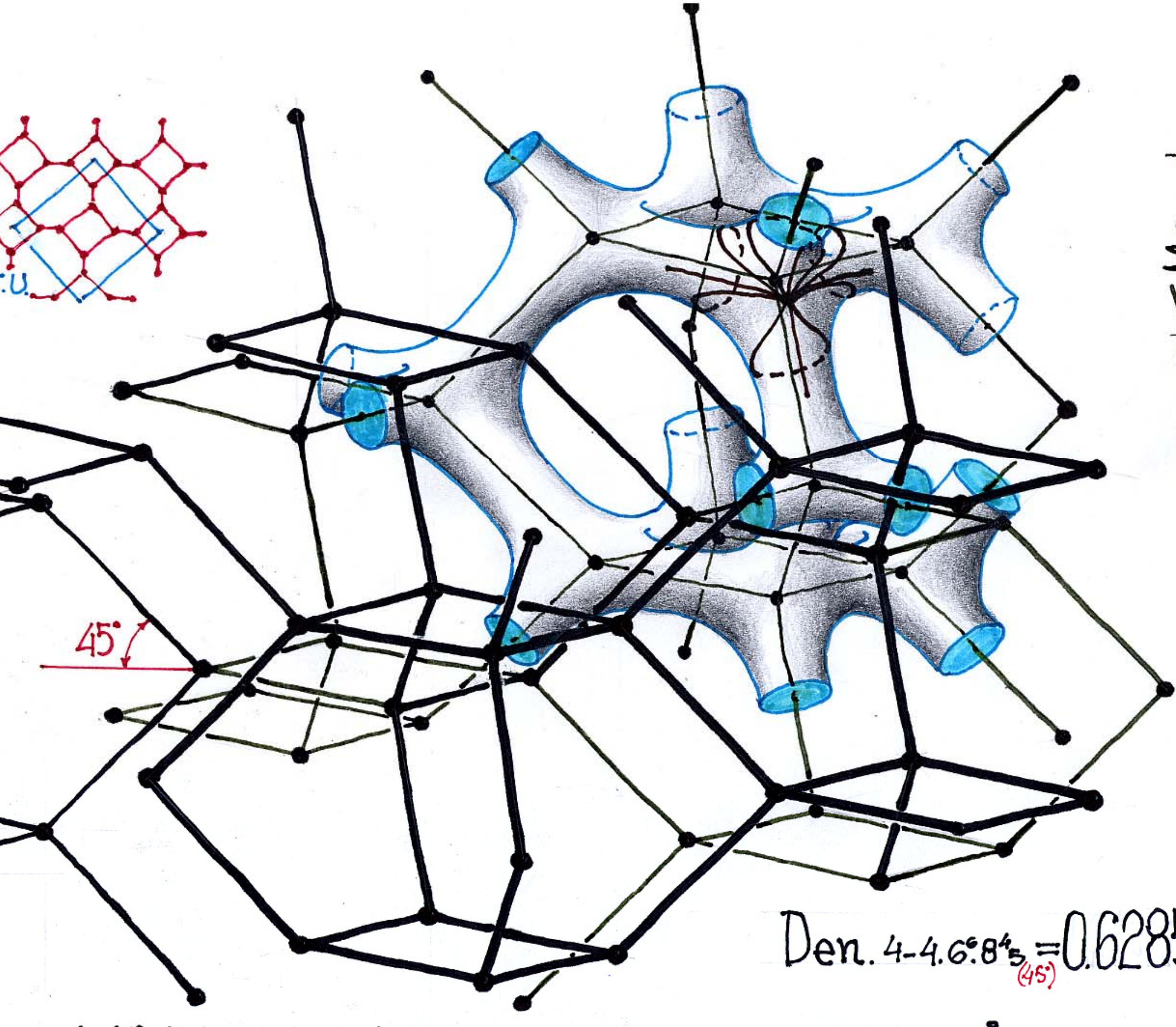
$CAD2\text{-}2n_{1}$ 5.

$CAD2\text{-}n_{1}$ 6.

ALL VERTICES OF THE 'BAND NETWORKS ARE EQUIDISTANT FROM A FIXED CENTRE-POINT, AXIS AND A PLANE SURFACE.

THE CAD CATEGORY GROUP INCLUDES INFINITE NUMBER OF MEMBERS.

DISTINCTIVE GROUP OF PRIMITIVE UNIFORM BAND-



$(4.4^2)^4$

$g_{T.U.} = 9$

$\Sigma\alpha = 6\pi$

Val. = 12

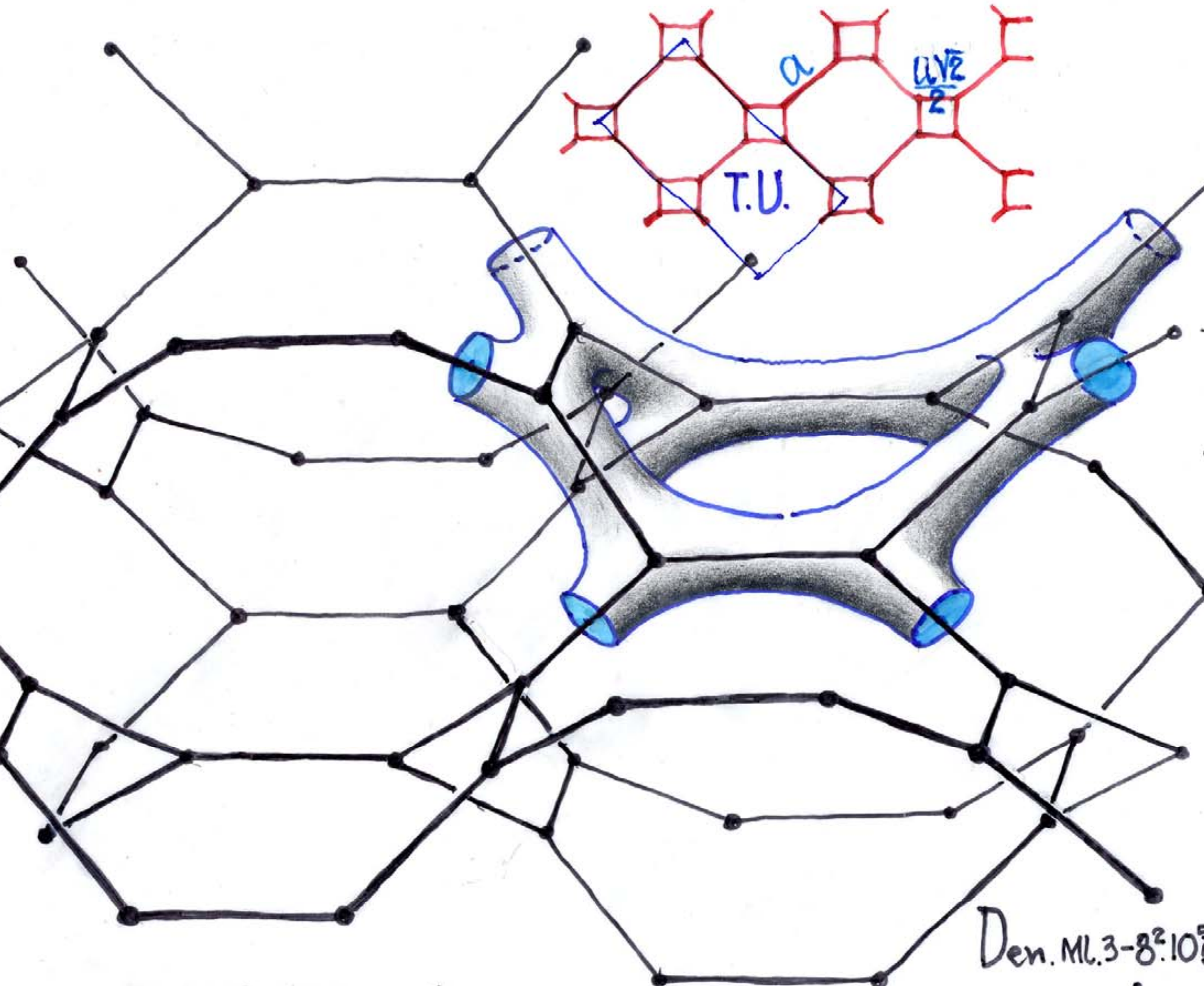
$V_{T.U.} = 8$

$E_{T.U.} = 48$

$F_{T.U.} = 24$

Den.$_{4\text{-}4.6^6 8^4 S \,(45°)}$ = 0.628539361 a/a³

ORM TETRAVALENT 4-4.6⁶8⁴ SPACE LATTICE AND



4^{6}_{5}

$g_{T.U.} = 5$

$\Sigma\alpha = 3\pi$

$Val. = 6$

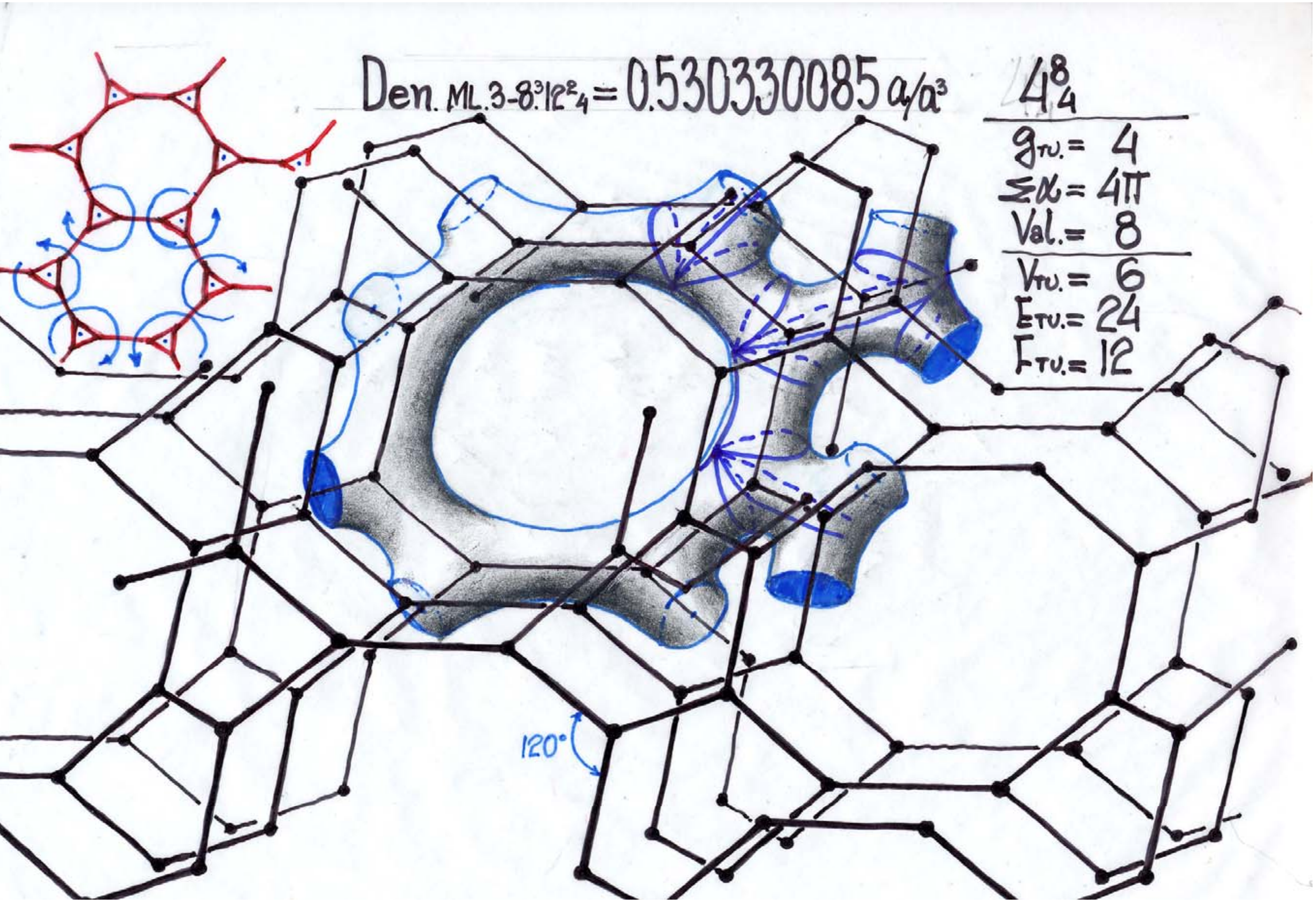
$V_{T.U.} = 16$

$E_{T.U.} = 48$

$F_{T.U.} = 24$

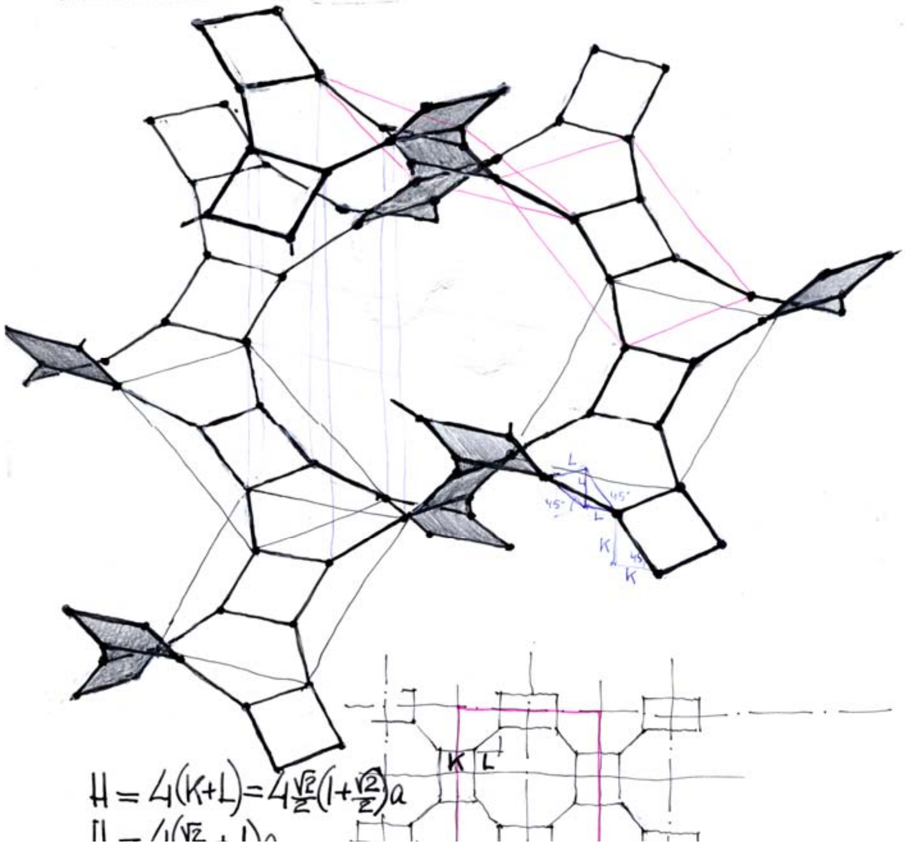
$D_{en.\,ML.3-8^2.10^5_5} = 0.618718433\, g/a^3$

LAYERED UNIFORM TRIVALENT ML3- SPACE LATTICE AND


Den. ML.3-8³·12²₄ = 0.530330085 a/a³
4⁸₄
g TU. = 4
Σα = 4π
Val. = 8
V TU. = 6
E TU. = 24
F TU. = 12
120°

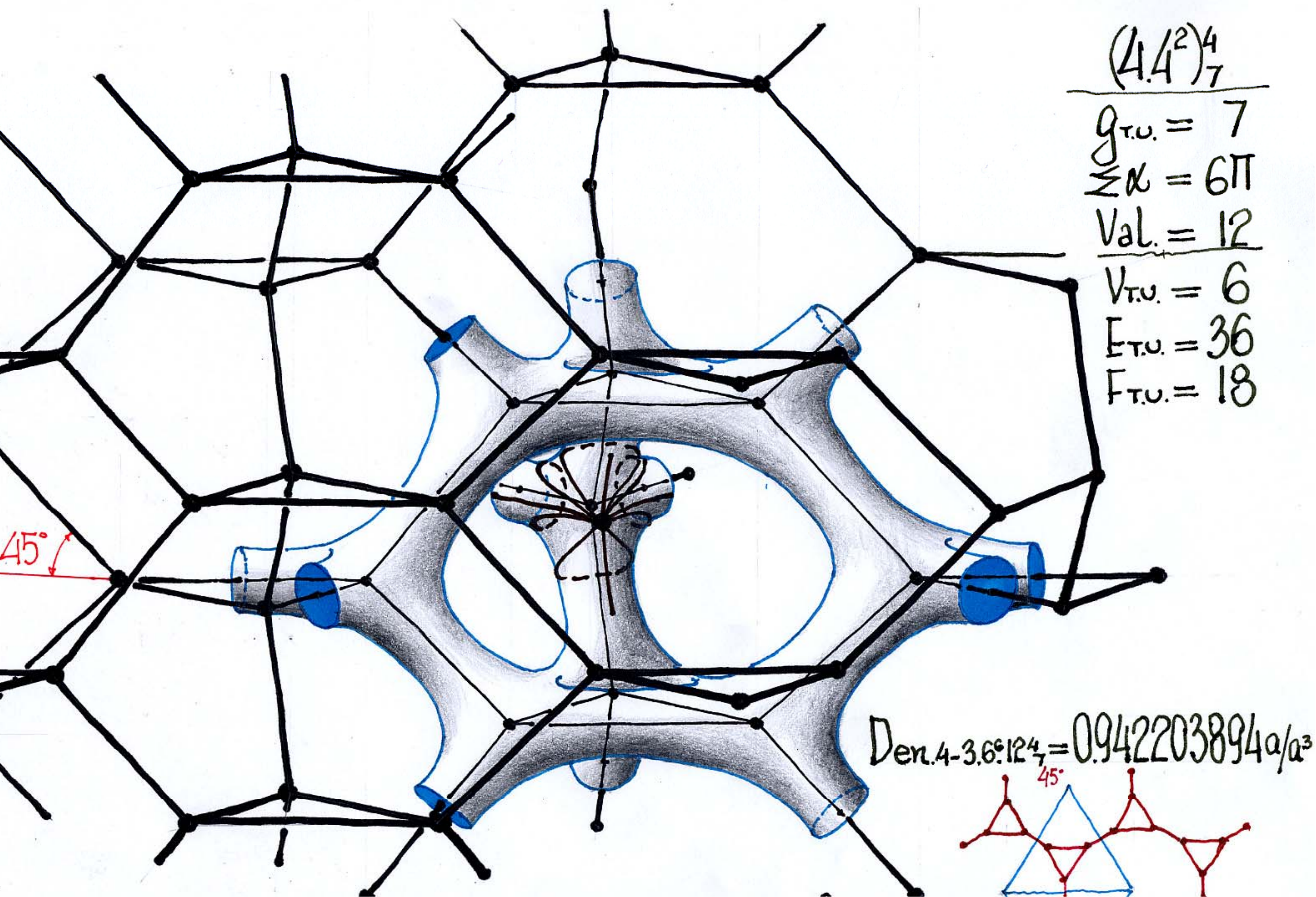
$$\text{Den.} = \frac{Na}{b^2 \cdot H} = \frac{12a}{4(K+L)(1+2L+K)^2} = \frac{3}{\frac{\sqrt{2}+1}{2}\left(2+\frac{\sqrt{2}}{2}\right)^2}$$

$$\boxed{\text{Den}_{4.8.16^2_3} = 0.285181197\ ^{a}/_{a^3}}$$

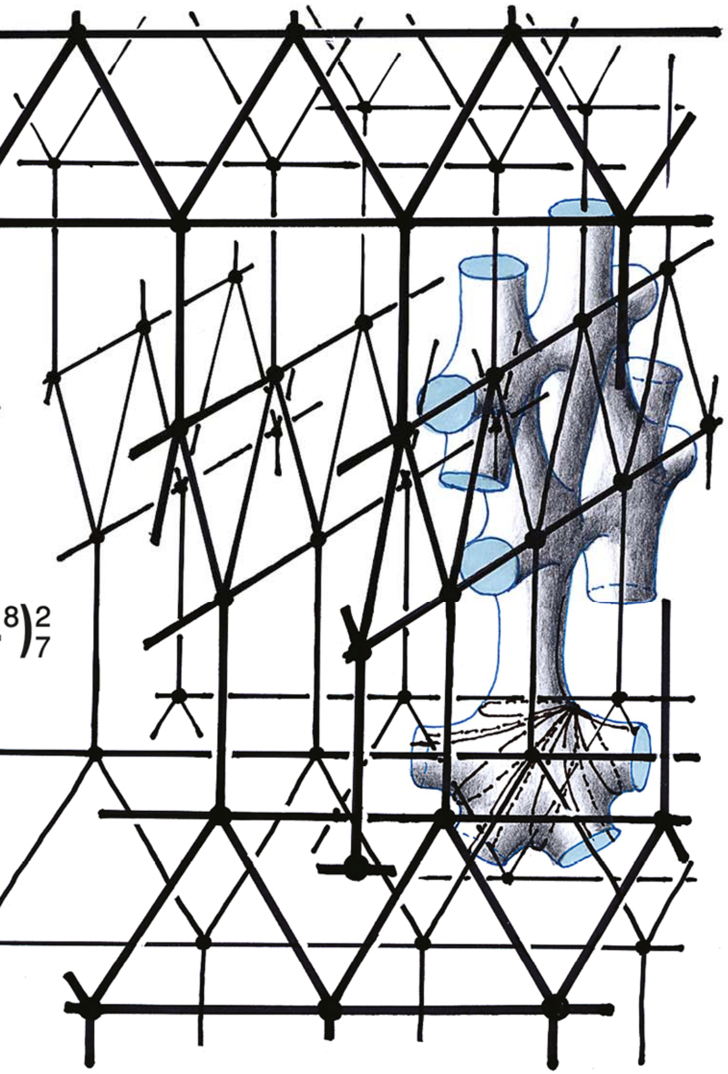


$$H = 4(K+L) = 4\frac{\sqrt{2}}{2}\left(1+\frac{\sqrt{2}}{2}\right)a$$

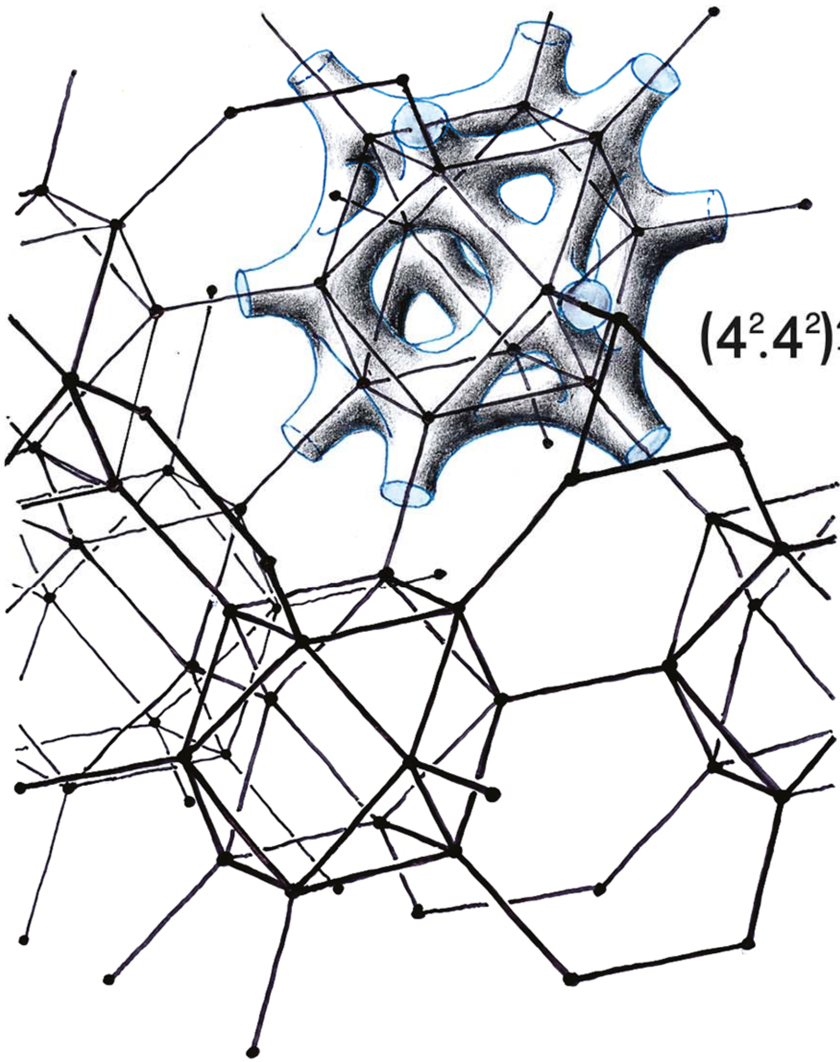
$$H = 4\left(\sqrt{2}+1\right)a$$



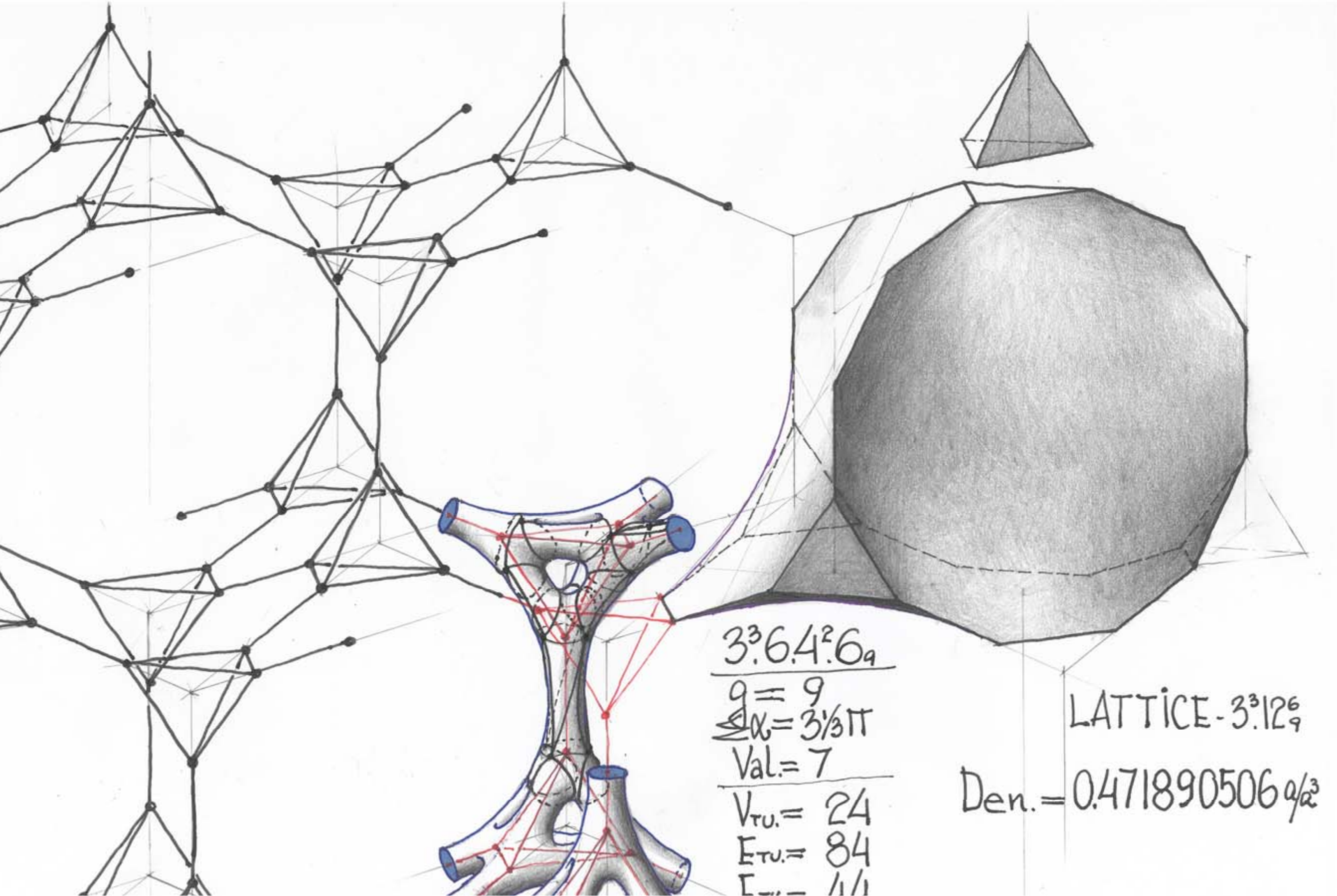
45°
$(4.4^2)^4_7$
$g_{T.U.} = 7$
$\sum\alpha = 6\pi$
Val. = 12
$V_{T.U.} = 6$
$E_{T.U.} = 36$
$F_{T.U.} = 18$
$Den._{4-3.6^6.12^4_7} = 0.942203894\ a/a^3$
45°

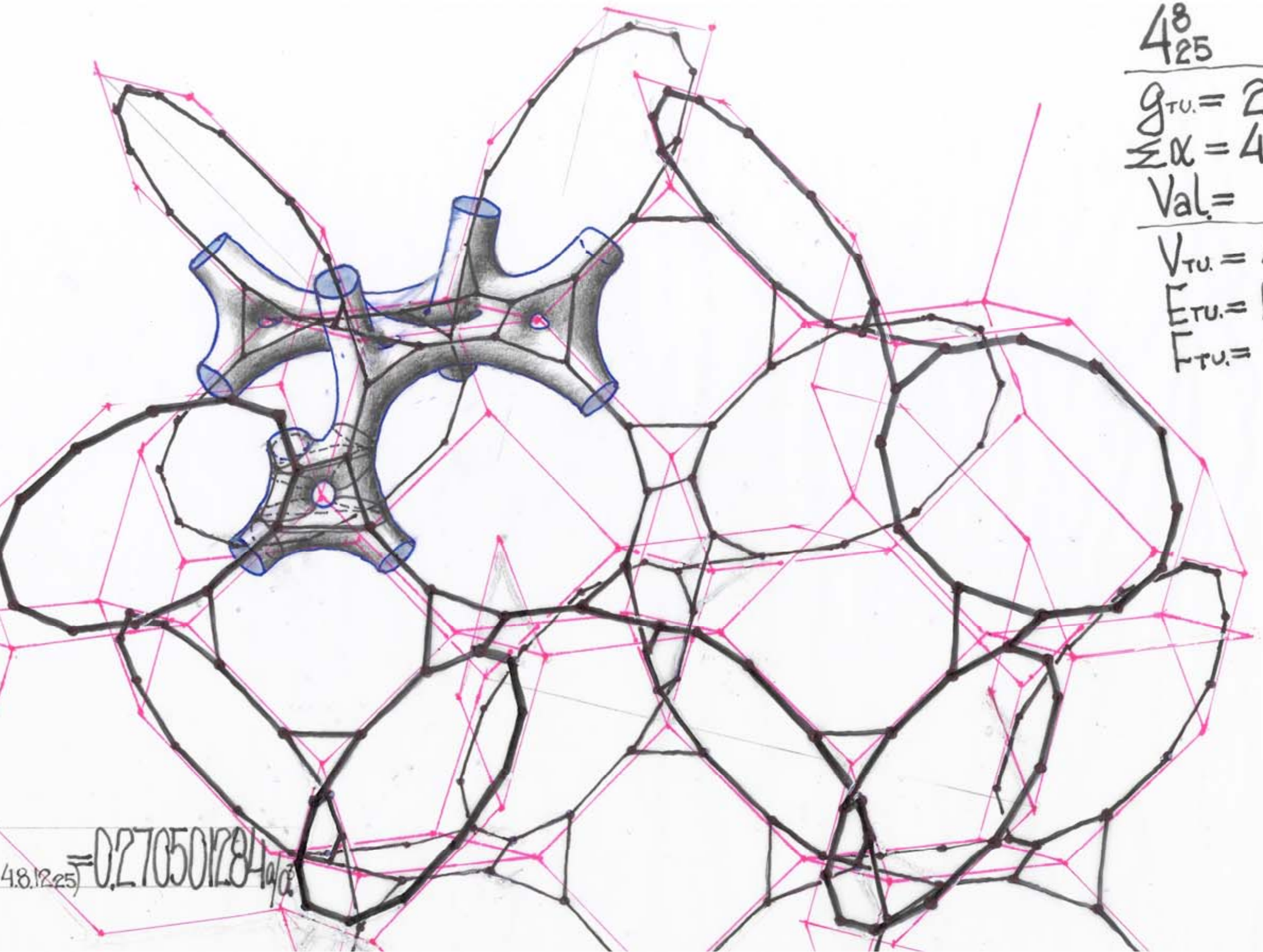


$^{8})^{2}_{7}$



$(4^{2}.4^{2})^{4}_{19}$


$3^3.6.4^2.6_9$
$g = 9$
$\sphericalangle\alpha = 3\frac{1}{3}\pi$
Val.= 7
$V_{TU.} = 24$
$E_{TU.} = 84$
LATTICE - $3^3.12^6_9$
Den. = 0.471890506 a/a^3



$$4^{8}_{25}$$

$g_{TU.} = 25$

$\Sigma\alpha = 4\pi$

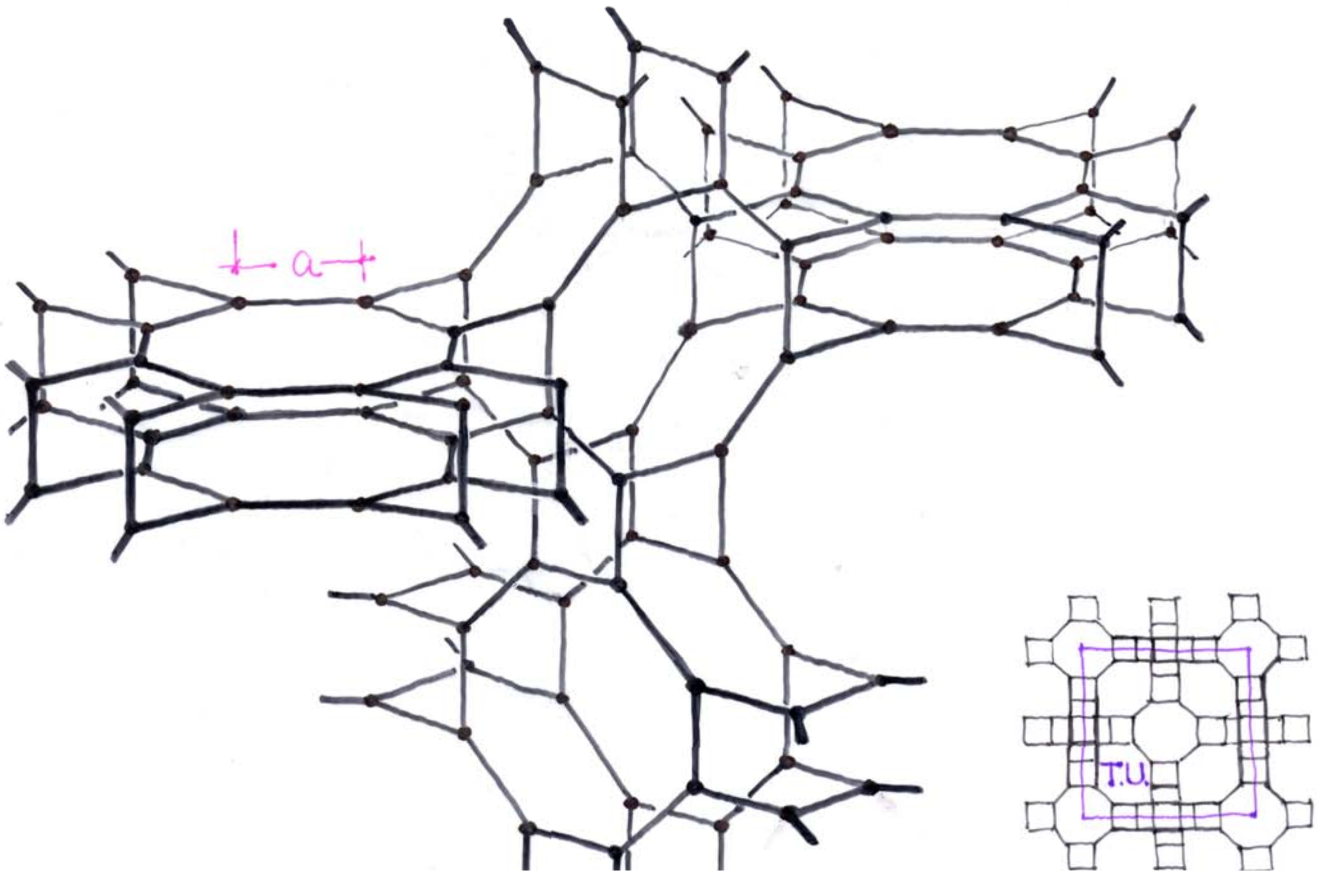
$Val. = 8$

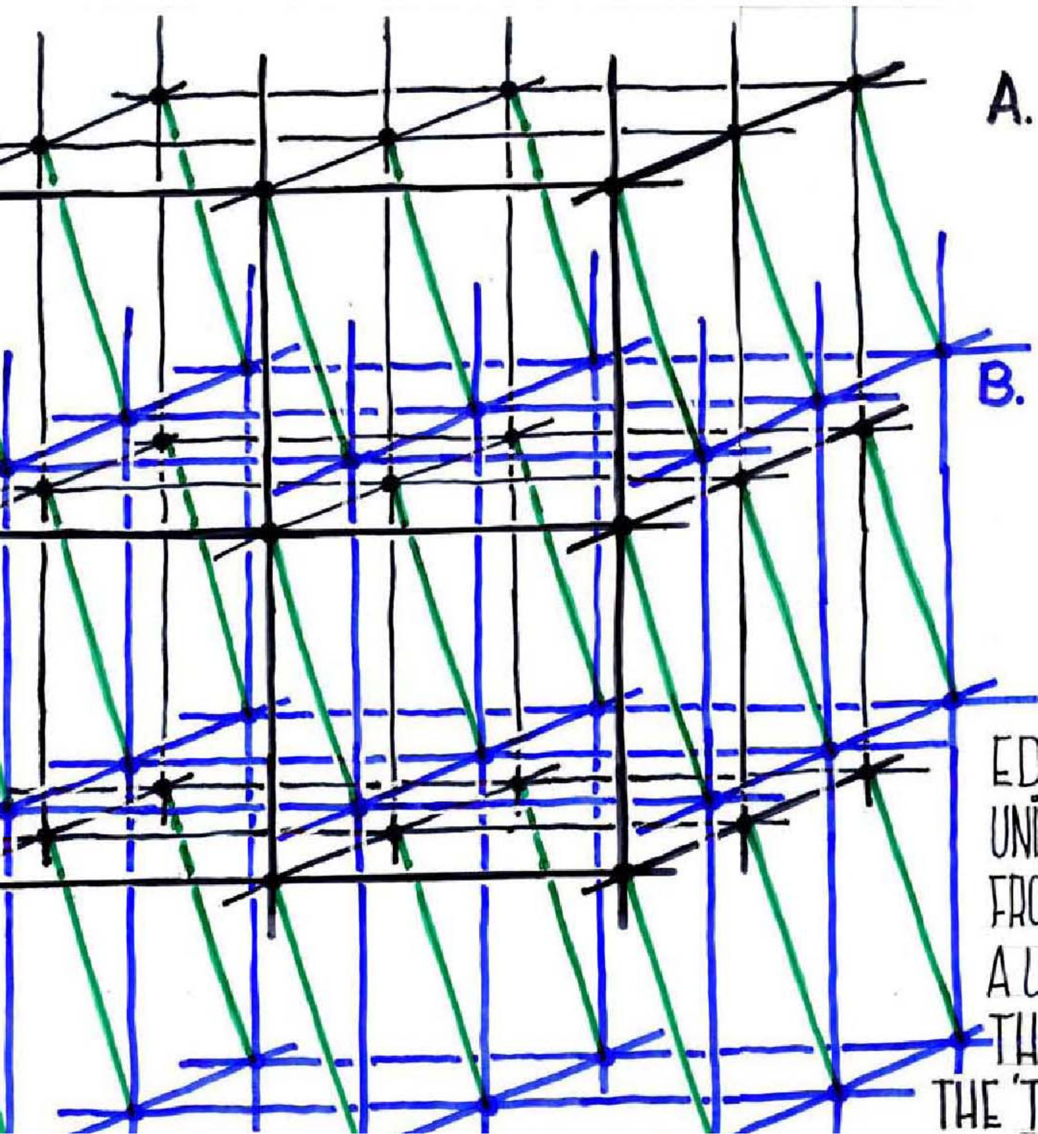
$V_{TU.} = 48$

$E_{TU.} = 192$

$F_{TU.} = 96$

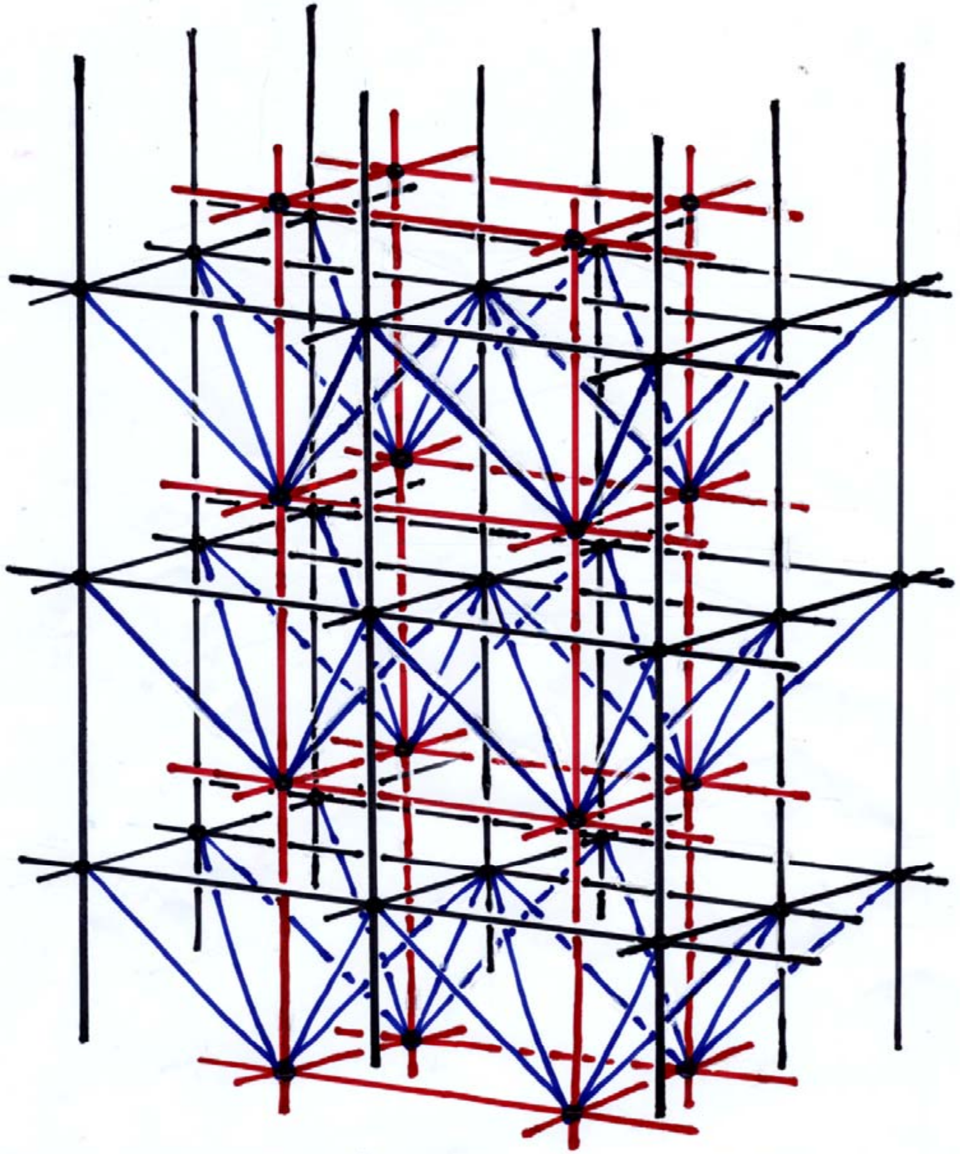
UNIFORM TRIVALENT SPACE LATTICE - $8^4.12_{25}$

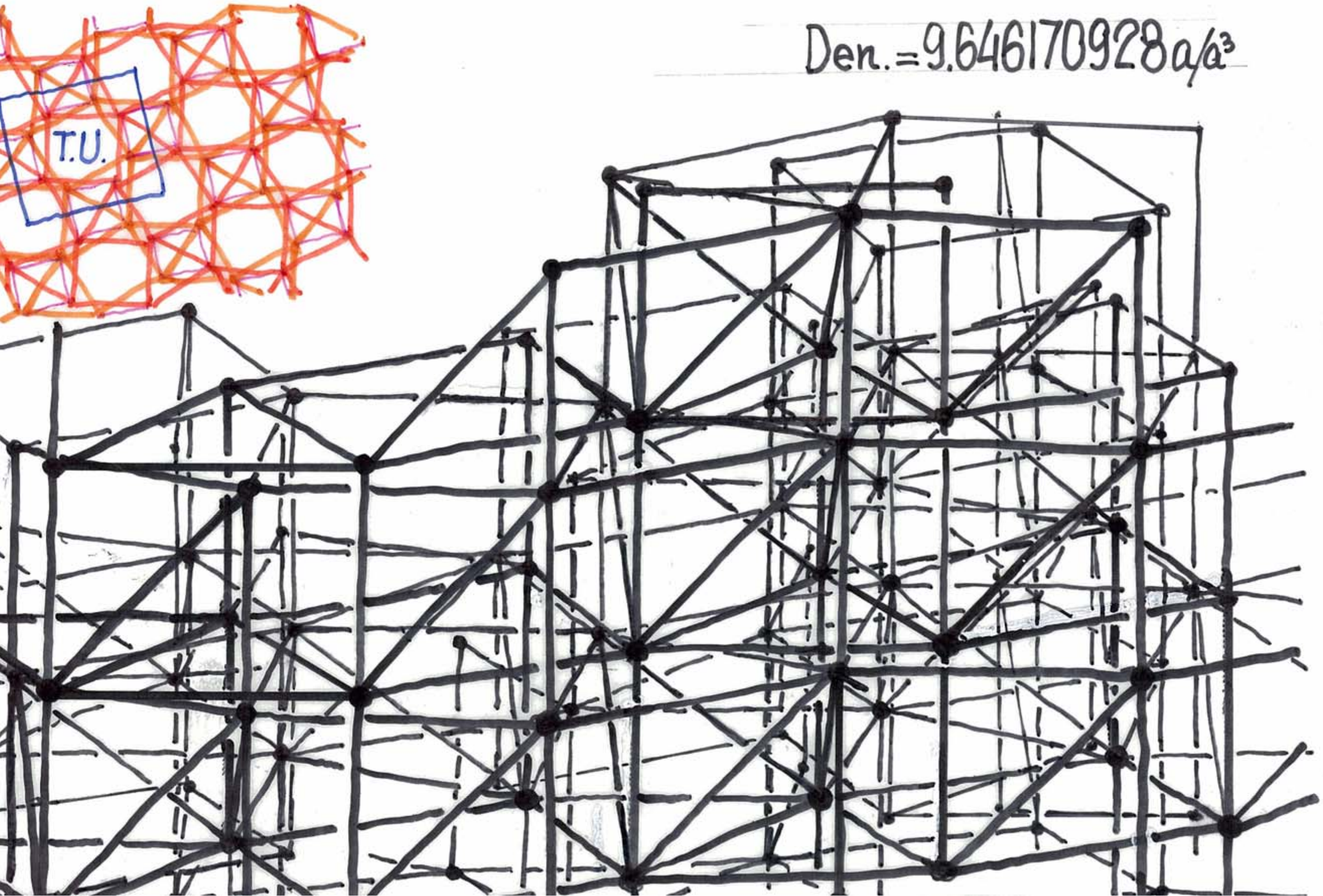




ENTANGLED NETWORKS

EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT (CUBIC) LATTICE FROM A-TO B-POSITION, RESULTING IN A UNIFORM SEPTAVALENT LATTICE, THE DENSITY OF WHICH IS 7.00 a/a^3. THE 'TRANSLATION LATTICE' IS A 3-D

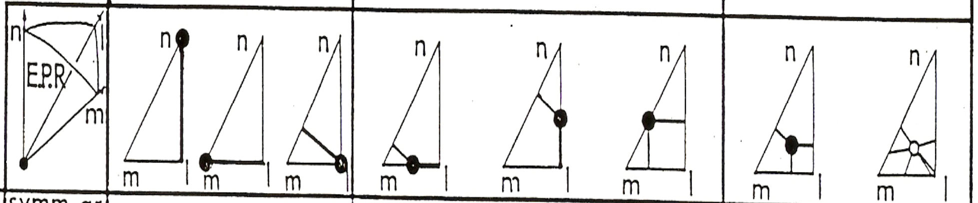



T.U.
Den. $=9.646170928\, a/a^3$

CONCEPTUAL DETERMINATION OF THE METHOD

1. **Morphological categorization of ‘uniform networks’**, whether finite or infinite, in 3-D space.
2. Basing the enquiry on the well founded and articulated symmetry research and the **established symmetry space groups**, concerning all the space networks of all a.m. categories.
3. On the basis of the **established symmetry space groups** accomplishing **exhaustive enumeration** and **stereometric representation** of all the **Elementary Periodic Regions (E.P.R.s)** in 3-D space and their **associated enveloping symmetry operation elements.**
4. Performing **exhaustive stereometric search** of all possible **vertex locations within the E.P.R.s** which constitute, **after the proper replication process**, **the uniform 3-D networks**, **with identical edges and vertex figures.**

M. Burt



		vertex-located pol.-V.L.P			edge-located polyhedra-E.L.P			face located pol.-F.L.P				
generalized notation	symm. gr. (l,m,n)	$(l.m)^n$	$(l.n)^m$	$(m.n)^l$	l.2n.m.2n	l.2m.n.2m	m.2l.n.2l	2l.2m.2n	l.3.m.3.n.3			
polydigons		$(2.2)^n$								Σα=0	r=2÷n	finite uniform polyhedra
dihedrons	(2,2,n)		$(2.n)^2$	$(2.n)^2$	2.2n.2.2n						r= 2; 3, 4	
prisms						2.4.n.4	2.4.n.4	4.4.2n		0≤Σα<2Π	r= 3 ; 4	
antiprisms									2.3.2.3.n.3		r= 4 ; 5; 6	
Platonic & Archimedean polyhedra	(2,3,3)	$(2.3)^3$	$(2.3)^3$	$(3.3)^2$	2.6.3.6	2.6.3.6	3.4.3.4	4.6.6	2.3.3.3.3.3	Π≤Σα<2Π	r=3; 4; 5.6	
	(2,3,4)	$(2.3)^4$	$(2.4)^3$	$(3.4)^2$	2.8.3.8	2.6.4.6	3.4.4.4	4.6.8	2.3.3.3.4.3			
	(2,3,5)	$(2.3)^5$	$(2.5)^3$	$(3.5)^2$	2.10.3.10	2.6.5.6	3.4.5.4	4.6.10	2.3.3.3.5.3			
plane-tessellations	(2,3,6)	$(2.3)^6$	$(2.6)^3$	$(3.6)^2$	2.12.3.12	2.6.6.6	3.4.6.4	4.6.12	2.3.3.3.6.3	Σα=2Π	r=3;4;5;6	
	(3,3,3)	$(3.3)^3$	$(3.3)^3$	$(3.3)^3$	3.6.3.6	3.6.3.6	3.6.3.6	6.6.6	3.3.3.3.3.3			
	(2,4,4)	$(2.4)^4$	$(2.4)^4$	$(4.4)^2$	2.8.4.8	2.8.4.8	4.4.4.4	4.8.8	2.3.4.3.4.3			

EXHAUSTIVE ENUMERATION PROCESS OF UNIFORM NETWORKS

To accomplish it, two main geometric processes have to be performed:

1. An exhaustive (stereometric) E.P.R.s representations of the **8 discreet groups of displacements in the plane** and the **230 infinite discreet groups of congruent (symmetrical) transformations** in 3D space.
2. An exhaustive (discreet) search within every considered E.P.R.**, of all the possible vertex locations**, potentially leading to a solution of a uniform network's motif-form

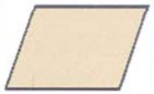
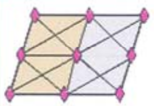
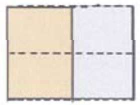
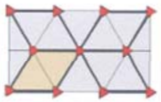
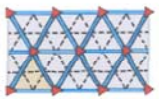
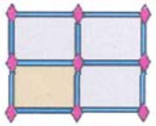
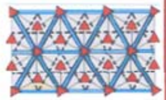
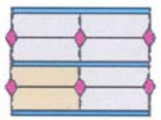
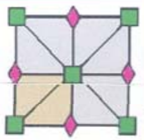
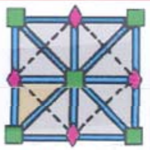
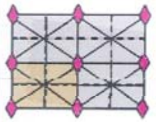
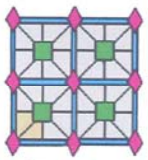
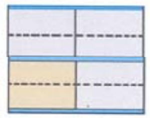
Plane groups

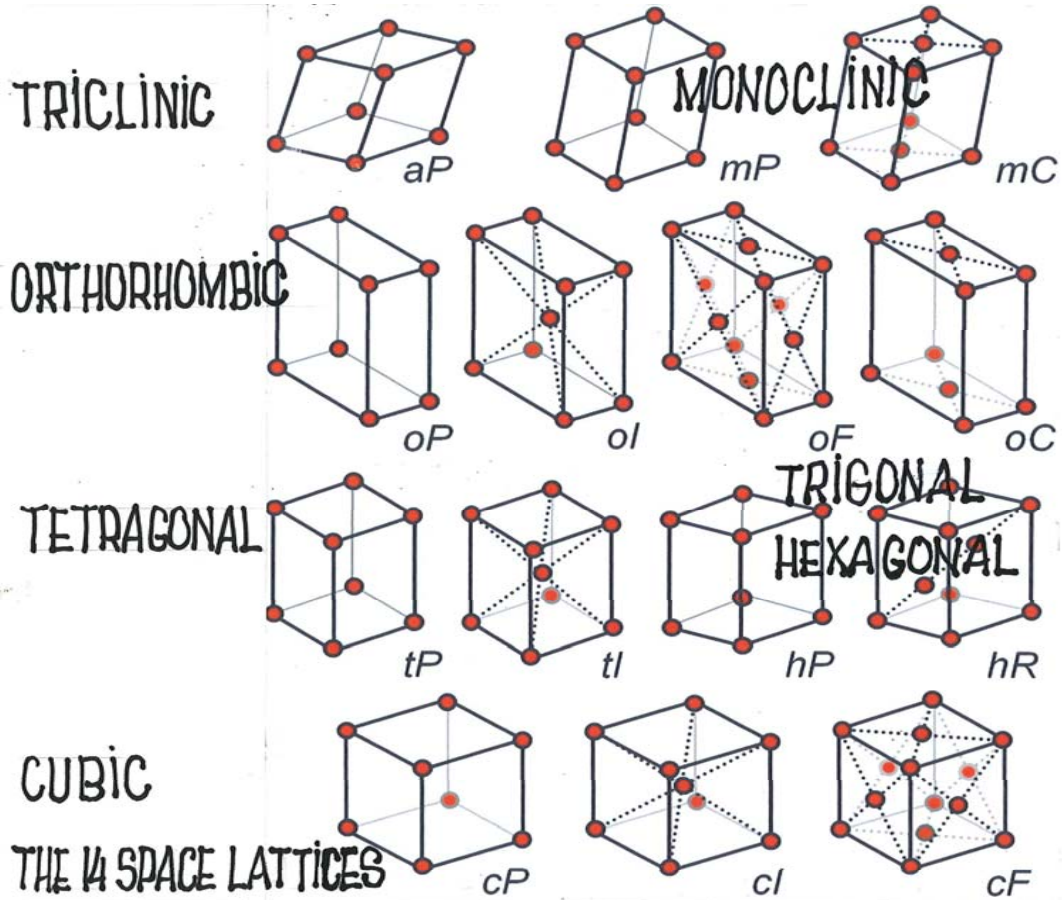
The following table shows the relationships between the 17 plane groups. The individual pictures are taken from the Wikipedia article on [Wallpaper groups](), which contains many examples of different tilings from architecture, arts and biology which represent these plane groups.

The groups are either based on rotations or reflections (mirror or glide axes) or a mixture of the two. Most are set in primitive cells, but two are based on the centred rectangular Bravais lattice.

The nomenclature of the groups follows a pattern

(the *Hermann-Mauguin scheme*): The first letter indicates whether the Bravais lattice is primitive (p) or centred (c). This is followed by the predominant symmetry element, *i.e.* the highest-order rotation (6, 4, 3, 2, or 1). Two intersecting mirror

pure rotations	rotation & reflection	pure reflections
 p1: 1-fold		 pm: mirror
 p2: 2-fold		 pg: glide
 p3: 3-fold	 p3m1	 pmm: 2 mirrors
	 p31m	 pmg
 p4: four-fold	 p4m	 pgg
	 p4g	Centred cells:  cm



THE 14 SPACE LATTICES

Class	Group
Triclinic	C_1 C_i
Monoclinic	C_2 C_s C_{2h}
Orthorhombic	D_2 C_{2v} D_{2h}
Tetragonal	C_4 S_4 C_{4h} D_4 C_{4v} D_{2d} D_{4h}

2

13

59

Symbols for the 230 space groups. Notation is described in the text

Triclinic

1. C_1^1 $(P1)$
2. C_i^1 $(P\bar{1})$

Monoclinic

3. C_2^1 $(P2)$
4. C_2^2 $(P2_1)$
5. C_2^3 $(B2)$
6. C_s^1 (Pm)
7. C_s^2 (Pb)
8. C_s^3 (Bm)
9. C_s^4 (Bb)
10. C_{2h}^1 $(P2/m)$
11. C_{2h}^2 $(P2_1/m)$
12. C_{2h}^3 $(B2/m)$
13. C_{2h}^4 $(P2/b)$
14. C_{2h}^5 $(P2_1/b)$
15. C_{2h}^6 $(B2/b)$

Orthorhombic

16. D_2^1 $(P222)$
17. D_2^2 $(P222_1)$
18. D_2^3 $(P2_12_12)$
19. D_2^4 $(P2_12_12_1)$
20. D_2^5 $(C222_1)$
21. D_2^6 $(C222)$
22. D_2^7 $(F222)$
23. D_2^8 $(I222)$
24. D_2^9 $(I2_12_12_1)$
25. C_{2v}^1 $(Pmm2)$
26. C_{2v}^2 $(Pmc2_1)$
27. C_{2v}^3 $(Pcc2)$
28. C_{2v}^4 $(Pma2)$
29. C_{2v}^5 $(Pca2_1)$
30. C_{2v}^6 $(Pnc2)$
31. C_{2v}^7 $(Pmn2_1)$
32. C_{2v}^8 $(Pba2)$
33. C_{2v}^9 $(Pna2_1)$
34. C_{2v}^{10} $(Pnn2)$
35. C_{2v}^{11} $(Cmm2)$
36. C_{2v}^{12} $(Cmc2_1)$
37. C_{2v}^{13} $(Ccc2)$
38. C_{2v}^{14} $(Amm2)$
39. C_{2v}^{15} $(Abm2)$
40. C_{2v}^{16} $(Ama2)$
41. C_{2v}^{17} $(Aba2)$
42. C_{2v}^{18} $(Fmm2)$
43. C_{2v}^{19} $(Fdd2)$
44. C_{2v}^{20} $(Imm2)$
45. C_{2v}^{21} $(Iba2)$
46. C_{2v}^{22} $(Ima2)$
47. D_{2h}^1 $(Pmmm)$
48. D_{2h}^2 $(Pnnn)$
49. D_{2h}^3 $(Pccm)$
50. D_{2h}^4 $(Pban)$
51. D_{2h}^5 $(Pmma)$
52. D_{2h}^6 $(Pnna)$
53. D_{2h}^7 $(Pmna)$
54. D_{2h}^8 $(Pcca)$
55. D_{2h}^9 $(Pbam)$
56. D_{2h}^{10} $(Pccn)$
57. D_{2h}^{11} $(Pbcm)$
58. D_{2h}^{12} $(Pnnm)$
59. D_{2h}^{13} $(Pmmn)$
60. D_{2h}^{14} $(Pbcn)$
61. D_{2h}^{15} $(Pbca)$
62. D_{2h}^{16} $(Pnma)$
63. D_{2h}^{17} $(Cmcm)$
64. D_{2h}^{18} $(Cmca)$
65. D_{2h}^{19} $(Cmmm)$
66. D_{2h}^{20} $(Cccm)$
67. D_{2h}^{21} $(Cmma)$
68. D_{2h}^{22} $(Ccca)$
69. D_{2h}^{23} $(Fmmm)$
70. D_{2h}^{24} $(Fddd)$
71. D_{2h}^{25} $(Immm)$
72. D_{2h}^{26} $(Ibam)$
73. D_{2h}^{27} $(Ibca)$
74. D_{2h}^{28} $(Imma)$

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Tetragonal

75. C_4^1 $(P4)$
76. C_4^2 $(P4_1)$
77. C_4^3 $(P4_2)$
78. C_4^4 $(P4_3)$
79. C_4^5 $(I4)$
80. C_4^6 $(I4_1)$
81. S_4^1 $(P\bar{4})$
82. S_4^2 $(I\bar{4})$
83. C_{4h}^1 $(P4/m)$
84. C_{4h}^2 $(P4_2/m)$
85. C_{4h}^3 $(P4/n)$
86. C_{4h}^4 $(P4_2/n)$
87. C_{4h}^5 $(I4/m)$
88. C_{4h}^6 $(I4_1/a)$
89. D_4^1 $(P422)$
90. D_4^2 $(P42_12)$
91. D_4^3 $(P4_122)$
92. D_4^4 $(P4_12_12)$
93. D_4^5 $(P4_222)$
94. D_4^6 $(P4_22_12)$
95. D_4^7 $(P4_322)$
96. D_4^8 $(P4_32_12)$
97. D_4^9 $(I422)$
98. D_4^{10} $(I4_122)$
99. C_{4v}^1 $(P4mm)$
100. C_{4v}^2 $(P4bm)$
101. C_{4v}^3 $(P4_2cm)$
102. C_{4v}^4 $(P4_2nm)$
103. C_{4v}^5 $(P4cc)$
104. C_{4v}^6 $(P4nc)$
105. C_{4v}^7 $(P4_2mc)$
106. C_{4v}^8 $(P4_2bc)$
107. C_{4v}^9 $(I4mm)$
108. C_{4v}^{10} $(I4cm)$
109. C_{4v}^{11} $(I4_1md)$
110. C_{4v}^{12} $(I4_1cd)$
111. D_{2d}^1 $(P\bar{4}2m)$
112. D_{2d}^2 $(P\bar{4}2c)$
113. D_{2d}^3 $(P\bar{4}2_1m)$
114. D_{2d}^4 $(P\bar{4}2_1c)$
115. D_{2d}^5 $(P\bar{4}m2)$
116. D_{2d}^6 $(P\bar{4}c2)$
117. D_{2d}^7 $(P\bar{4}b2)$
118. D_{2d}^8 $(P\bar{4}n2)$
119. D_{2d}^9 $(I\bar{4}m2)$
120. D_{2d}^{10} $(I\bar{4}c2)$
121. D_{2d}^{11} $(I\bar{4}2m)$
122. D_{2d}^{12} $(I\bar{4}2d)$
123. D_{4h}^1 $(P4/mmm)$
124. D_{4h}^2 $(P4/mcc)$
125. D_{4h}^3 $(P4/nbm)$
126. D_{4h}^4 $(P4/nnc)$
127. D_{4h}^5 $(P4/mbm)$
128. D_{4h}^6 $(P4/mnc)$
129. D_{4h}^7 $(P4/nmm)$
130. D_{4h}^8 $(P4/ncc)$
131. D_{4h}^9 $(P4_2/mmc)$
132. D_{4h}^{10} $(P4_2/mcm)$
133. D_{4h}^{11} $(P4_2/nbc)$
134. D_{4h}^{12} $(P4_2/nnm)$
135. D_{4h}^{13} $(P4_2/mbc)$
136. D_{4h}^{14} $(P4_2/mnm)$
137. D_{4h}^{15} $(P4_2/nmc)$
138. D_{4h}^{16} $(P4_2/ncm)$
139. D_{4h}^{17} $(I4/mmm)$
140. D_{4h}^{18} $(I4/mcm)$
141. D_{4h}^{19} $(I4_1/amd)$
142. D_{4h}^{20} $(I4_1/acd)$

25

27

Trigonal

143. C_3^1 $(P3)$
144. C_3^2 $(P3_1)$
145. C_3^3 $(P3_2)$
146. C_3^4 $(R3)$
147. C_{3i}^1 $(P\bar{3})$
148. C_{3i}^2 $(R\bar{3})$
149. D_3^1 $(P312)$
150. D_3^2 $(P321)$
151. D_3^3 $(P3_112)$
152. D_3^4 $(P3_121)$
153. D_3^5 $(P3_212)$
154. D_3^6 $(P3_221)$
155. D_3^7 $(R32)$
156. C_{3v}^1 $(P3m1)$
157. C_{3v}^2 $(P31m)$
158. C_{3v}^3 $(P3c1)$
159. C_{3v}^4 $(P31c)$
160. C_{3v}^5 $(R3m)$
161. C_{3v}^6 $(R3c)$
162. D_{3d}^1 $(P\bar{3}1m)$
163. D_{3d}^2 $(P\bar{3}1c)$
164. D_{3d}^3 $(P\bar{3}m1)$
165. D_{3d}^4 $(P\bar{3}c1)$
166. D_{3d}^5 $(R\bar{3}m)$
167. D_{3d}^6 $(R\bar{3}c)$

Hexagonal

168. C_6^1 $(P6)$
169. C_6^2 $(P6_1)$
170. C_6^3 $(P6_5)$
171. C_6^4 $(P6_2)$
172. C_6^5 $(P6_4)$
173. C_6^6 $(P6_3)$
174. C_{3h}^1 $(P\bar{6})$
175. C_{6h}^1 $(P6/m)$
176. C_{6h}^2 $(P6_3/m)$
177. D_6^1 $(P622)$
178. D_6^2 $(P6_122)$
179. D_6^3 $(P6_522)$
180. D_6^4 $(P6_222)$
181. D_6^5 $(P6_422)$
182. D_6^6 $(P6_322)$
183. C_{6v}^1 $(P6mm)$
184. C_{6v}^2 $(P6cc)$
185. C_{6v}^3 $(P6_3cm)$
186. C_{6v}^4 $(P6_3mc)$
187. D_{3h}^1 $(P\bar{6}m2)$
188. D_{3h}^2 $(P\bar{6}c2)$
189. D_{3h}^3 $(P\bar{6}2m)$
190. D_{3h}^4 $(P\bar{6}2c)$
191. D_{6h}^1 $(P6/mmm)$
192. D_{6h}^2 $(P6/mcc)$
193. D_{6h}^3 $(P6_3/mcm)$
194. D_{6h}^4 $(P6_3/mmc)$

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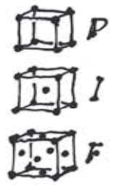
Cubic



tetrahedral 23

195. T^1 $(P23)$
196. T^2 $(F23)$
197. T^3 $(I23)$
198. T^4 $(P2_13)$
199. T^5 $(I2_13)$

tetrahedral m3

200. T_h^1 $(Pm3)$
201. T_h^2 $(Pn3)$
202. T_h^3 $(Fm3)$
203. T_h^4 $(Fd3)$
204. T_h^5 $(Im3)$
205. T_h^6 $(Pa3)$
206. T_h^7 $(Ia3)$

octahedral 432

207. O^1 $(P432)$
208. O^2 $(P4_232)$
209. O^3 $(F432)$
210. O^4 $(F4_132)$
211. O^5 $(I432)$
212. O^6 $(P4_332)$
213. O^7 $(P4_132)$
214. O^8 $(I4_132)$

tetrahedral 43m

215. T_d^1 $(P\bar{4}3m)$
216. T_d^2 $(F\bar{4}3m)$
217. T_d^3 $(I\bar{4}3m)$
218. T_d^4 $(P\bar{4}3n)$
219. T_d^5 $(F\bar{4}3c)$
220. T_d^6 $(I\bar{4}3d)$

octahedral m3m

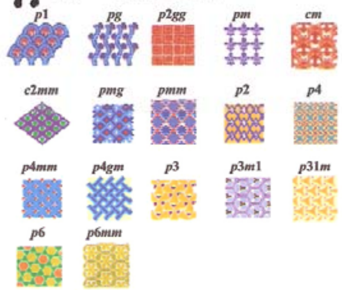
221. O_h^1 $(Pm3m)$
222. O_h^2 $(Pn3n)$
223. O_h^3 $(Pm3n)$
224. O_h^4 $(Pn3m)$
225. O_h^5 $(Fm3m)$
226. O_h^6 $(Fm3c)$
227. O_h^7 $(Fd3m)$
228. O_h^8 $(Fd3c)$
229. O_h^9 $(Im3m)$
230. O_h^{10} $(Ia3d)$

EXHAUSTIVE ENUMERATION OF UNIFORM NETWORKS:

DOUBLE LAYER MULTI LAYER & POLYVECTORIAL

17-

Plane Groups



230-

Space Groups

Translational elements + 32 crystal point groups; 230 space groups.

230 **distinct** ways of packing repeating object in **3-D**.

Space Groups

2

13

59

Triclinic

1	P1							
$\bar{1}$	$P\bar{1}$	Centrosymmetric space groups						

Monoclinic

2	P2	$P2_1$	C2			
m	P*m*	P*c*	C*m*	C*c*		
2/*m*	P2/*m*	$P2_1/m$	C2/*m*	P2/*c*	$P2_1/c$	C2/*c*

Orthorhombic

222	P222	$P222_1$	$P2_12_12$	$P2_12_12_1$	$C222_1$	C222	F222	I222
	$I2_12_12_1$							
*mm*2	P*mm*2	$Pmc2_1$	P*cc*2	P*ma*2	$Pca2_1$	P*nc*2	$Pmn2_1$	P*ba*2
	$Pna2_1$	P*nn*2	C*cc*2	A*mm*2	A*bm*2	A*ma*2	A*ba*2	F*mm*2
	C*mm*2	$Cmc2_1$	F*dd*2	I*mm*2	I*ba*2	I*ma*2		
mmm	P*mmm*	P*nnn*	P*ccm*	P*ban*	P*mma*	P*nna*	P*mna*	P*cca*
	P*bam*	P*ccn*	P*bcm*	P*nnm*	P*mmn*	P*bcn*	P*bca*	P*nma*
	C*mcm*	C*mca*	C*mmm*	C*ccm*	C*mma*	C*cca*	F*mmm*	F*ddd*
	I*mmm*	I*bam*	I*bca*	I*mma*				

Space Groups

68

Tetragonal

4	P4	$P4_1$	$P4_2$	$P4_3$	I4	$I4_1$
$\bar{4}$	$P\bar{4}$	$I\bar{4}$				
4/*m*	P4/*m*	$P4_2/m$	P4/*n*	$P4_2/n$	I4/*m*	$I4_1/a$
422	P422	$P42_12$	$P4_122$	$P4_12_12$	$P4_222$	$P4_22_12$
	$P4_322$	$P4_32_12$	I422	$I4_122$		
4*mm*	P4*mm*	P4*bm*	$P4_2cm$	$P4_2nm$	P4*cc*	P4*nc*
	$P4_2mc$	$P4_2bc$	I4*mm*	I4*cm*	$I4_1md$	$I4_1cd$
$\bar{4}2m$	$P\bar{4}2m$	$P\bar{4}2c$	$P\bar{4}2_1m$	$P\bar{4}2_1c$	$P\bar{4}m2$	$P\bar{4}c2$
	$P\bar{4}2b$	$P\bar{4}n2$	$I\bar{4}m2$	$I\bar{4}c2$	$I\bar{4}2m$	$I\bar{4}2d$
4/*mmm*	P4/*mmm*	P4/*mcc*	P4/*nbm*	P4/*nnc*	P4/*mbm*	P4/*mnc*
	P4/*nmm*	P4/*ncc*	$P4_2/mmc$	$P4_2/mcm$	$P4_2/nbc$	$P4_2/nnm$
	$P4_2/mbc$	$P4_2/mnm$	$P4_2/nmc$	$P4_2/ncm$	I4/*mmm*	I4/*mcm*
	$I4_1/amd$	$I4_1/acd$				

Space Groups

21

31

Trigonal/Rhombohedral

3	P3	$P3_1$	$P3_2$	R3			
$\bar{3}$	$P\bar{3}$	$R\bar{3}$					
32	P312	P321	$P3_112$	$P3_121$	$P3_212$	$P3_221$	R32
3*m*	P3*m*1	P31*m*	P3*c*1	P31*c*	R3*m*	R3*c*	
$\bar{3}m$	$P\bar{3}1m$	$P\bar{3}1c$	$P\bar{3}m1$	$P\bar{3}c1$	$R\bar{3}m$	$R\bar{3}c$	

Hexagonal

6	P6	$P6_1$	$P6_5$	$P6_2$	$P6_4$	$P6_3$
$\bar{6}$	$P\bar{6}$					
6/*m*	P6/*m*	$P6_3/m$				
622	P622	$P6_122$	$P6_522$	$P6_222$	$P6_422$	$P6_322$

Space Groups

36

11

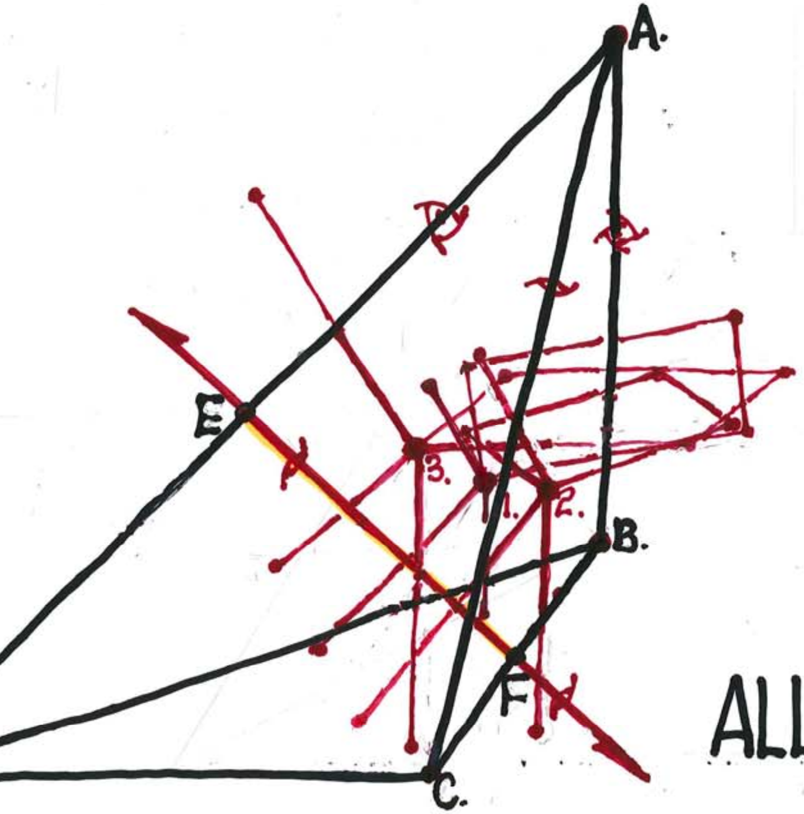
Cubic

23	P23	F23	I23	$P2_13$	$I2_13$	
*m*3	P*m*3	P*n*3	F*m*3	F*d*3	I*m*3	P*a*3
432	P432	$P4_232$	F432	$F4_132$	I432	$P4_332$
	$P4_132$	$I4_132$				
$\bar{4}3m$	$P\bar{4}3m$	$F\bar{4}3m$	$I\bar{4}3m$	$P\bar{4}3n$	$F\bar{4}3c$	$I\bar{4}3d$
*m*3*m*	P*m*3*m*	P*n*3*n*	P*m*3*n*	P*n*3*m*	F*m*3*m*	F*m*3*c*
	F*d*3*m*	F*d*3*c*	I*m*3*m*	I*a*3*d*		

C SPACE GROUPS

E.P.R. – 2 REFLECTIONS: ACD & BCD; EF – 2-FOLD AXIS

ORDER (OF THE SYMM. GR.) = 96



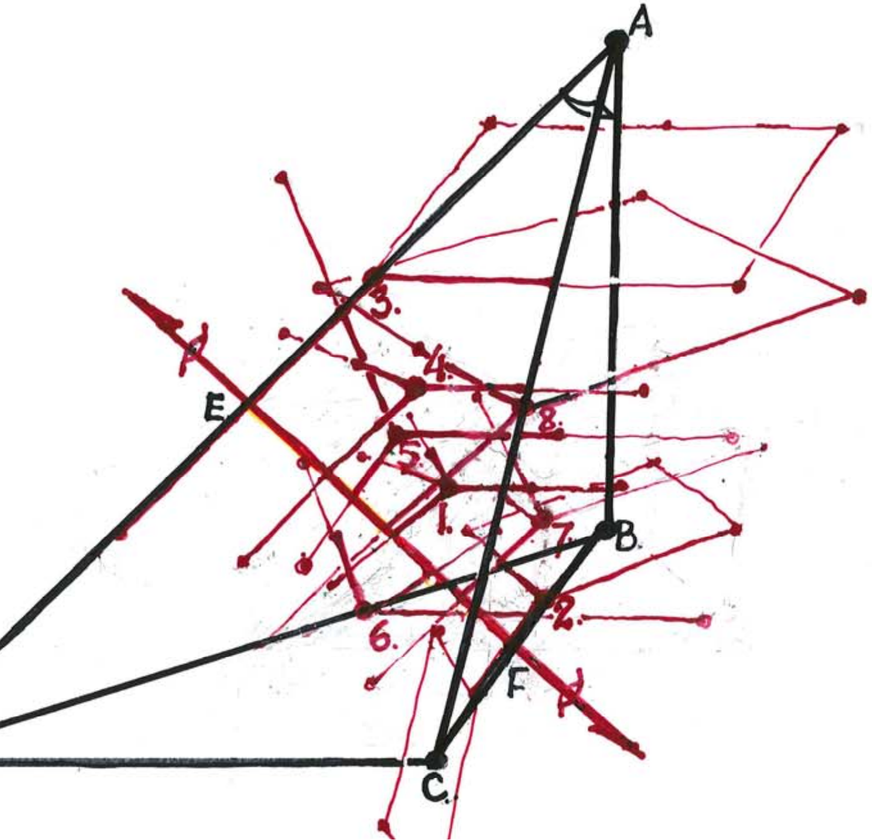
1. IN-VOLUME LOCATION
2. IN-FACE (ABC) LOCATION
3. IN-FACE (ABD) LOCATION

ALL UNIFORM POLYVECTORIAL PENTAVALENT NETWORKS

CUBIC SPACE GROUPS, 1/2 E.P.R. UNIT ORDER=96

3 REFLECTIONS, 1 2-fold AXIS

ABC; ABD; ACD.; EF-ROTATION 2-fold AXIS

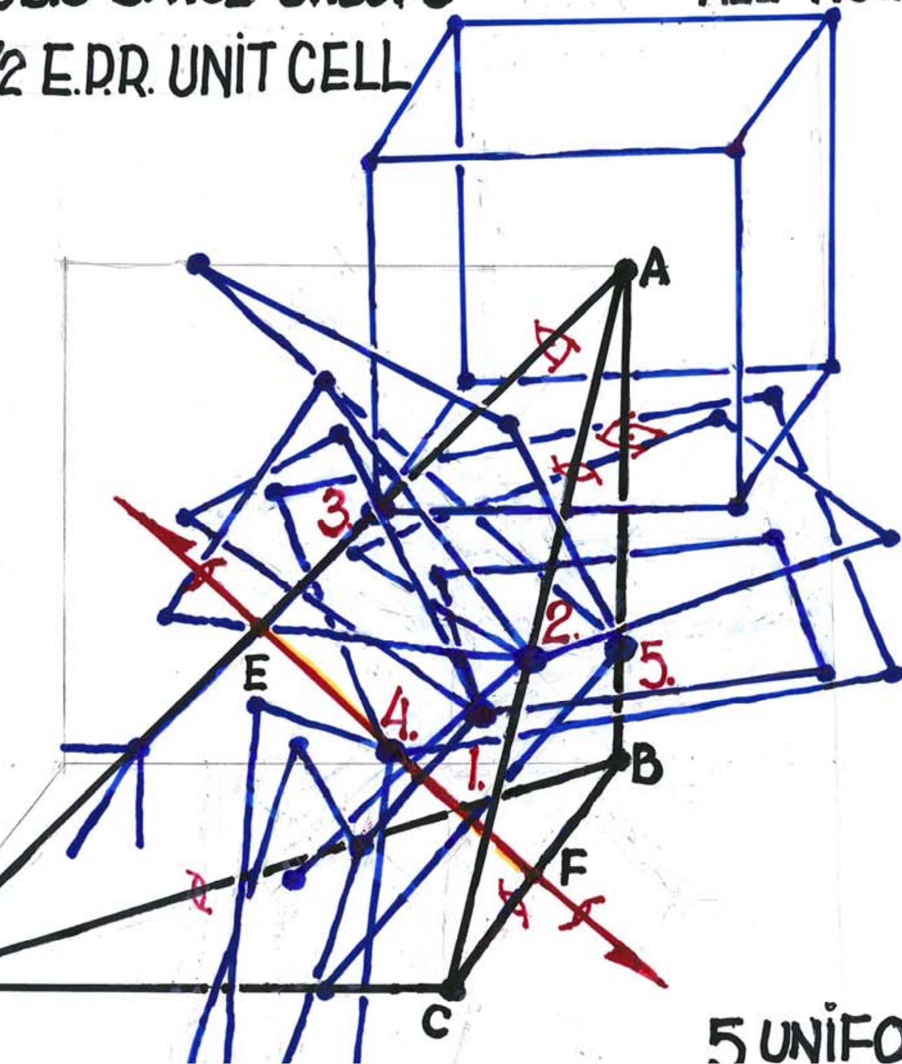


1. INVOLUME LOCATION
2. IN-EDGE BC
3. IN-EDGE AE(D)
4. IN-FACE ACD
5. IN-FACE ABD
6. IN-EDGE BD
7. IN-FACE ABC
8. IN-EDGE AC

8 UNIFORM SPACE NETWORKS

UBIC SPACE GROUPS -
2 E.P.R. UNIT CELL

ALL ROTATION AXES + EF- 2-fold AXIS.



1. IN-VOLUME - PENTAVALENT

2. IN-EDGE(AC) - PENTAVALENT

3. IN-EDGE(AD) - TETRAVALENT

4. IN-VOLUME - TETRAVALENT
MID POINT OF EF, 2-fold AXIS

5. IN-EDGE(AB) - OCTAVALENT

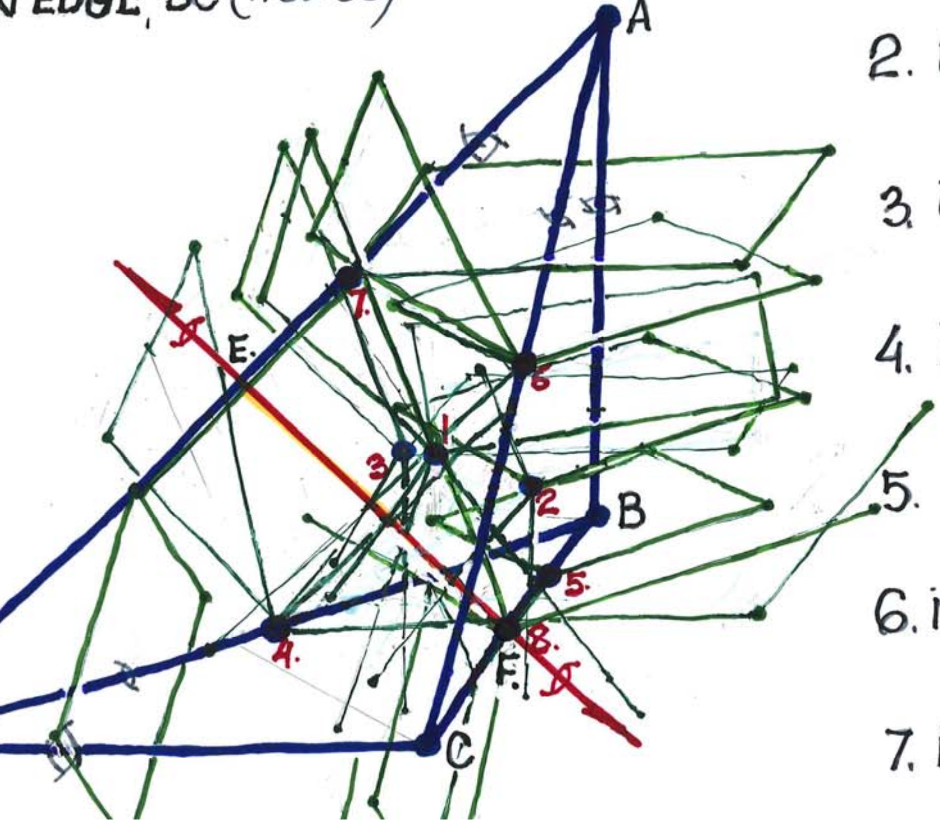
5 UNIFORM SPACE NETWORKS

UBIC SPACE GROUPS

E.P.R UNIT CELL. ALL ROTATION AXES

ER = 96

N EDGE, BC (AB&CD) - 4-FOLDS



VERTEX LOCATIONS:

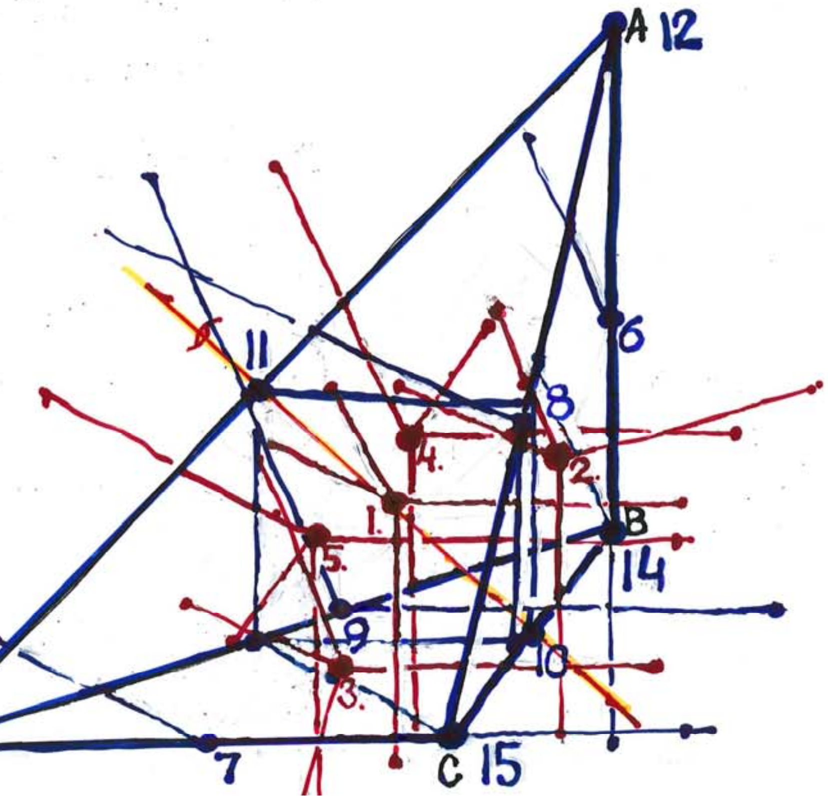
1. IN-VOLUME; (BC; BD; EF) - 2-FOLD ROTATIONS
   AB - 4-FOLD "
   AD - 3-FOLD "
2. IN-FACE ABC; (BC; EF) - 2-FOLDS
   AB - 4-FOLD
   AD - 3-FOLD
3. IN-FACE ABD; (BD; EF) - 2-FOLDS
   AB - 4-FOLD
   AD - 3-FOLD
4. IN-EDGE BD; EF - 2-FOLD
   AD - 3-FOLD
   AB - 4-FOLD
5. IN-EDGE BC; EF - 2-FOLD
   AB - 4-FOLD
6. IN-EDGE AC; EF - 2-FOLD
   AD - 3-FOLD
   AB - 4-FOLD
7. IN-EDGE AD; (AC; EF) 2-FOLDS
   AD - 3-FOLD

UBIC SPACE GROUPS

E.P.R UNIT - 4 REFLECTIONS

DER=48

- VERTEX LOCATIONS THAT LEAD TO GENERATION OF UNIFORM NETWORKS IN 3D SPACE.



1. 1. IN VOLUME LOCATION

2. [2. MID-FACE ABC
3. MID-FACE BCD]

3. [4. MID-FACE ABD
5. MID-FACE ACD]

4. [6. MID-EDGE AB
7. MID-EDGE CD]

5. [8. MID-EDGE AC
9. MID-EDGE BD]

6. 10. MID-EDGE BC

7. 11. MID-EDGE AD

DUAL [CUBIC NET / CUBIC NET]

8. [12. VERTEX LOCATION A
13. VERTEX LOCATION D]

9. [14. VERTEX LOCATION B
15. VERTEX LOCATION C]

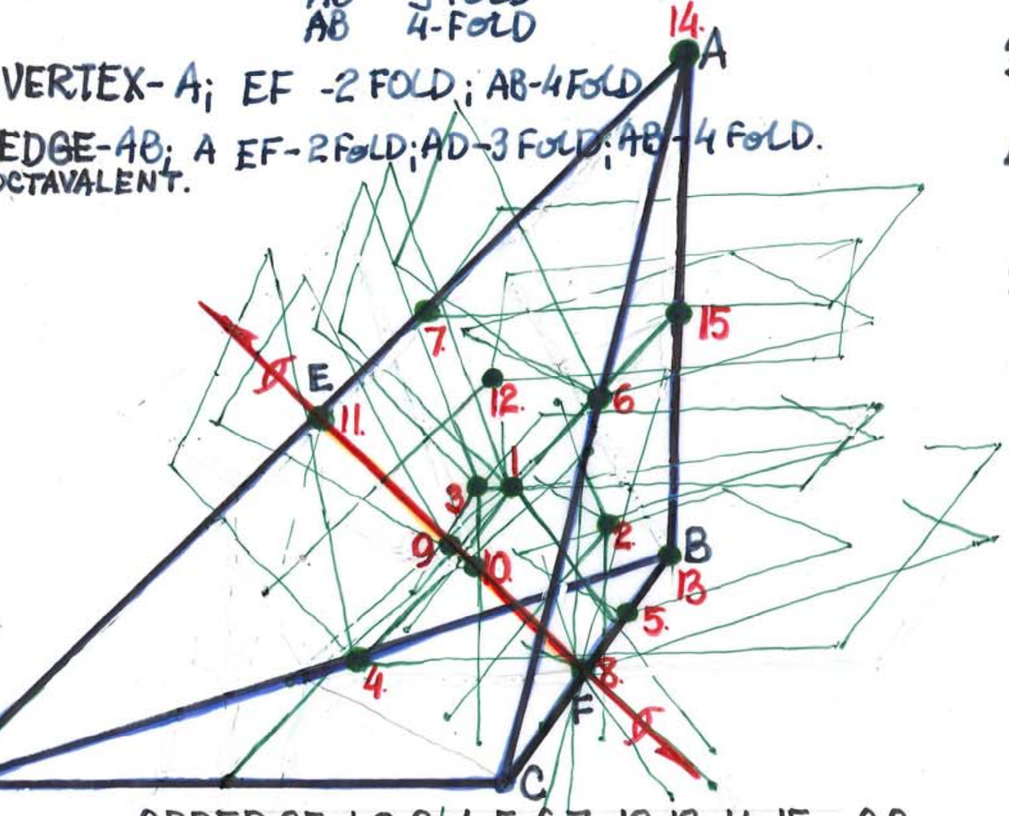
CUBIC SPACE GROUPS

I E.P.R. UNIT CELL. ALL ROTATION AXES; NO REFLECTIONS.

VERTEX-B; EF; 2-FOLD
AD 3-FOLD
AB 4-FOLD

VERTEX- A; EF -2 FOLD; AB-4 FOLD

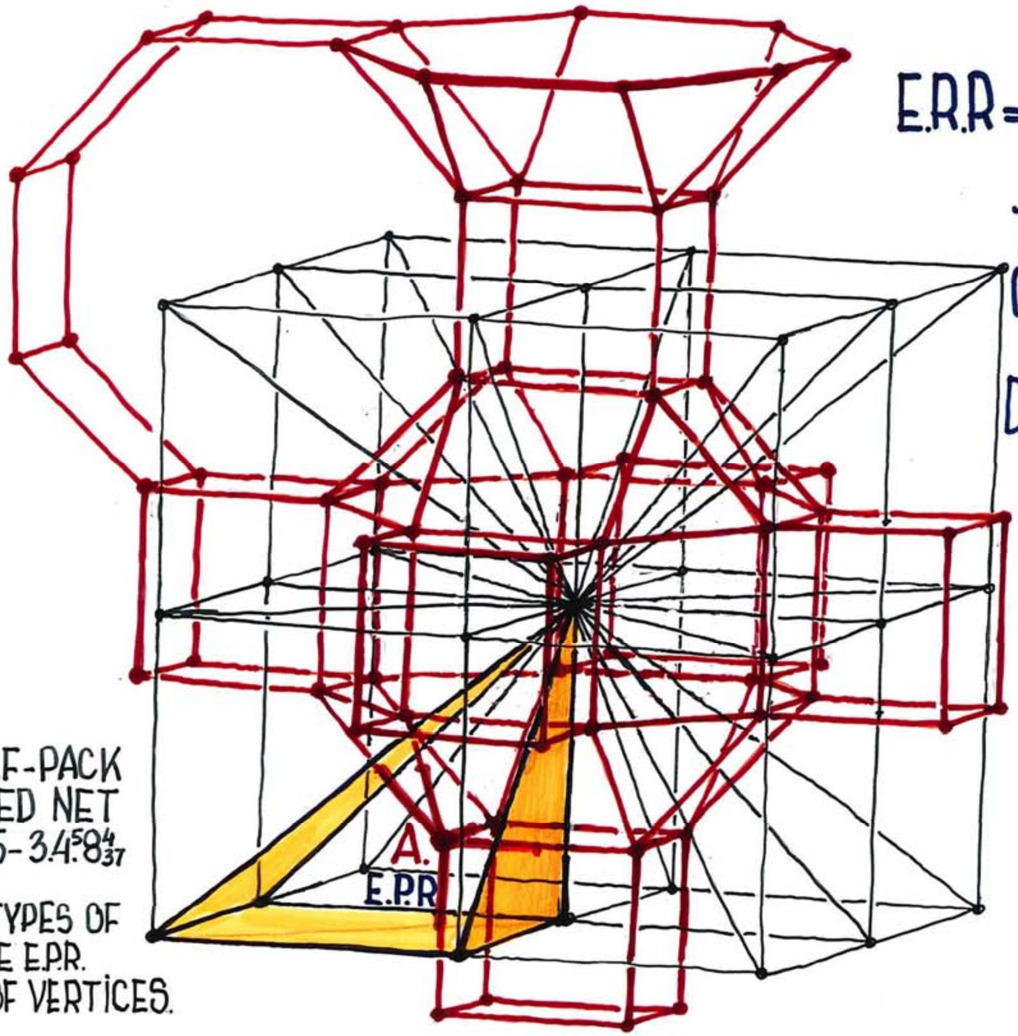
EDGE-AB; A EF- 2 FOLD; AD - 3 FOLD; AB - 4 FOLD.
OCTAVALENT.



ORDER OF - 1;2;3;4;5;6;7; 12;13;14;15 = 96

VERTEX LOCATIONS:
1. IN-VOLUME; (BC; BD; EF) - 2 FOLDS
AB - 4 FOLD
AD - 3 FOLD
2. IN-FACE ABC; (BC; AC; EF) - 2 FOLDS
AB - 4 FOLD
3. IN-FACE ABD; (BD; EF) - 2 FOLDS
AD - 3 FOLD
AB - 4 FOLD
4. IN-EDGE BD; (EF; BC) - 2 FOLDS
AD - 3 FOLD
AB - 4 FOLD
5. IN-EDGE BC; (EF; BD) - 2 FOLDS
AB - 4 FOLD
6. IN-EDGE AC; EF - 2 FOLD
AD - 3 FOLD
AB - 4 FOLD
7. IN-EDGE AD; (AC; EF) - 2 FOLDS
AB - 4 FOLD
AD - 3 FOLD
8. IN-EDGE BC; (AB; CD) - 4 FOLDS
9. IN-VOLUME; AD - 3 FOLD
(AB; CD) - 4 FOLDS
10. IN-VOLUME; BC - 2 FOLD
(AB; CD) - 4 FOLDS
11. IN-EDGE AD; (AC; BD) - 2 FOLDS

B.



E.R.R = 1/24 OF THE CUBE'S VOLUME.

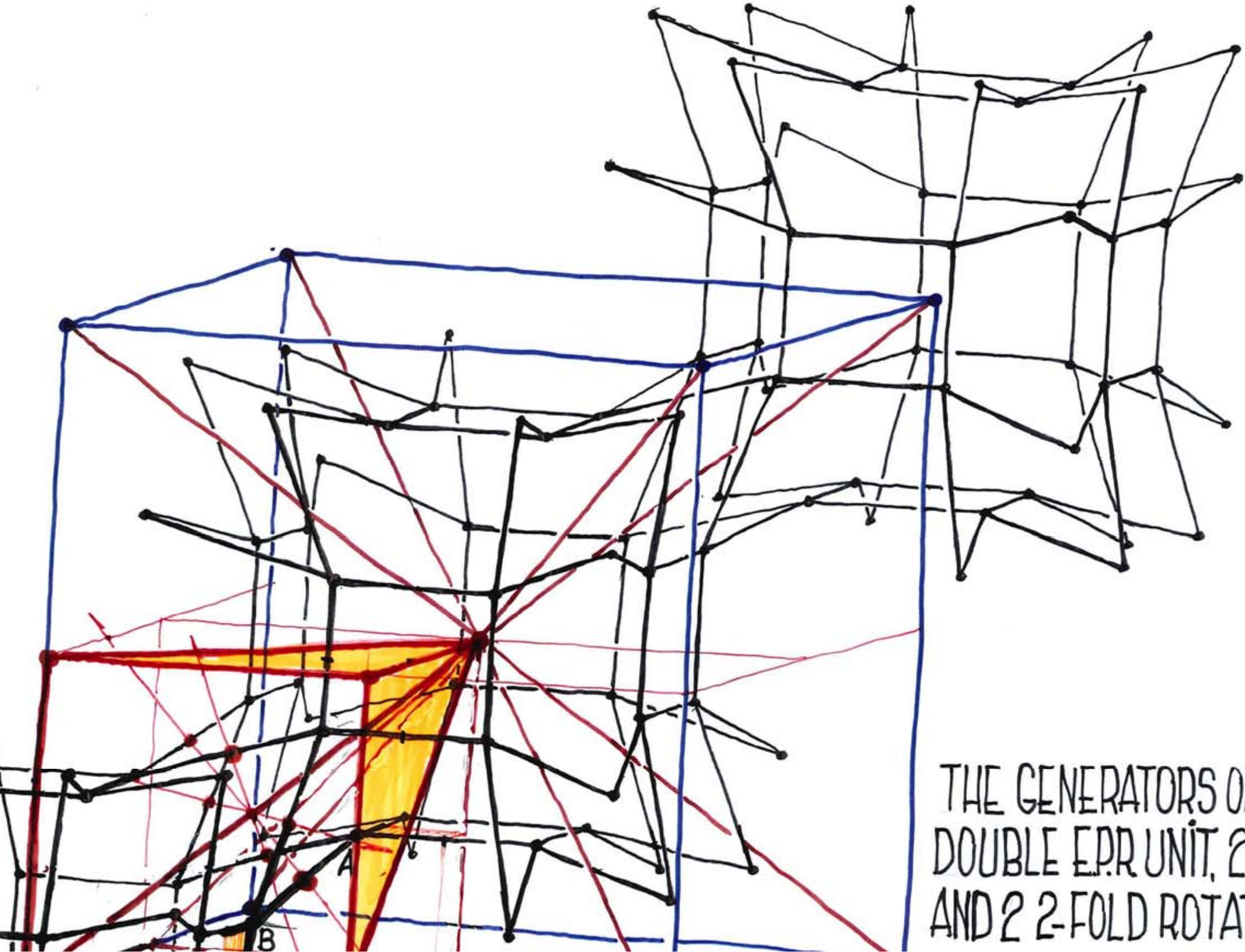
THE NETWORK DESCRIBES CLOSE PACKING OF 4^3_o; 3.4^3_o; $4^2.8_o$

Den. PV.-5-$3.4^5.8^4_{37}$=1.5075759551 a/a^3

ORDER = 24

IS A SELF-PACK
GENERATED NET
L OF PV.-5-$3.4^5.8^4_{37}$

$_7$ HAS 3 TYPES OF
OLIDS; THE E.P.R.
3 TYPES OF VERTICES.



THE GENERATORS OF THE NET:
DOUBLE E.P.R UNIT, 2 REFLECTIONS
AND 2 2-FOLD ROTATION AXES.

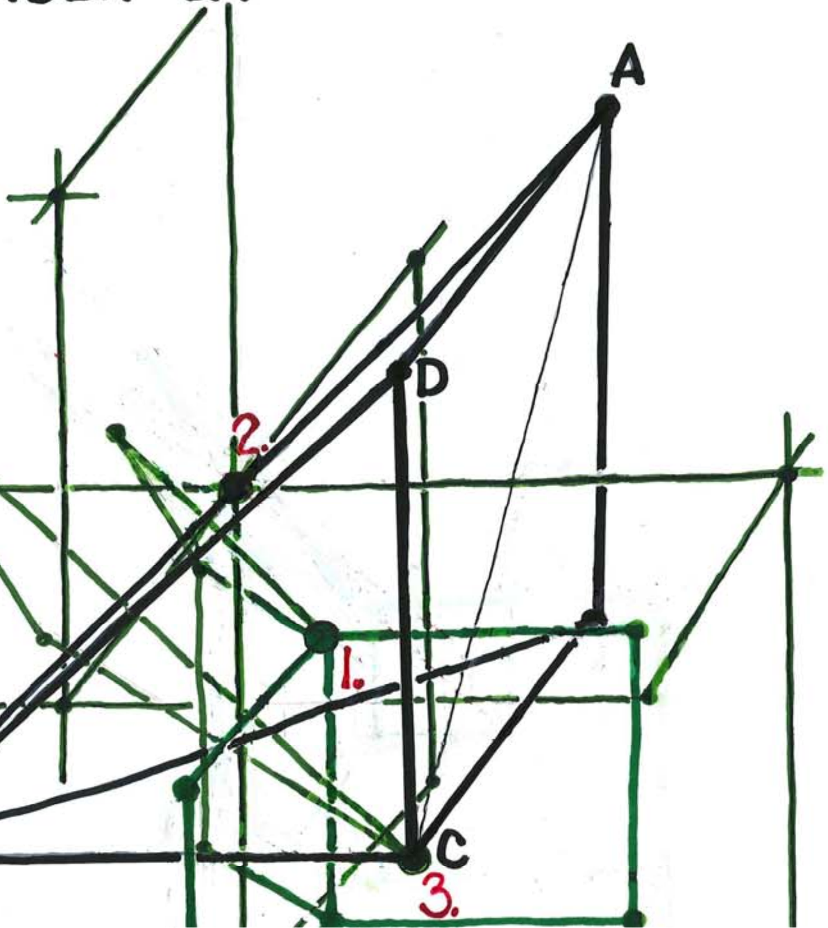
BIC SPACE GROUPS - ALL(5) REFLECTIONS-ABCD; ABE; ADE; BCE; CDE.
E.P.R. UNIT CELL.

VERTEX LOCATIONS: 1. IN-VOLUME - PENTAVALENT

RDER=24

2. IN-EDGE (AE)- HEXAVALENT

3. IN-VETEX(C)-DODECAVALENT



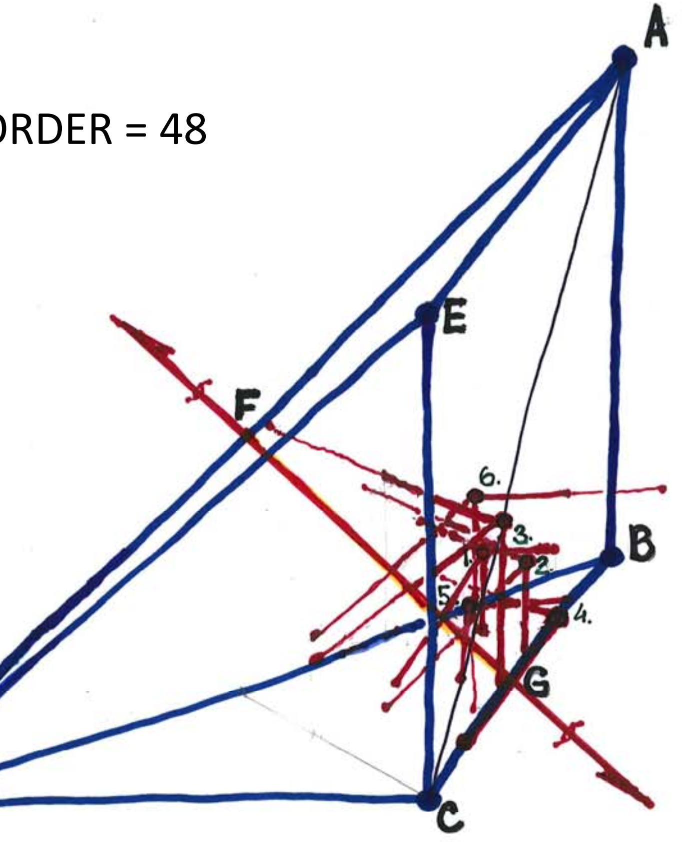
UBIC SPACE GROUPS

E.P.R UNIT CELL.

REFLECTIONS: ABCE; BCD; ECD; ABD; ADE.

-VOLUME 2-FOLD ROTATION AXIS-FG

RDER = 48



VERTEX LOCATIONS:

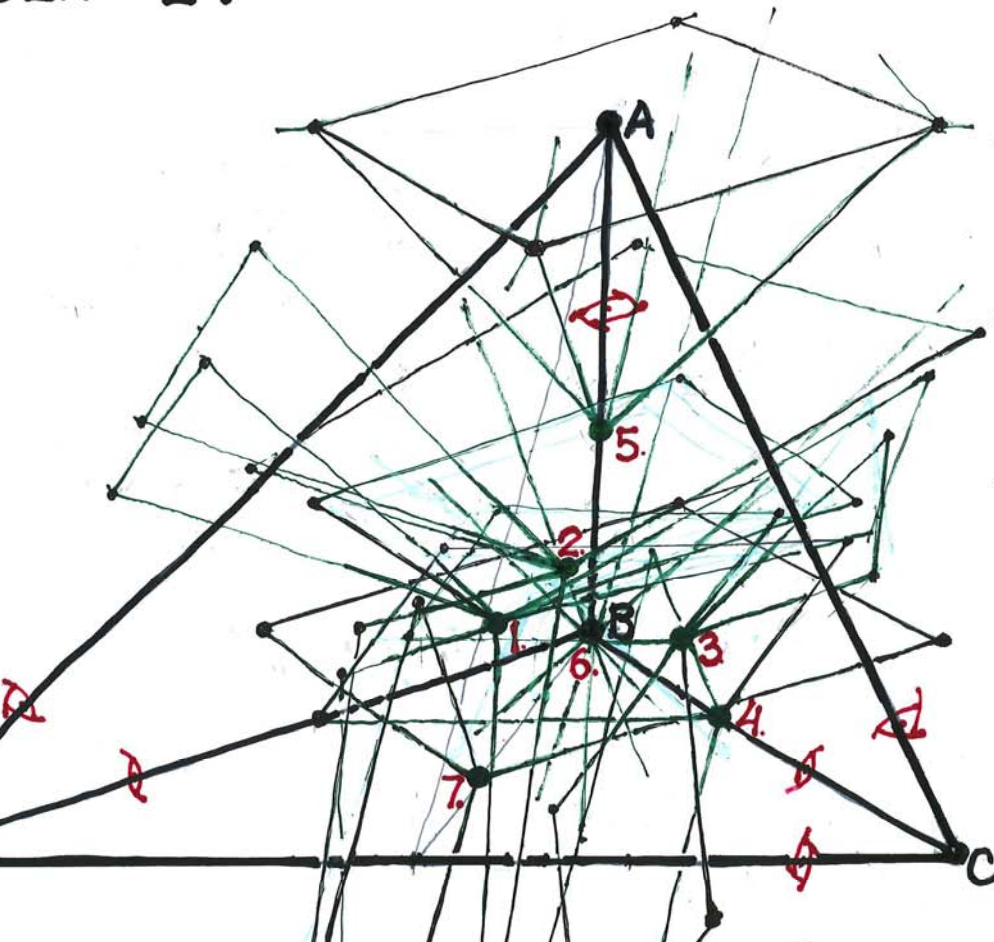
1. IN-VOLUME -TETRAVALENT ORDER=96
2. IN-FACE, ABC. - TETRAVALENT
3. IN-EDGE, AC. - TETRAVALENT
4. IN-EDGE, BC. - TRIVALENT
5. IN-EDGE, BD. - TRIVALENT
6. IN-FACE, ABD. - PENTAVALENT

SIX UNIFORM SPACE NETWORKS

ƃIC SPACE GROUPS - ALL ROTATIONS
P.P UNIT CELL
DER = 24

(AB; CD) - 4 FOLDS; (AC; AD) - 3 FOLDS; (BC; BD) - 2-FOLDS.

VERTEX LOCATIONS:



1. IN-VOLUME - OCTAVALENT ×
(AC; AD) - 3 FOLDS
(AB; CD) - 4 FOLDS

2. IN-VOLUME (NON-SYMMETRICAL) - HEXAVALENT
AD - 3 FOLD
(AB; CD) - 4 FOLDS

3. IN-FACE ABC. - HEXAVALENT ×
AC - 3 FOLD
(AB; CD) - 4 FOLDS

4. IN-EDGE BC. - HEXAVALENT ×
(BC; BD) - 2 FOLDS
(AB; CD) - 4 FOLDS

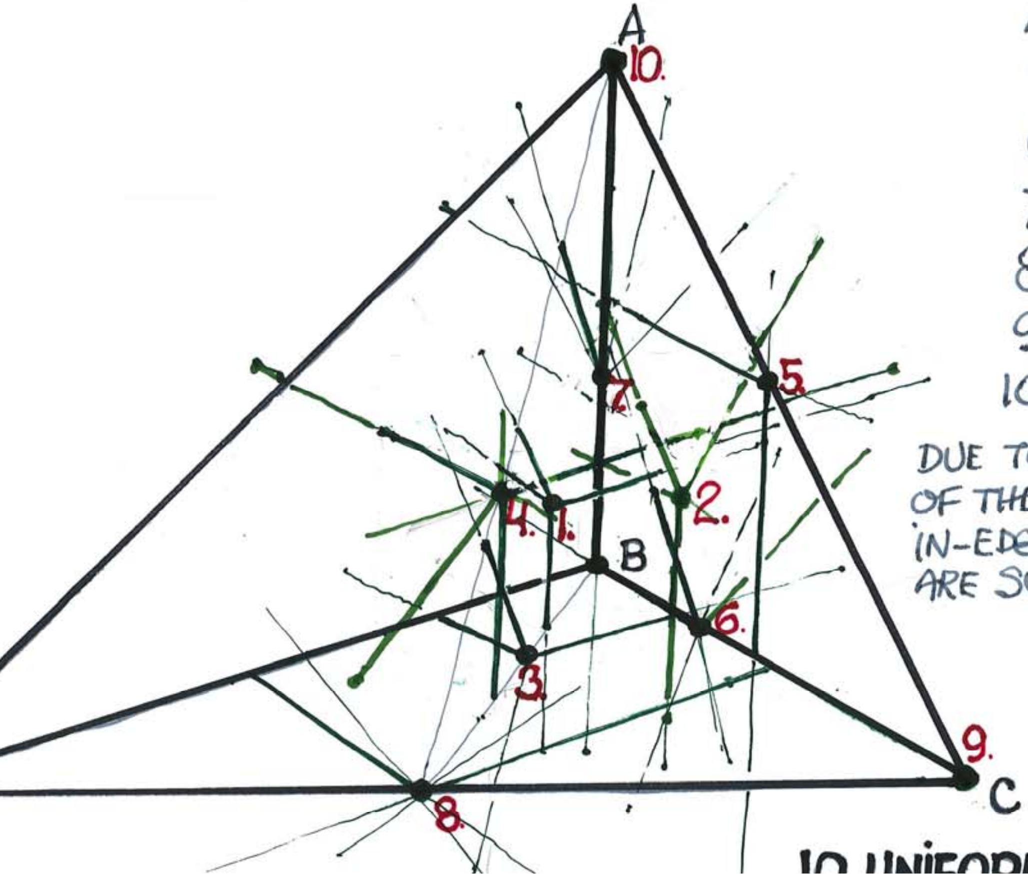
5. IN-EDGE - AB - PENTAVALENT ×
(BC; BD) - 2 FOLDS
(AC; AD) - 3 FOLDS

6. IN-VERTEX - B. - OCTAVALENT ×
(AC; AD) - 3 FOLDS
(AB; CD) - 4 FOLDS

BIC SPACE GROUPS - 4 REFLECTIONS - ABC; ABD; ACD; BCD.
EP.R UNIT CELL

RDER = 24

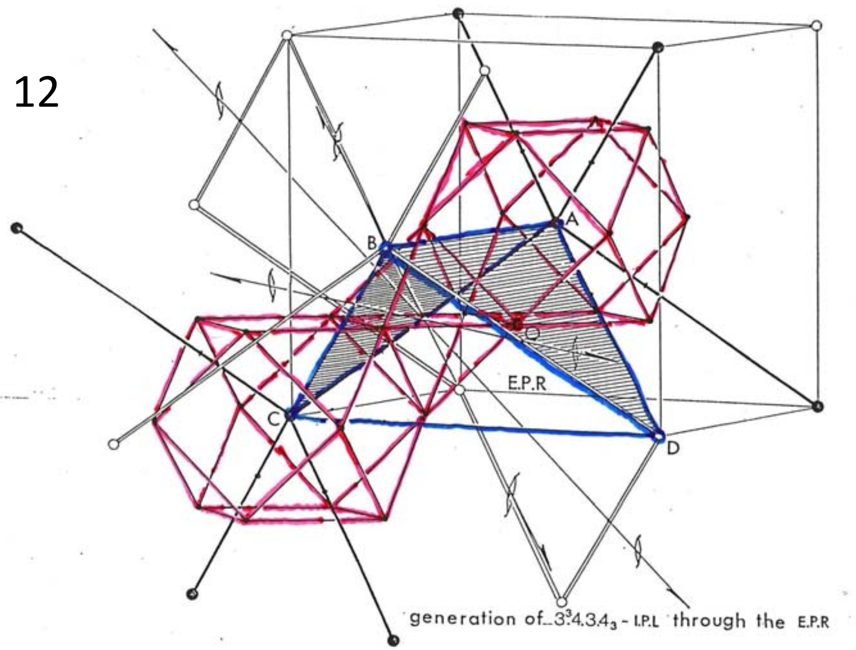
VERTEX LOCATIONS:
1. IN-VOLUME - TETRAVALENT ×
2. IN-FACE ABC - TETRAVALENT
3. IN-FACE BCD. - TETRAVALENT ×
4. IN-FACE ACD. - TETRAVALENT ×
5. IN-EDGE AC. - HEXAVALENT
6. IN-EDGE BC. - PENTAVALENT
7. IN-EDGE AB. - PENTAVALENT ×
8. IN-EDGE CD. - HEXAVALENT ×
9. IN-VERTEX C. - HEXAVALENT ×
10. IN-VERTEX A. - HEXAVALENT ×



DUE TO THE INHERENT SYMMETRY OF THE UNIT CELL IN-FACE ABD; IN-EDGE AD & BD AND IN-VERTEX D ARE SUPERFLUOUS.

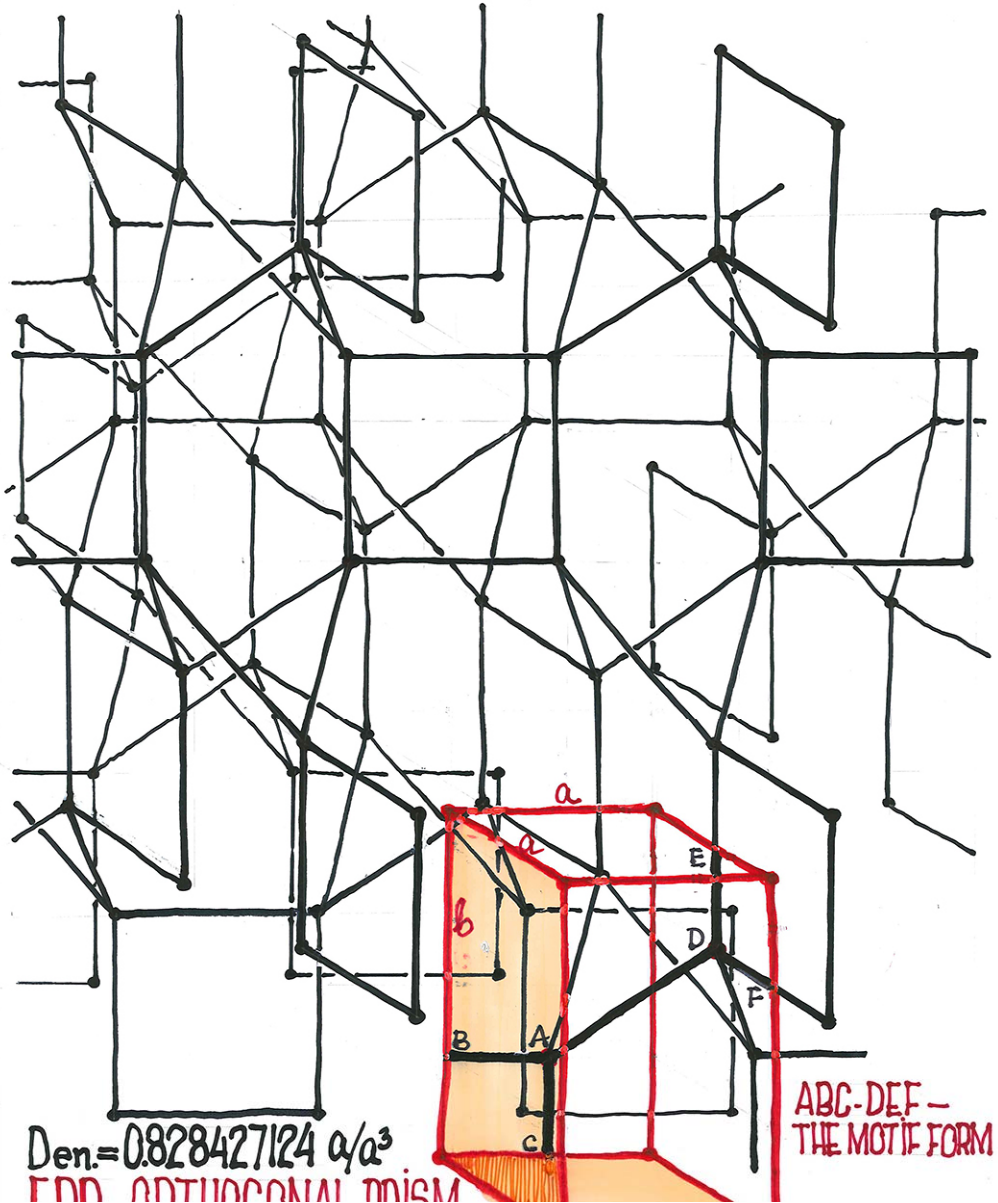
10 UNIFORM NETWORKS

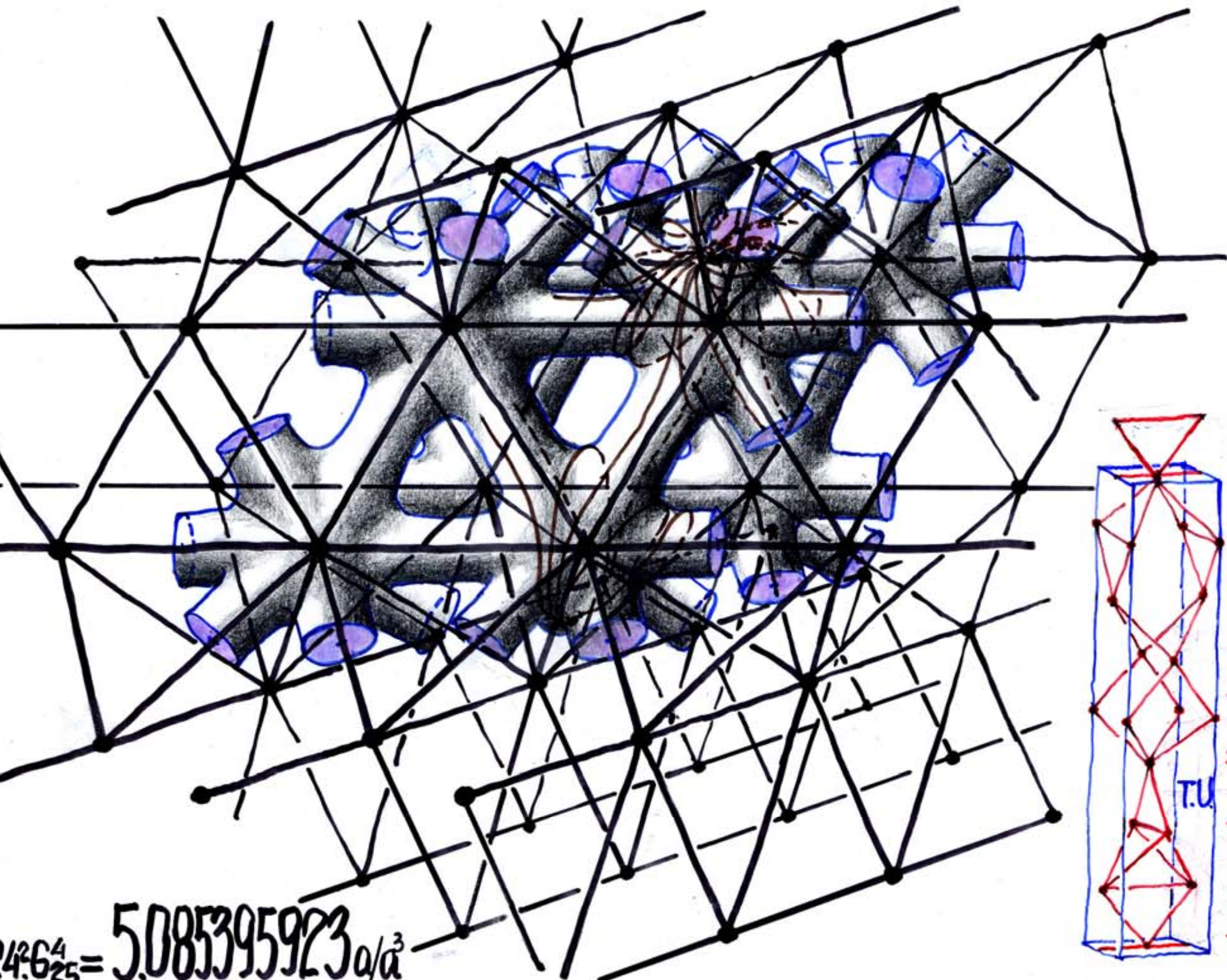
ORDER = 12



generation of $3^3.4.3.4_3$ - I.P.L through the E.P.R

E.P.R. UNIT CELL, COMPOSED OF 4 CUBIC E.P.R UNITS.

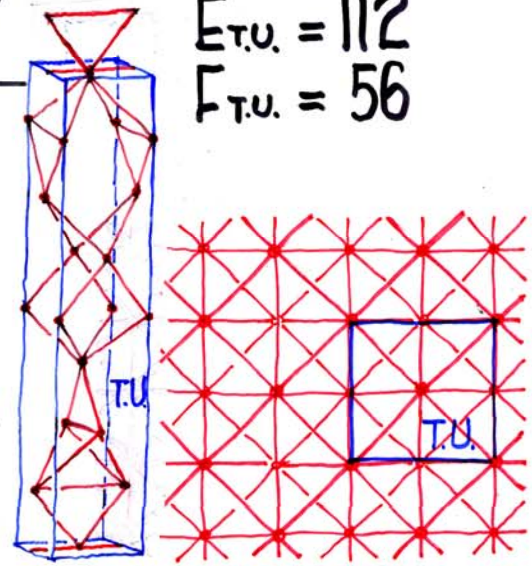

a
a
b
E
D
F
B
A
C
Den.=0.828427124 a/a³
ABC-DEF –
THE MOTIF FORM



$$\left(4^{14}\right)^2_{25}$$

$$g_{T.U.} = 25$$
$$\Sigma\alpha = 14\pi$$
$$Val. = 28$$

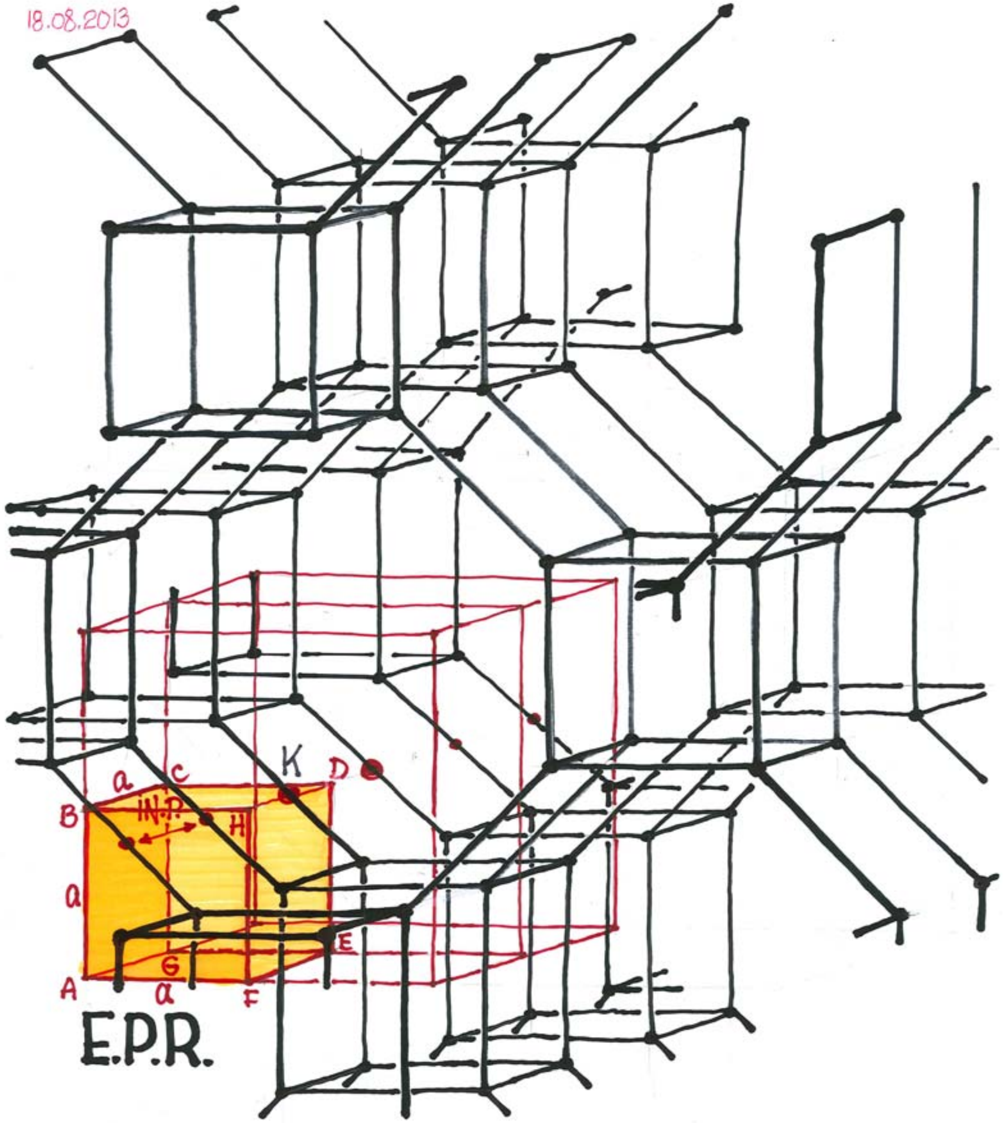
$$V_{T.U.} = 8$$
$$E_{T.U.} = 112$$
$$F_{T.U.} = 56$$



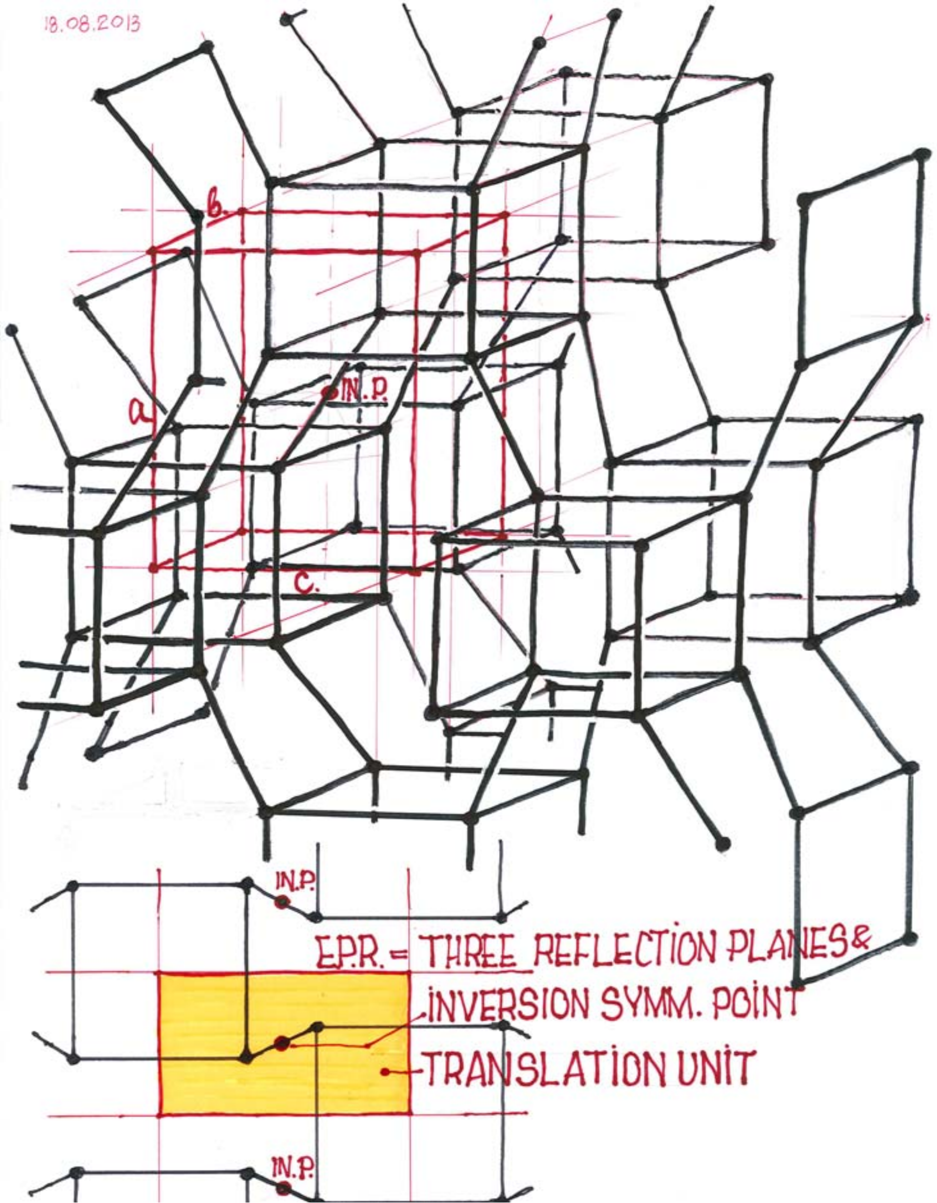
$$4^2 6^4_{25} = 5.085395923\, a/a^3$$

M MULTİLAYER OCTAVALENT ML8 $3^4 4^2 6^4$ SPACE LATTİCE

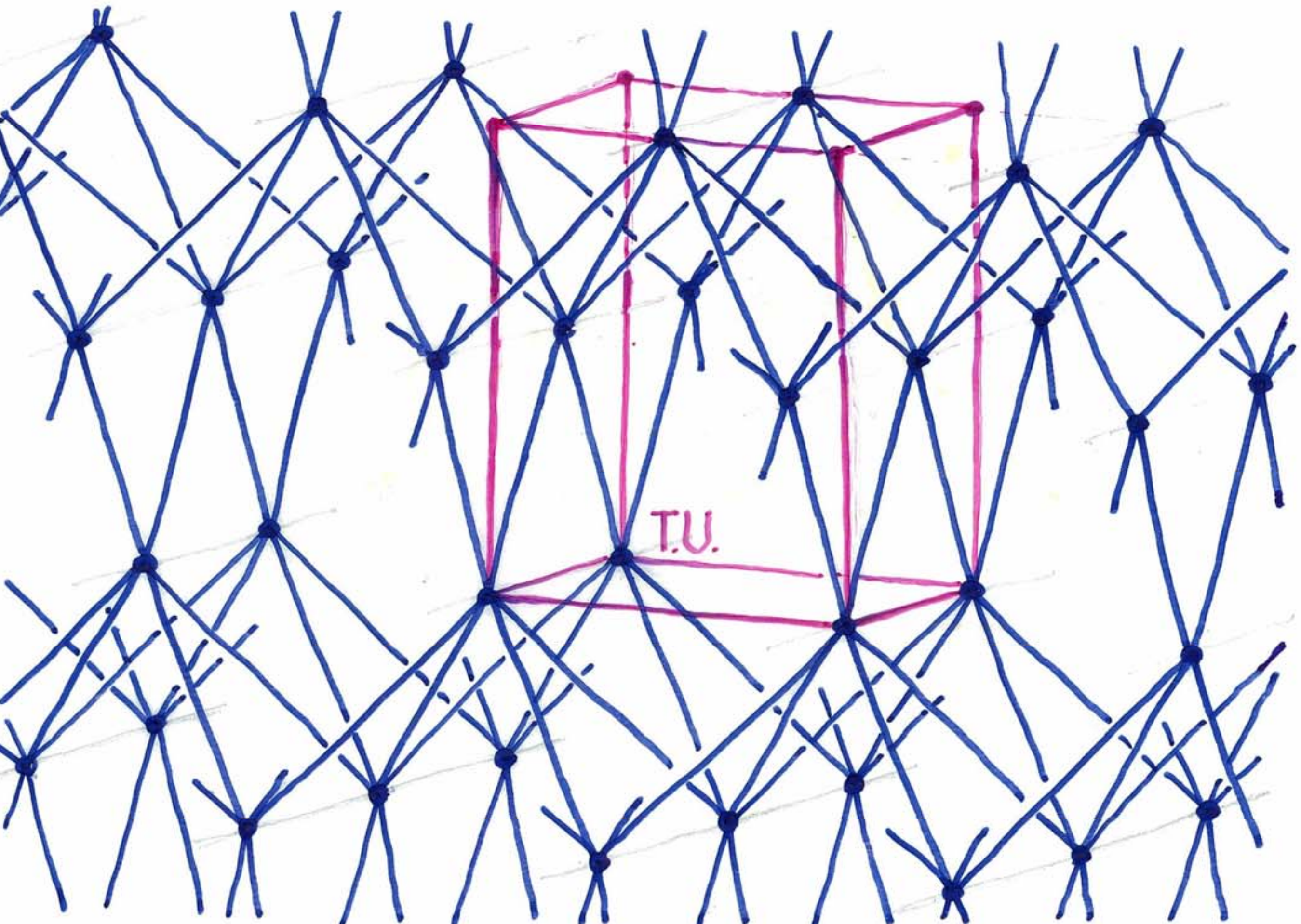
18.08.2013



ABCG
AGEF] REFLECTION PLANES; K INVERSION SYMM POINT

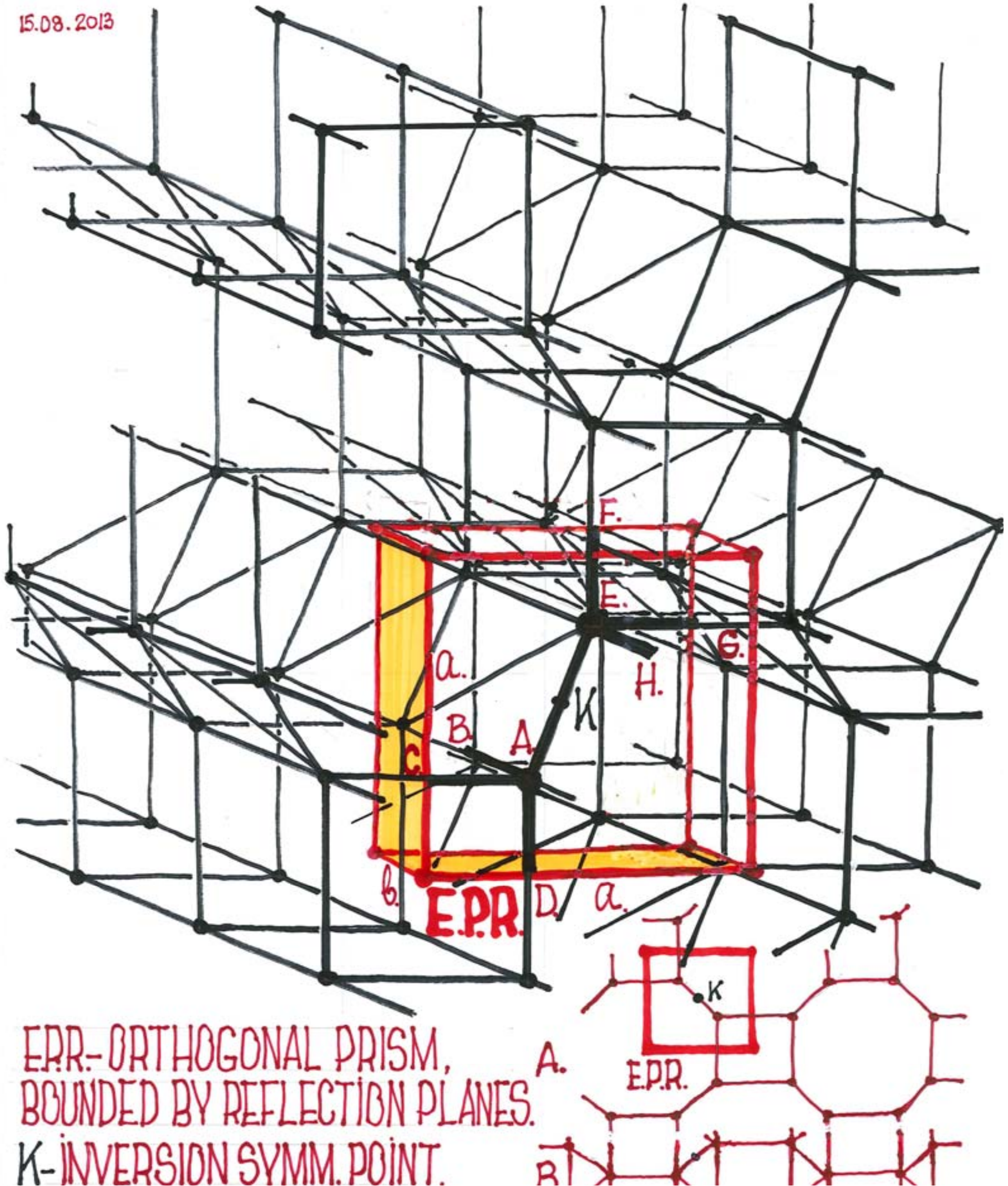


18.08.2013
b
a
IN.P.
c
IN.P.
E.P.R. = THREE REFLECTION PLANES &
INVERSION SYMM. POINT
TRANSLATION UNIT
IN.P.



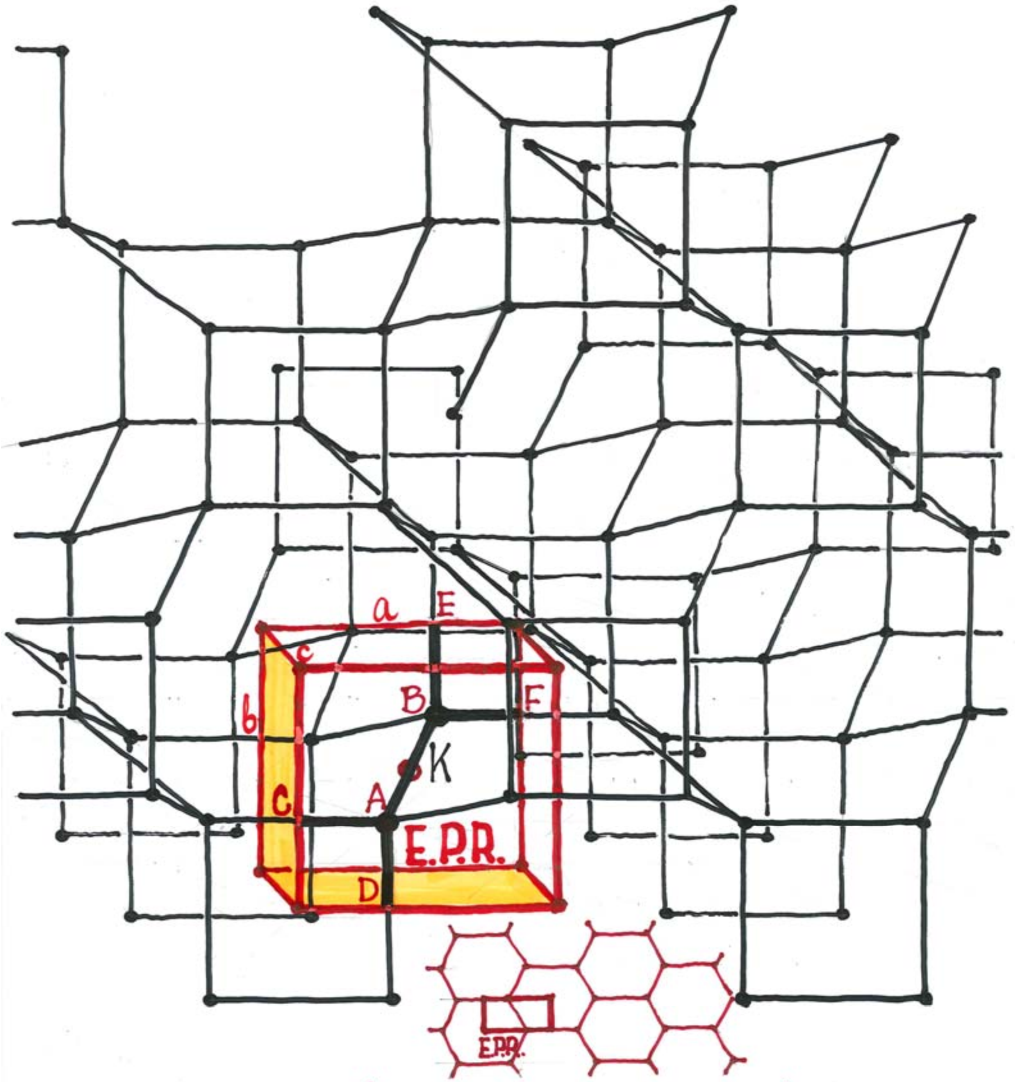
T.U.

15.08.2013

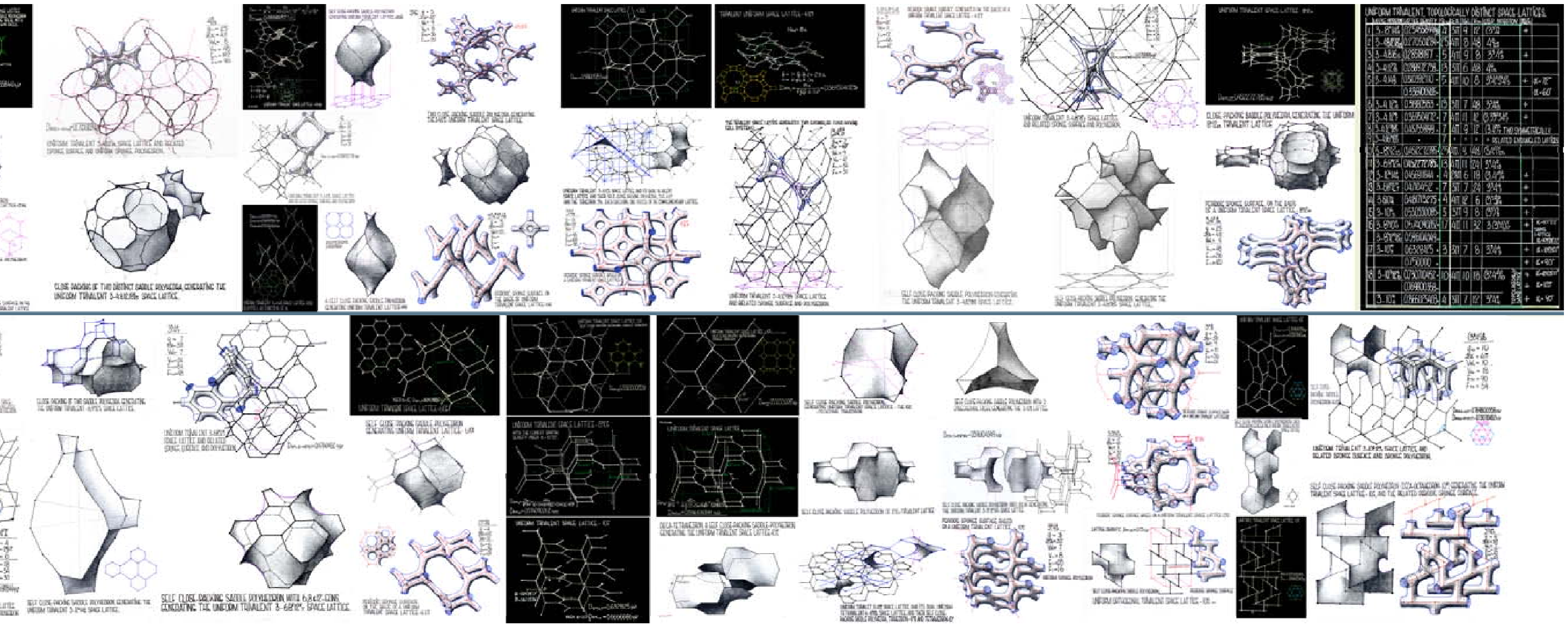


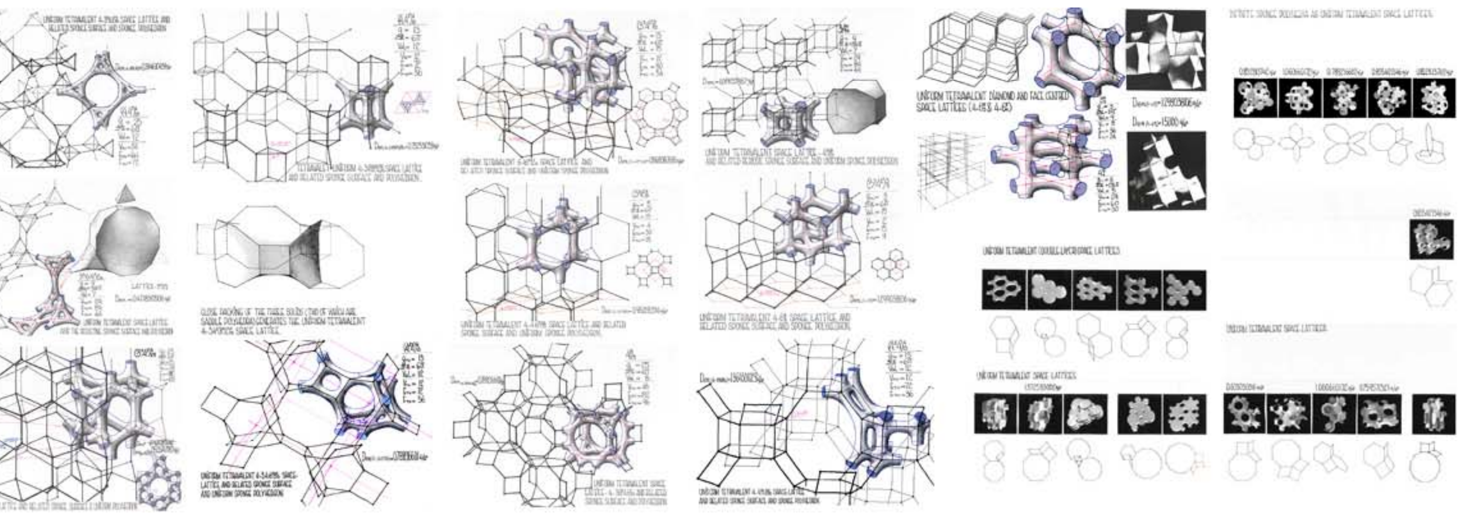
E.P.R.- ORTHOGONAL PRISM, BOUNDED BY REFLECTION PLANES.

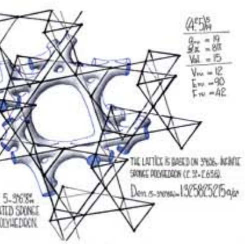
K- INVERSION SYMM. POINT.



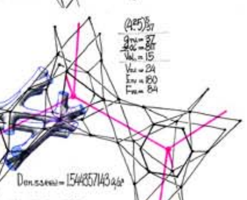
E.P.R-ORTHOGONAL PRISM BOUNDED BY REFLECTION PLANES


UNIFORM TRIVALENT, TOPOLOGICALLY DISTINCT SPACE LATTICES.

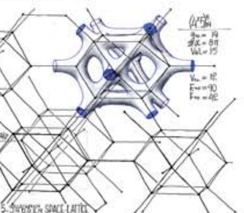

UNIFORM TETRAVALENT DIAMOND AND FACE CENTRED SPACE LATTICES
UNIFORM TETRAVALENT SPACE LATTICES



5-3.4.6.3.6 ... RELATED SPONGE ... POLYHEDRON.

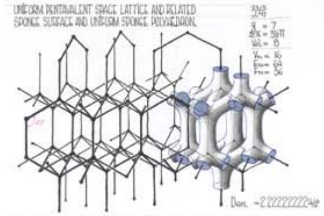


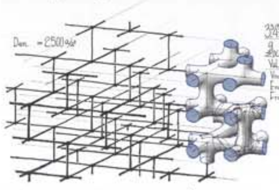
... SPACE-LATTICE ... AND LATTICE SYMMETRY ... SURFACE AND POLYHEDRON.



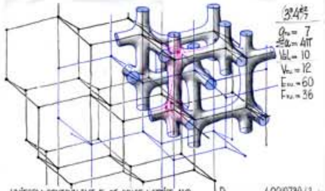
... SPACE LATTICE ... SURFACE AND SPONGE POLYHEDRON

UNIFORM PENTAVALENT SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

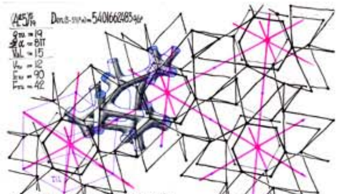




UNIFORM (ORTHOGONAL) PENTAVALENT SPACE LATTICE, AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

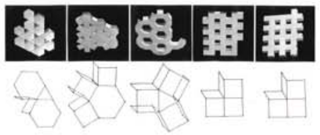


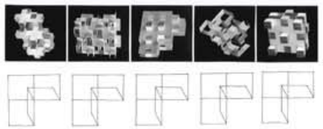
UNIFORM PENTAVALENT 5-6 SPACE LATTICE AND RELATED SPONGE SURFACE AND SPONGE POLYHEDRON.



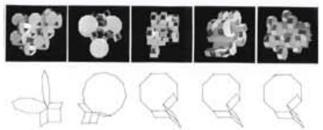
UNIFORM PENTAVALENT 5-5.6 SPACE-LATTICE WITH GLOBAL CUBE-CENTRED LATTICE SYMMETRY AND RELATED SPONGE SURFACE AND POLYHEDRON.

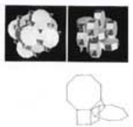
UNIFORM DOUBLE LAYER PENTAVALENT SPACE LATTICES

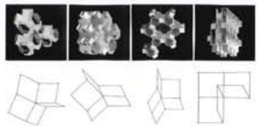




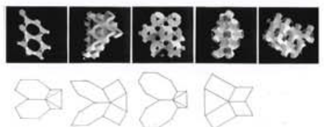
UNIFORM PENTAVALENT SPACE LATTICES.



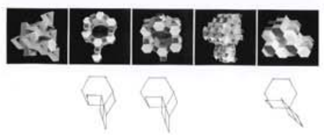


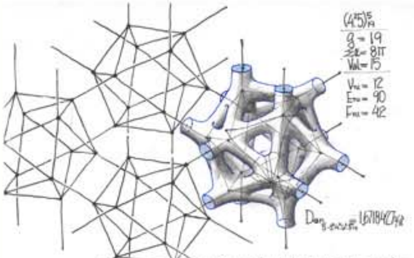


UNIFORM PENTAVALENT SPACE LATTICES.



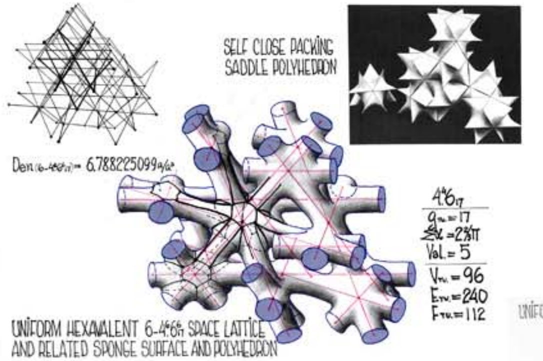
UNIFORM PENTAVALENT SPACE LATTICES.





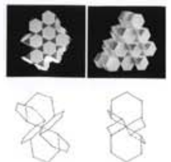
UNIFORM PENTAVALENT 5-3⁴4²6... SPACE LATTICE AND RELATED SPONGE SURFACE AND SPONGE POLYHEDRON

THREE SOLIDS, CLOSE PACKING OF WHICH GENERATES THE UNIFORM HEXAVALENT 6-3⁴4... SPACE LATTICE.

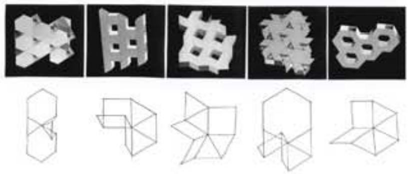


UNIFORM HEXAVALENT 6-4⁶6⁶₁₇ SPACE LATTICE AND RELATED SPONGE SURFACE AND POLYHEDRON

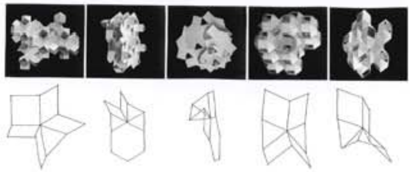
UNIFORM HEXAVALENT SPACE-LATTICES.

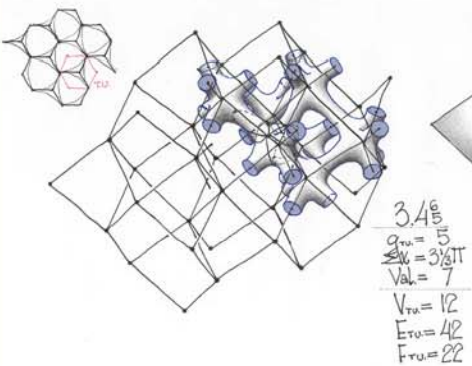


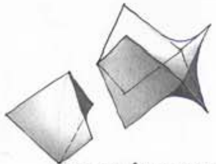
UNIFORM DOUBLE LAYER HEXAVALENT SPACE-LATTICES.



UNIFORM HEXAVALENT SPACE LATTICES.

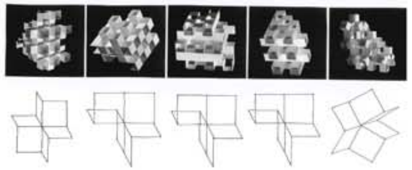


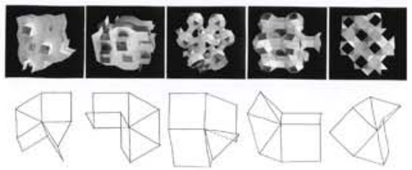




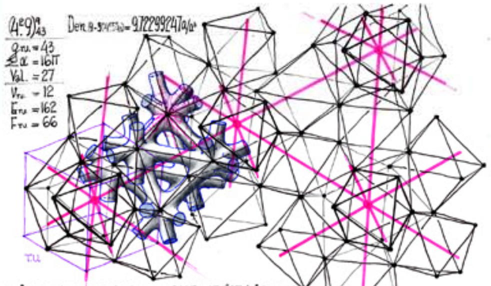
CLOSE PACKING SADDLE POLYHEDRA, GENERATING THE SPACE LATTICE.

UNIFORM HEXAVALENT SPACE-LATTICES.

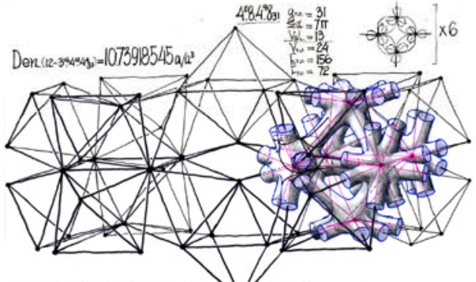




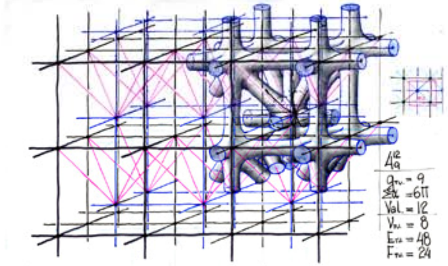
UNIFORM HEXAVALENT 6-(4.6)⁶₅ SPACE LATTICE AND RELATED PERIODIC SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.



UNIFORM 9-VALENT 9-$3^4 4^5 5_{43}$ SPACE-LATTICE (WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL CUBE-CENTRED LATTICE SYMMETRY) AND REL.



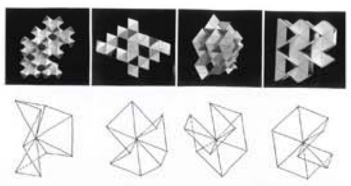
UNIFORM DODECAVALENT 12-$3^4 4^8$ SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

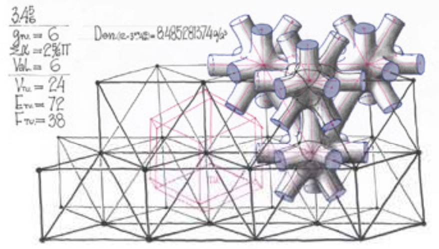


UNIFORM DECAVALENT 10-$3^8 4^8$ SPACE LATTICE AND RELATED SPONGE SURFACE AND SPONGE POLYHEDRA

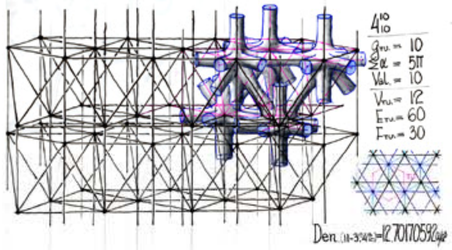
Den. = 10.000 a/a³

UNIFORM 9-VALENT SPACE-LATTICES.



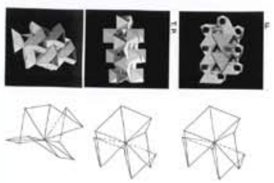


UNIFORM DODECAVALENT 12-$3^8 4^6$ SPACE LATTICE (OCTET LATTICE) AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

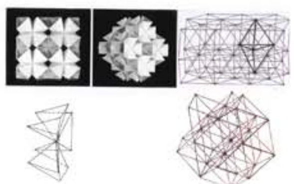


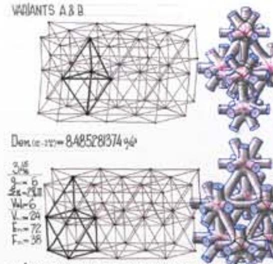
UNIFORM 11-VALENT 11-$3^8 4^3$ SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON

UNIFORM DECAVALENT SPACE-LATTICES.

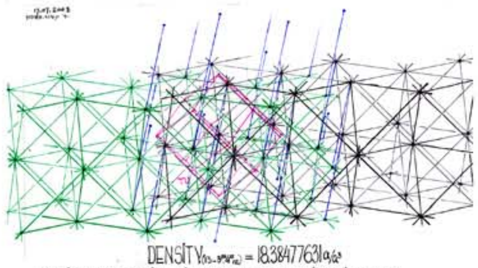


UNIFORM DODECAVALENT SPACE-LATTICES.





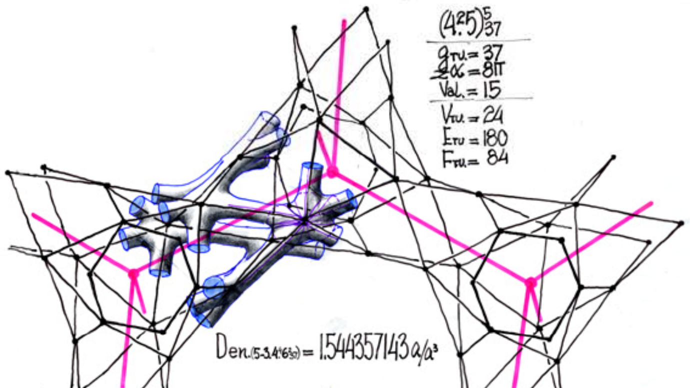
UNIFORM DODECAVALENT 12-3^8 SPACE LATTICE (OCTET) AND RELATED SPONGE SURFACES AND SPONGE POLYHEDRA.



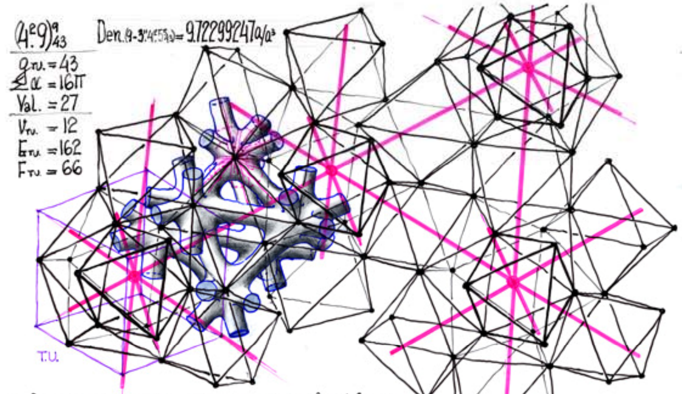
DENSITY = 18.38477631 a/a³

TWO INTER-PENETRATING UNIFORM OCTET SPACE LATTICES WITH EDGE-a, WHEN JOINED TOGETHER WITH A SET OF PARALLEL a-EDGES, GENERATE A UNIFORM 13-VALENT SPACE LATTICE.

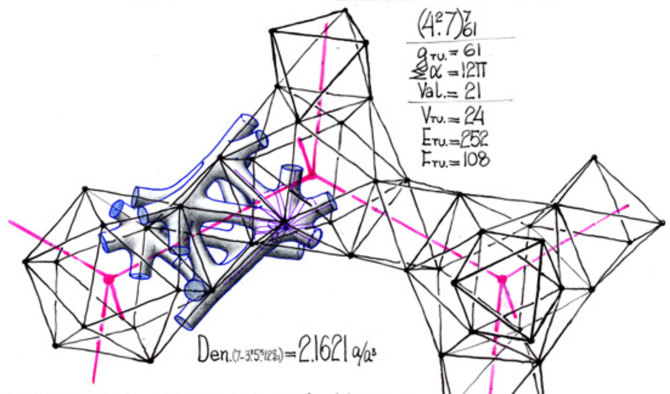
CAL REPEATING ICOSAHEDRAL SYMMETRY



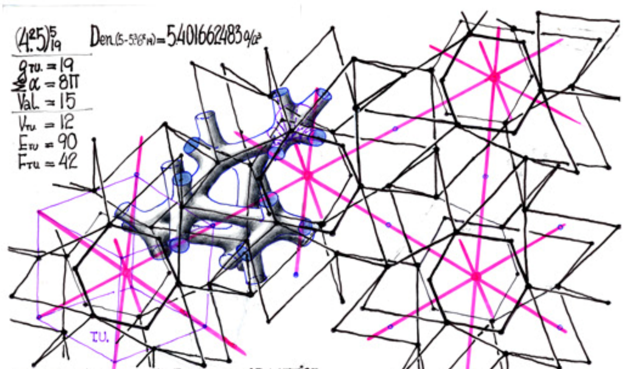
UNIFORM 5-VALENT 5-3.4.4.6 37 SPACE-LATTICE
WITH GLOBAL DIAMOND LATTICE SYMMETRY
AND RELATED SPONGE SURFACE AND POLYHEDRON.



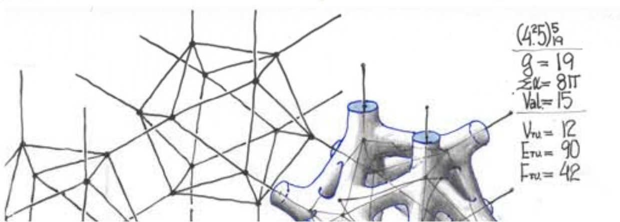
UNIFORM 9-VALENT 9-3.4.5 43 SPACE-LATTICE (WITH LOCAL
ICOSAHEDRAL SYMMETRY AND GLOBAL CUBE-CENTRED LATTICE
SYMMETRY) AND RELATED SPONGE SURFACE AND POLYHEDRON.

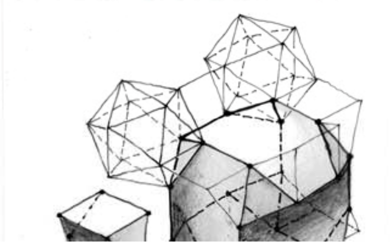


UNIFORM 7-VALENT 7-3.5.12 61 SPACE-LATTICE (WITH LOCAL
ICOSAHEDRAL SYMMETRY AND GLOBAL DIAMOND LATTICE
SYMMETRY) AND RELATED SPONGE SURFACE AND POLYHEDRON.



UNIFORM PENTAVALENT 5-5.6 19 SPACE-LATTICE
WITH GLOBAL CUBE-CENTRED LATTICE SYMMETRY
AND RELATED SPONGE SURFACE AND POLYHEDRON.





In Conclusion

It is supposed that if the staged enquiry approach as formulated is followed ,the exhaustive enumeration of the uniform networks in 3-D space will, eventually, be achieved and accomplished.