# UNIFORM NETWORKS, SPONGE SURFACES AND UNIFORM SPONGE POLYHEDRA IN 3-D SPACE

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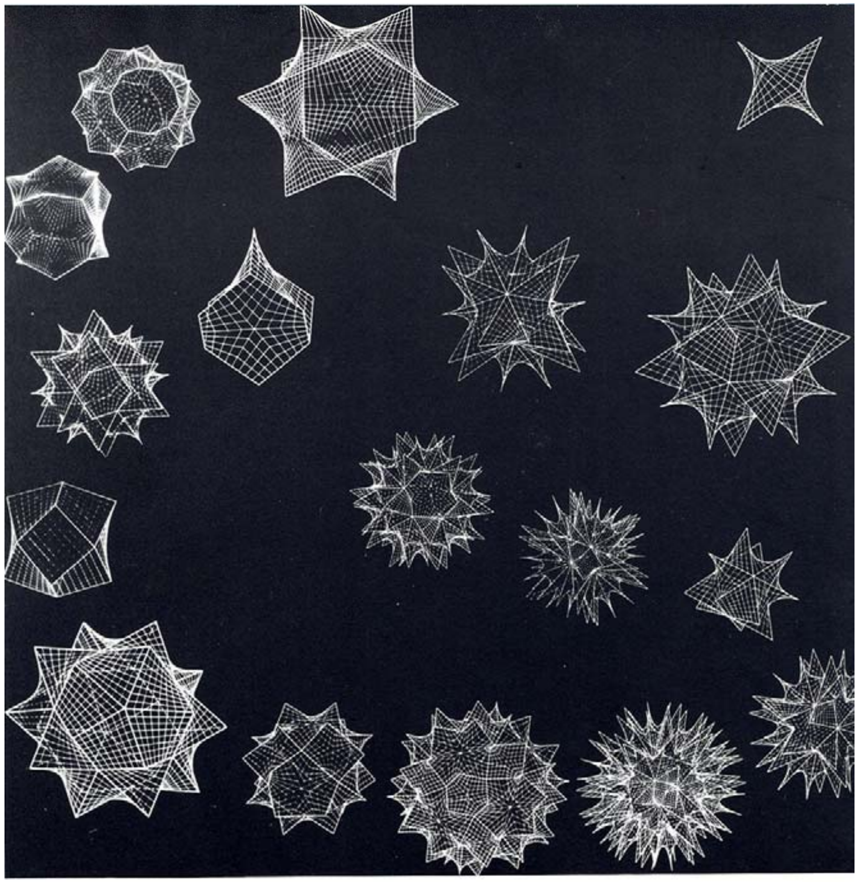
# A

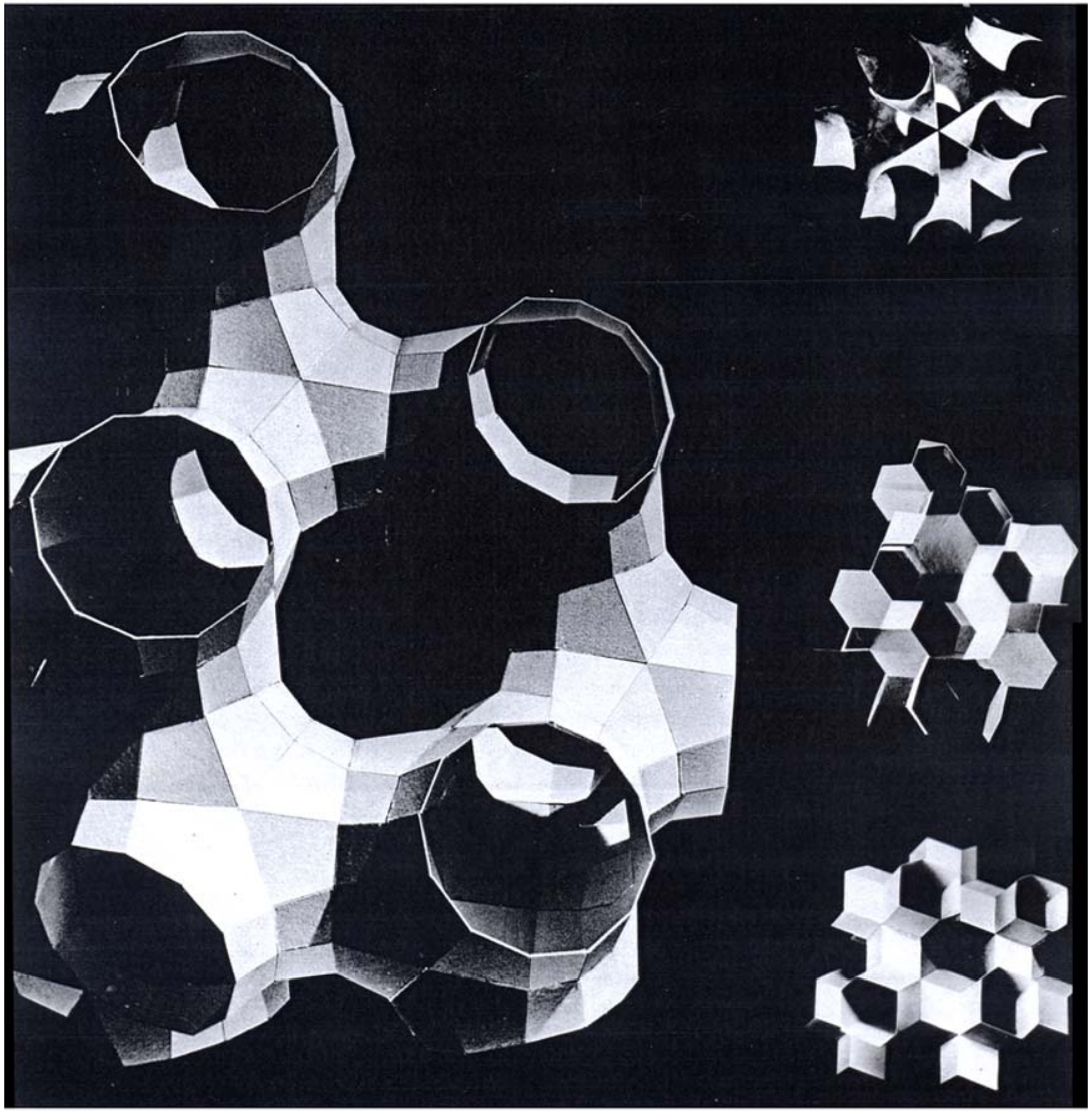
The diversity of shapes and forms which meets the eye is overwhelming. They shape our environment: physical, mental, intellectual. Theirs is a dynamic milieu; time induced transformation, flowing with the change of light, with the relative movement of the eye, with physical and biological transformation and the evolutionary development of the perceiving mind.

B

"Our study of natural form, "the essence of morphology", is part of that wider science of form which deals with the forms assumed by nature under all aspects and conditions, and in a still wider sense, with forms which are theoretically imaginable…..(On Growth and Form – D'Arcy Thompson), "Theoretically" to imply that we are dealing with causal- rational forms.

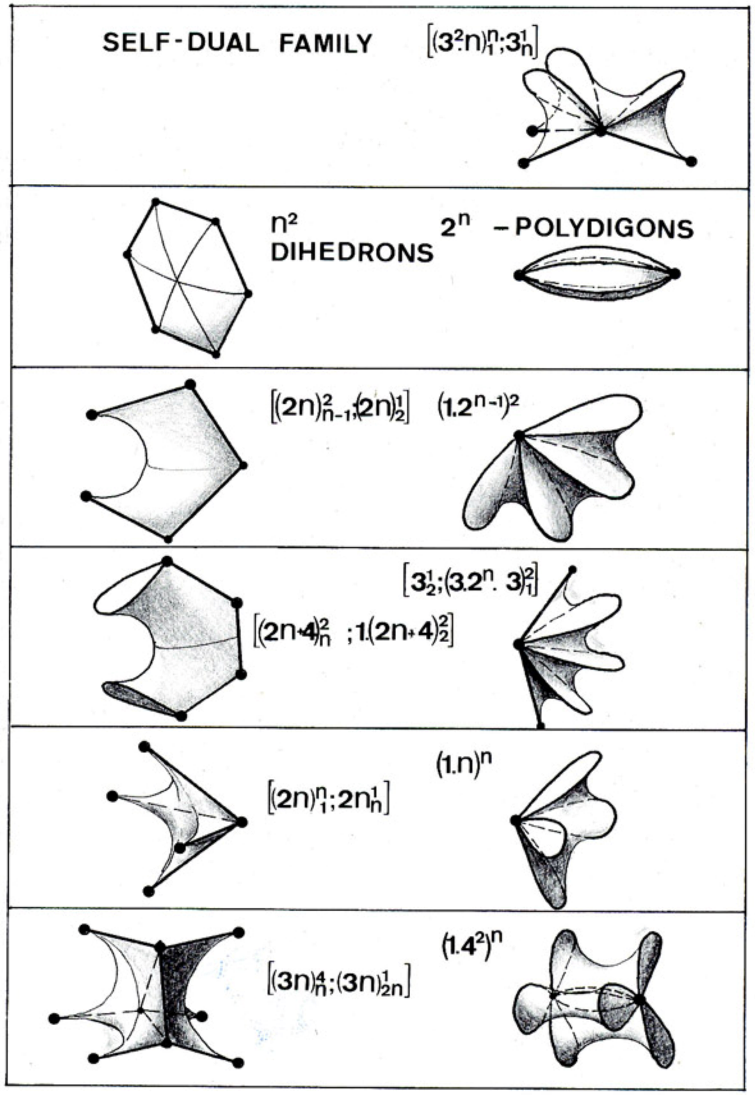


Finite Saddle Polyhedra.

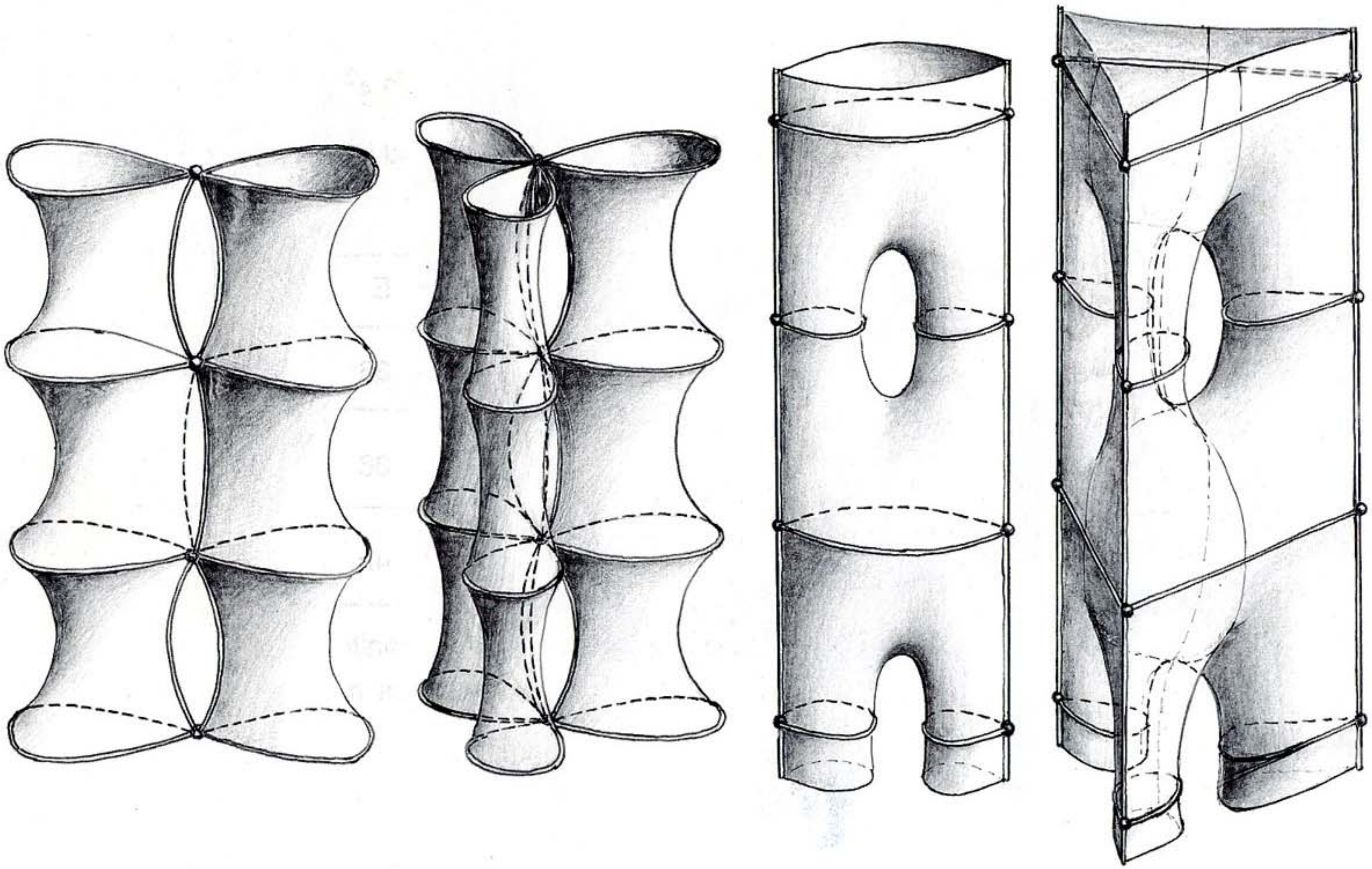


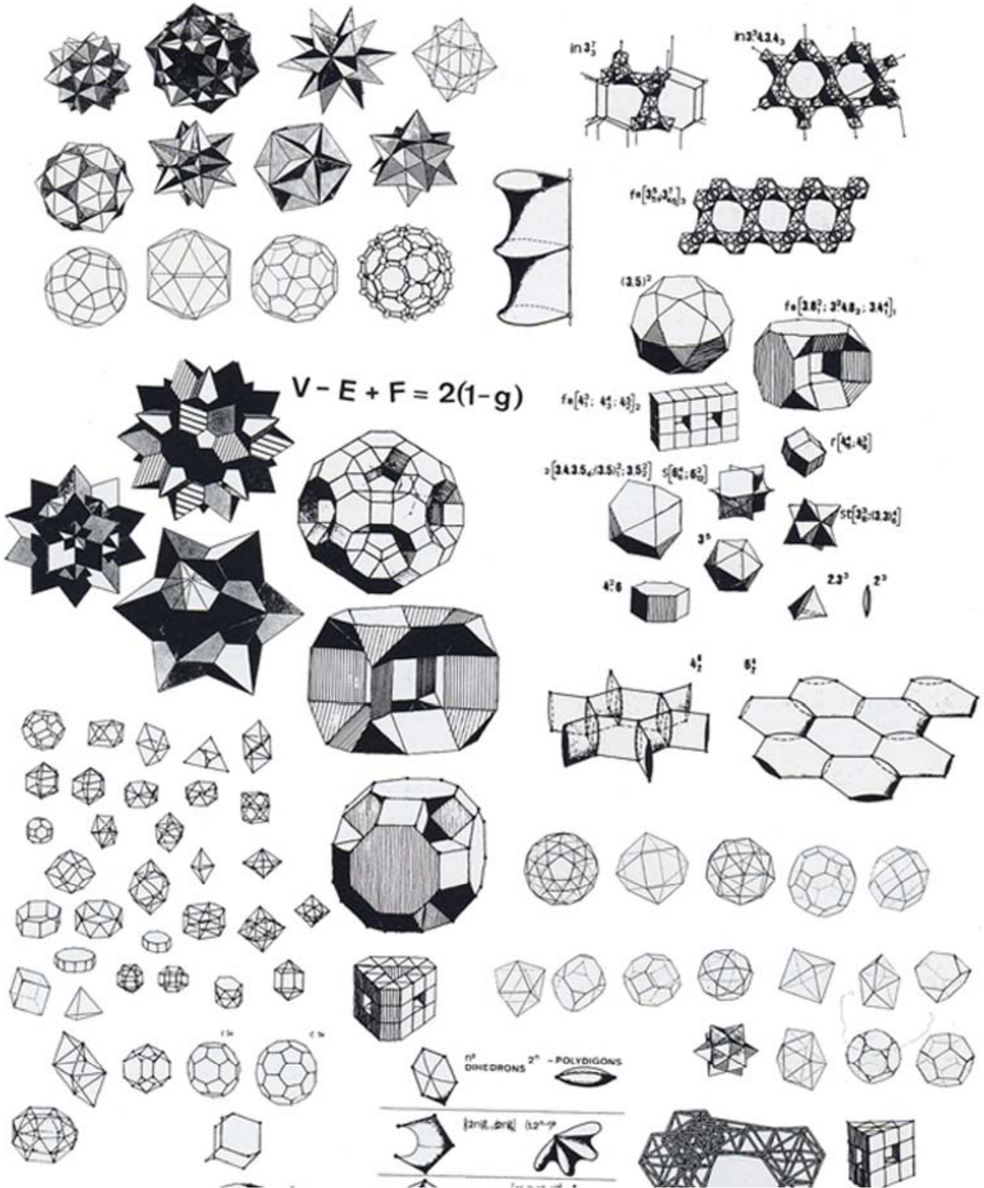
Periodic, uniform and non-uniform infinite sponge polyhedra related to a minimal (hyperbolic) surface of g =3 which subdivides space between two diamond lattices.

Floral polyhedra, families within the g=0 domain Aaranged according to their dual pairs.



Periodic Floral Infinite
Polyhedra.




V – E + F = 2(1-g)
DIHEDRONS
– POLYDIGONS

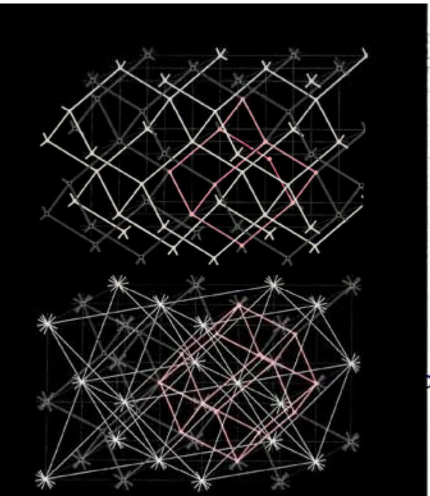
C

Abstract and physical 3-D space is not a passive vacuum. It is populated with inter-relating and inter-connected entities, generating configurations represented as diagrams with a network characteristics and hyperbolic 'force fields', and surface partitions, aptly described as sponge surface configurations. Diagrams of this kind can represent the structure of almost any plurality that may exist; from a reality of any battle field, cultural economical or political, to transportation and communication systems, to social patterns, cosmological arrays and patterns of perception, knowledge and thought .

D

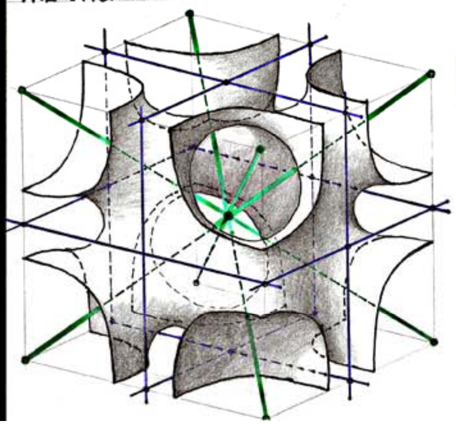
A periodic ordered network is formed by extended repetition of a locally symmetrical association of vertex figures. The resulting configuration of vertices and axes-edges could be described as a polyhedral network, with edges terminated at one or two vertices, each, and vertices joining n-edges together, featuring its bonding valency, so as to conform with the following relation:  = , with E: V&Val.av. standing for the number of Edges, Vertices and average Valency value in a vertex, respectively. In the case of a periodic network, the same applies to its translation unit (T.U.):

=

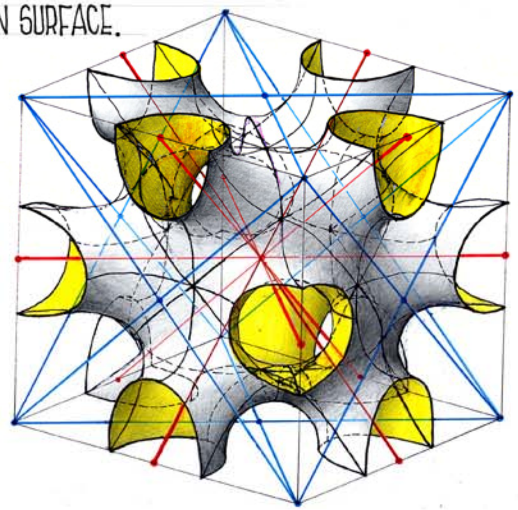


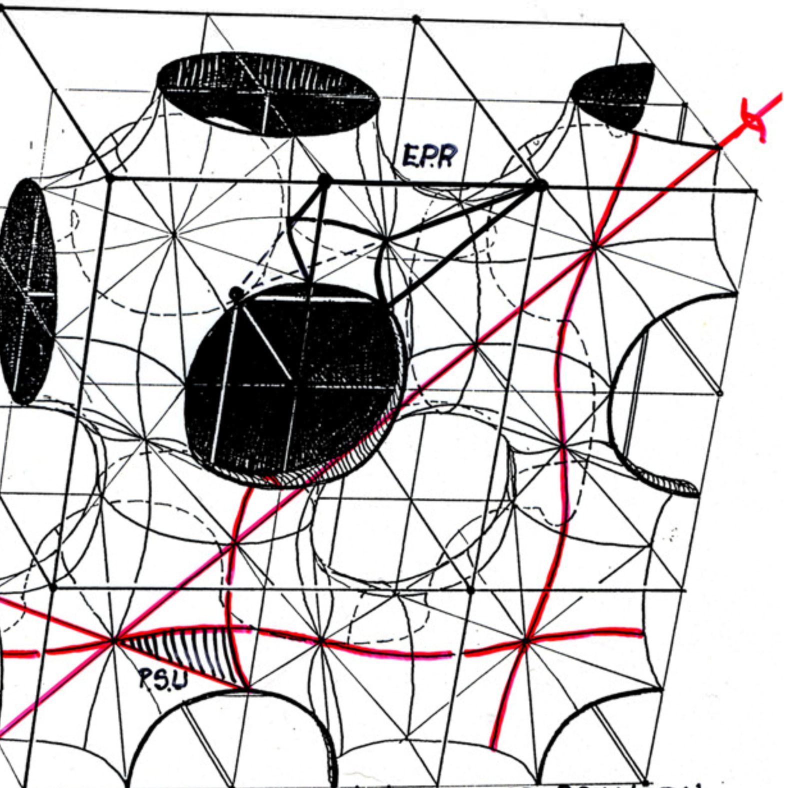
UNIFORM TETRAVALENT 'DIAMOND' LATTICE AND
UNIFORM DODECAVALENT 'OCTET' LATTICE AND DUALS.

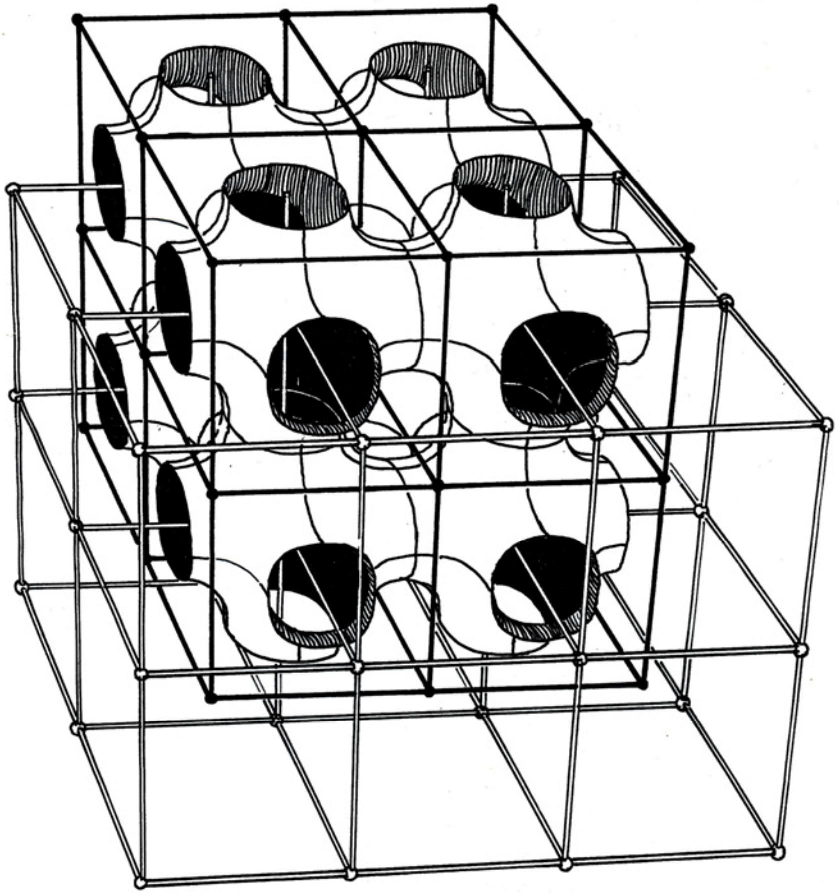
THE TRINITY: TWO DUAL-COMPLEMENTARY NETWORKS AND THE RECIPROCAL SURFACE-PARTITION, SBDIVIDING THE SPACE BETWEEN THE TWO.



THE TRINITY OF THE DUAL PAIR OF NETWORKS AND THE ASSOCIATED PARTITION SURFACE.





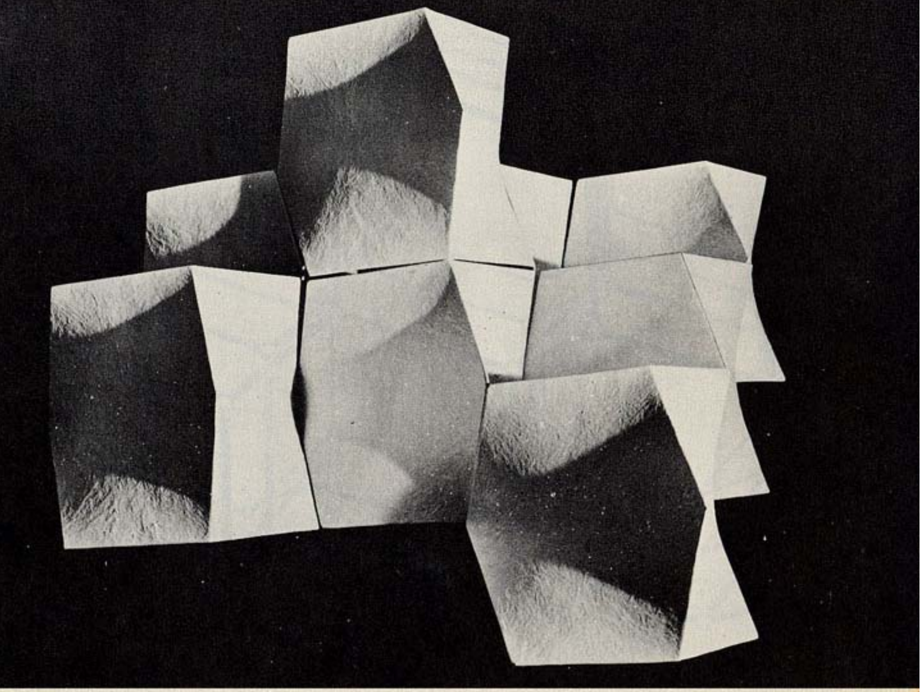



BOLIC SURFACE SUBDIVIDING SPACE BETWEEN
COMPLEMENTARY (DUAL) SPACE NETWORKS.

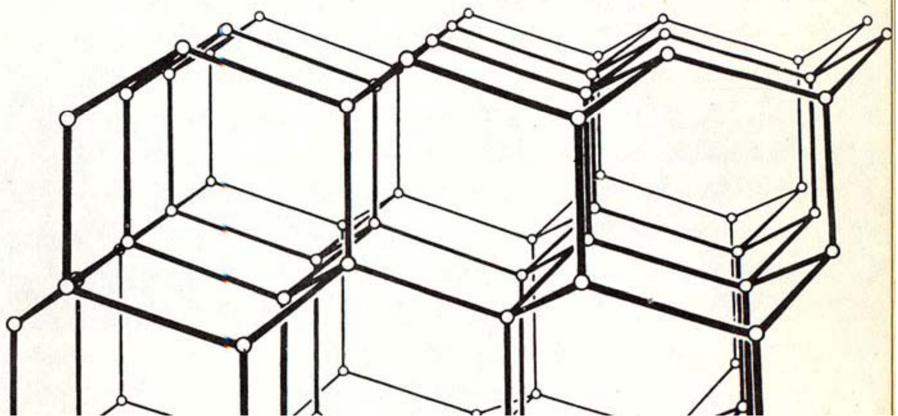
E OF THE PERIODIC HYPERBOLIC SURFACES
WITH VERY HIGH DEGREE OF REGULARITY.

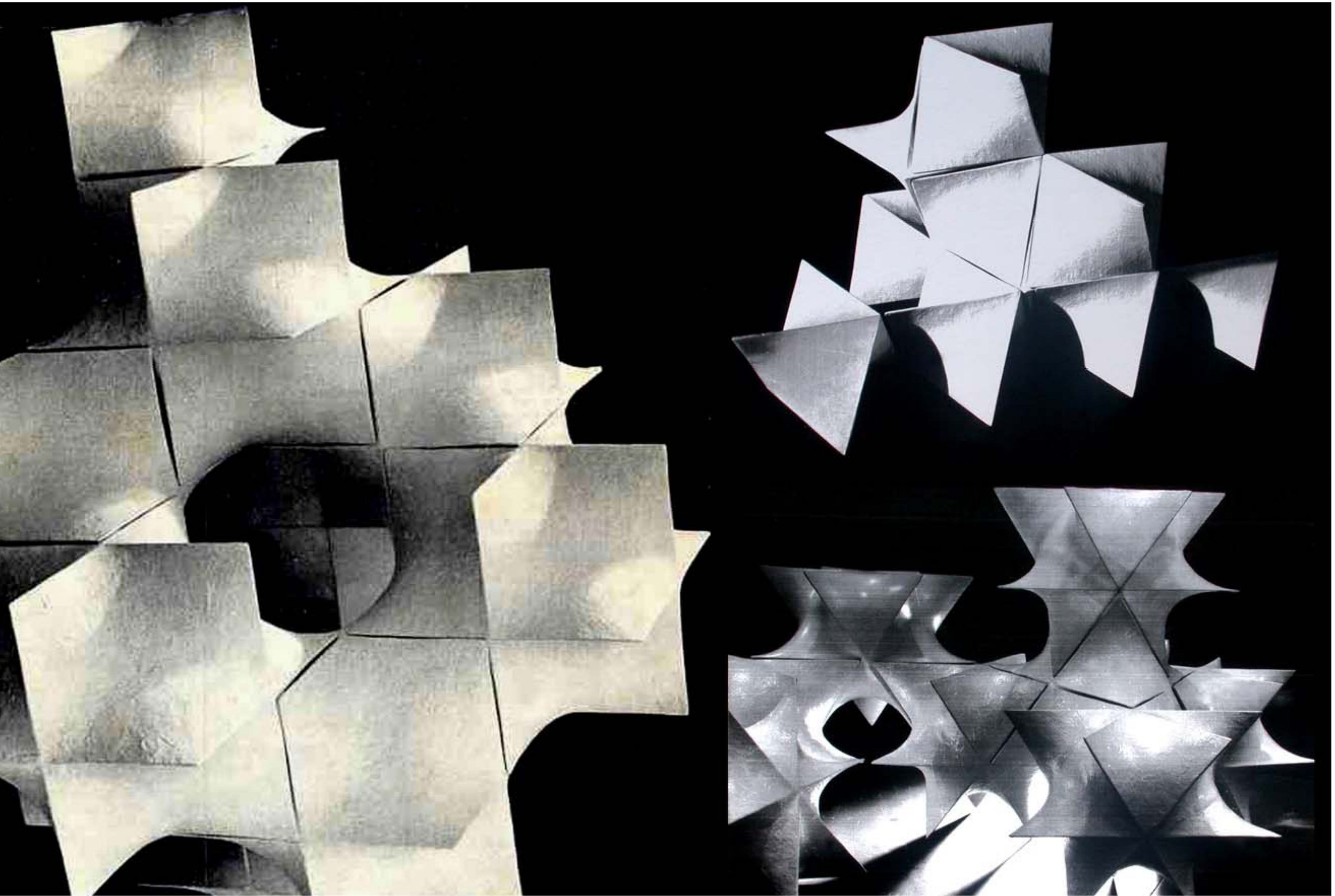
– A REPETITIVE <u>PERIODIC SURFACE UNIT.</u>

1



1



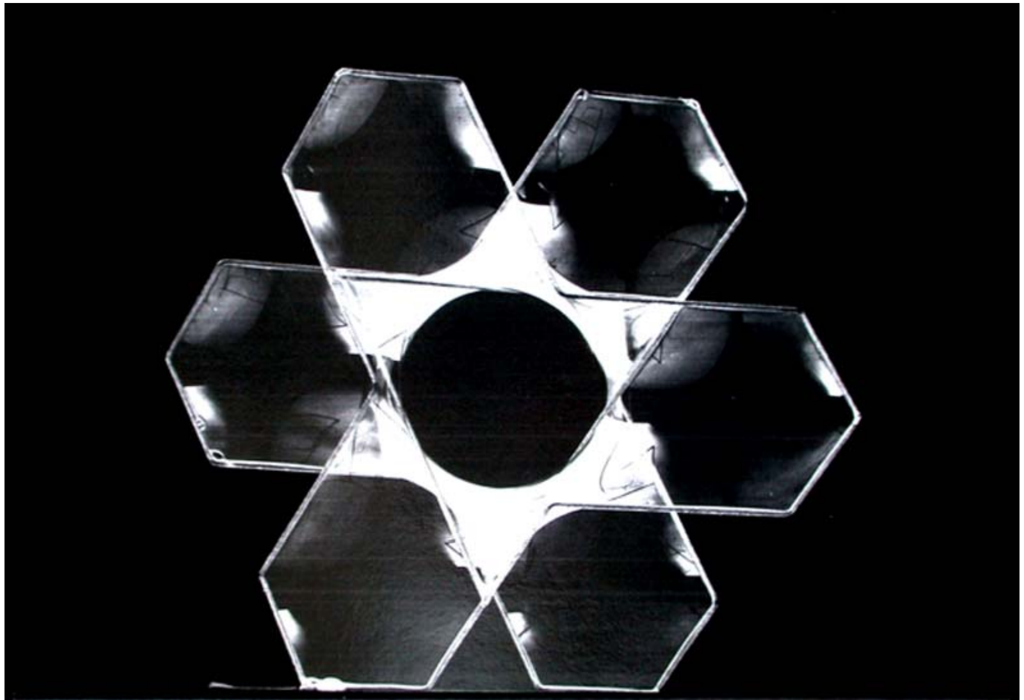


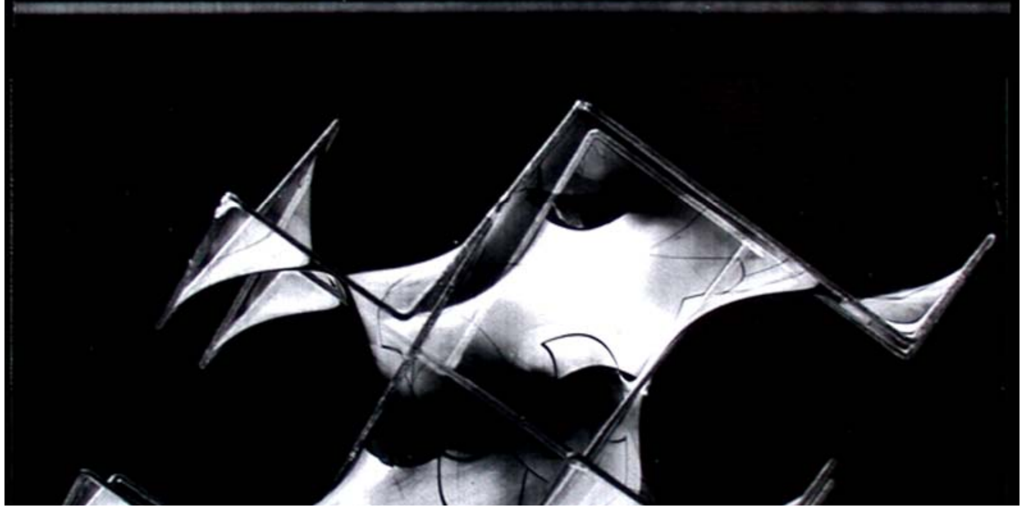
E

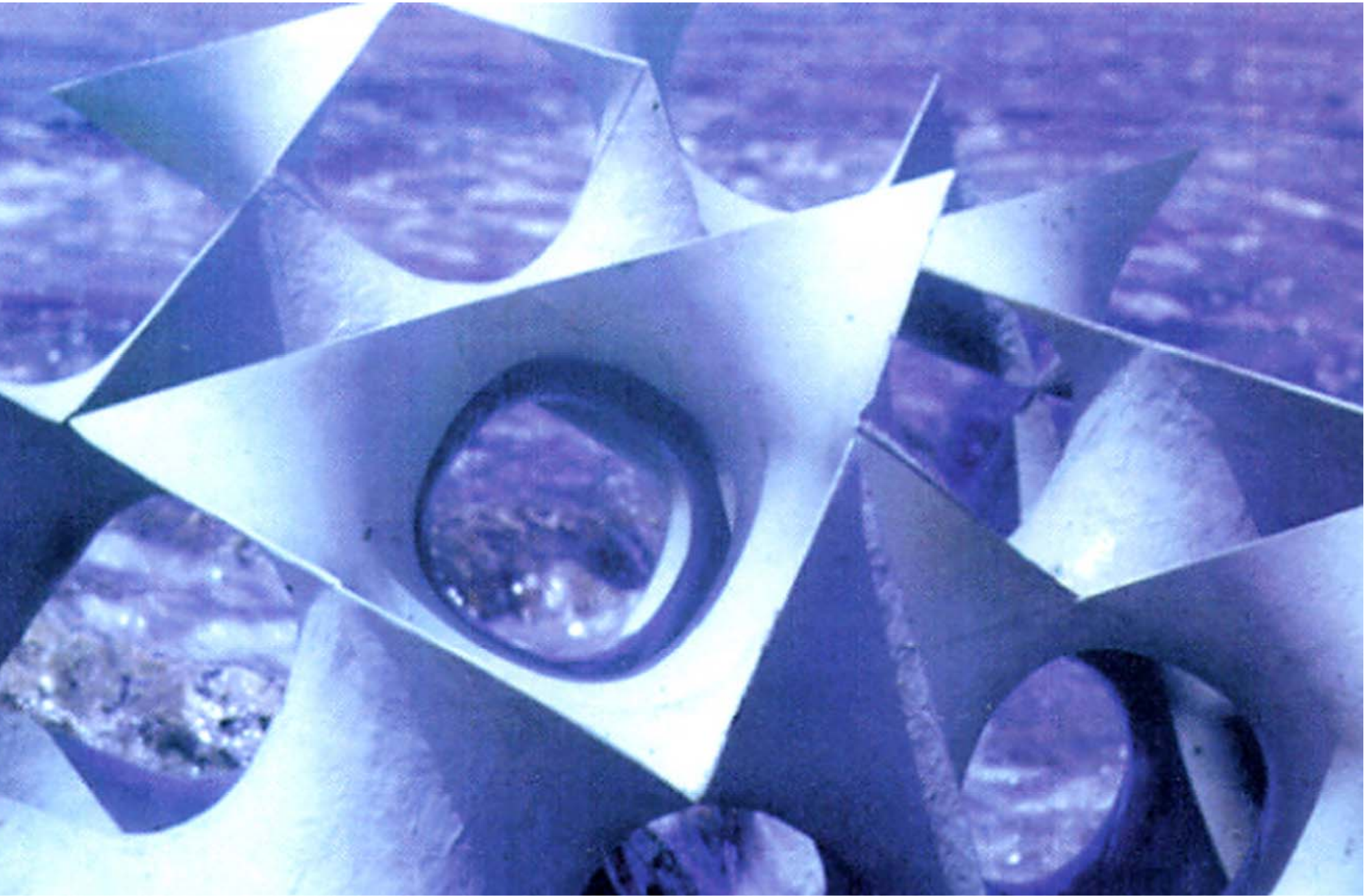
**networks come in dual pairs.** Each network is uniquely determined by, and reciprocal of its dual (complementary) companion.

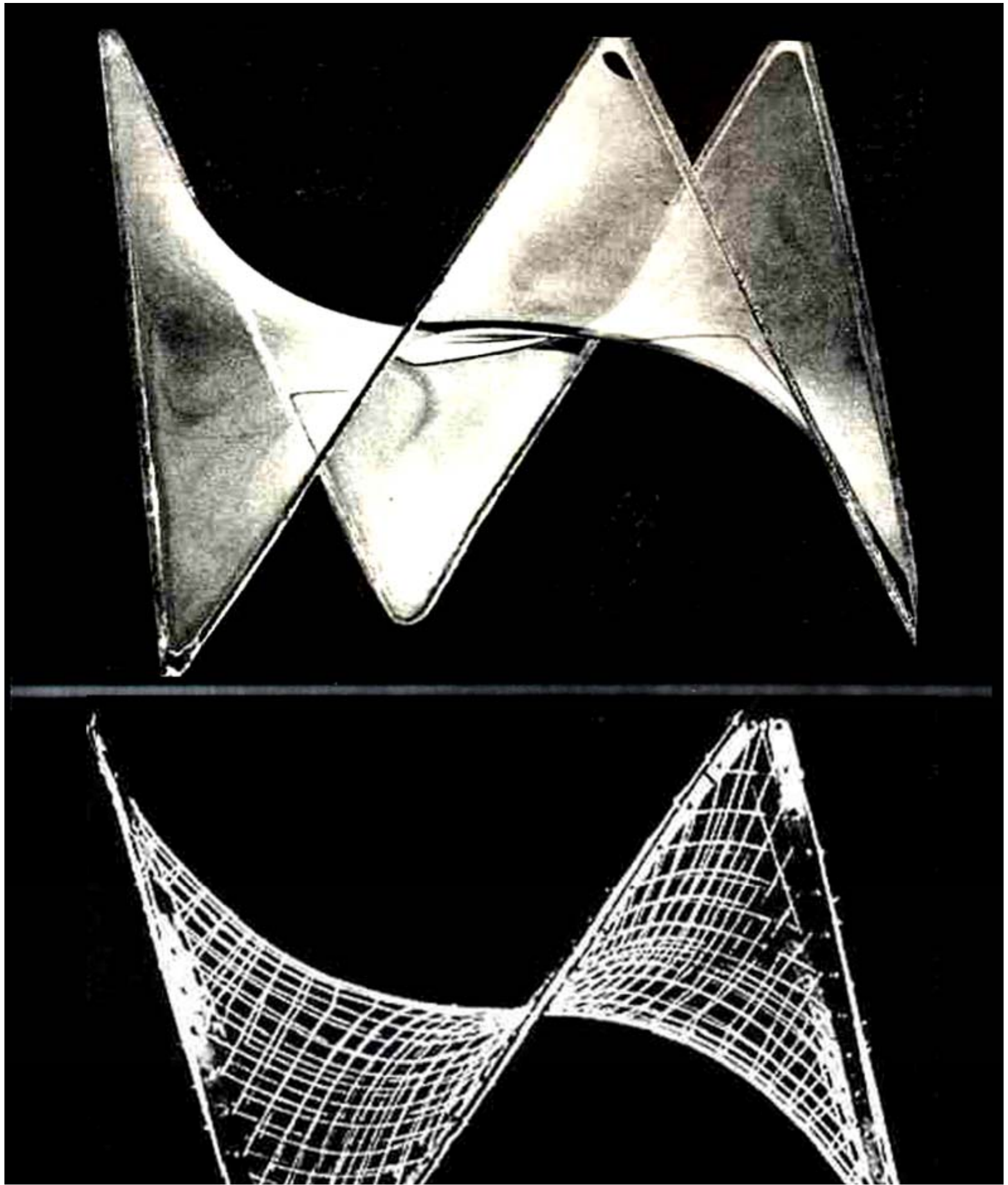
ery dual pair of networks is associated with a continuous hyperbolical sponge face which subdivides the space between the two, into two complementary -spaces.

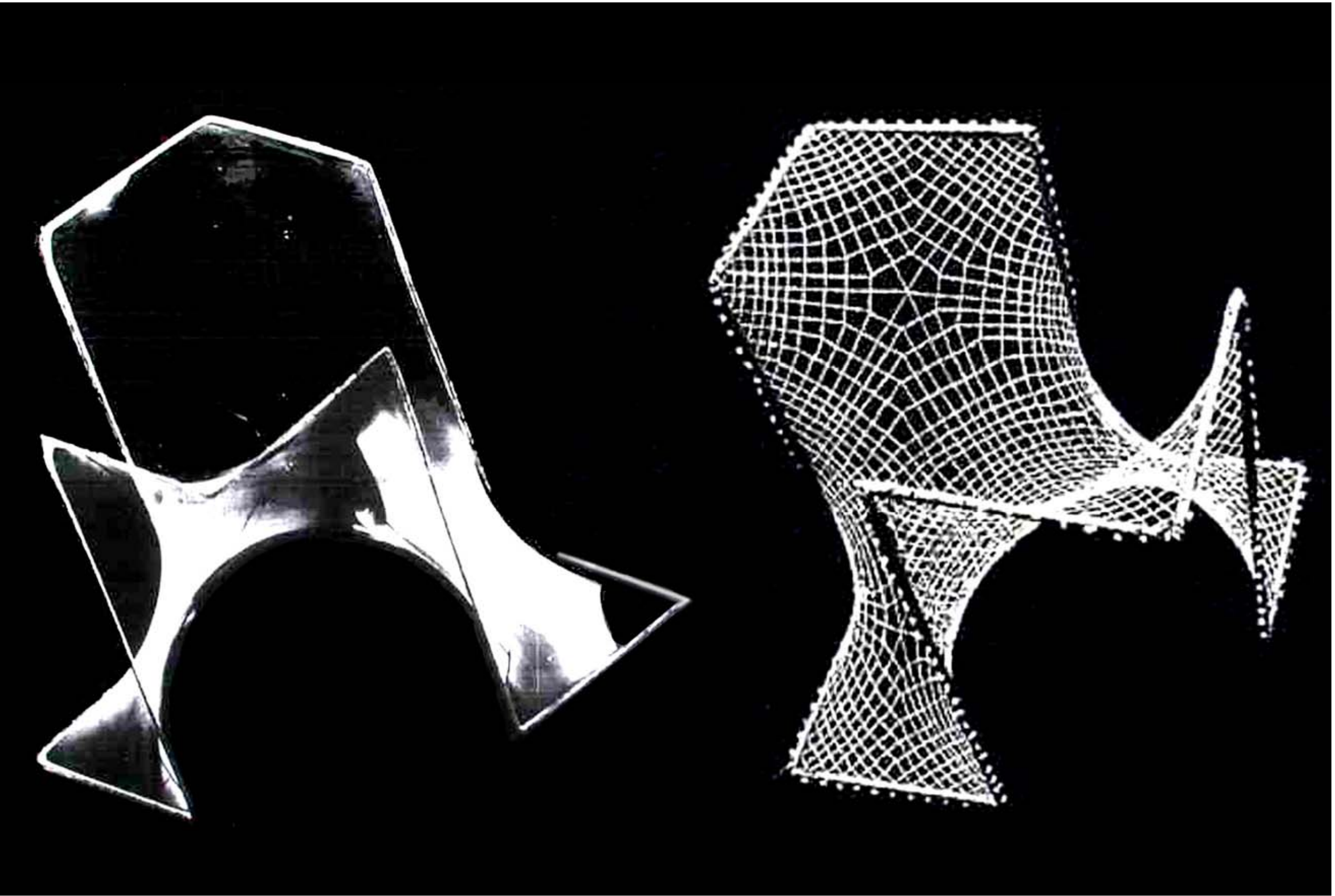
s **trinity of the dual pair and the associated-reciprocal sponge surface** is most conspicuous, all pervading geometric-topological phenomenon of our 3-pace, associated with its order and organization and more than anything else ermines the way we perceive and comprehend its structure

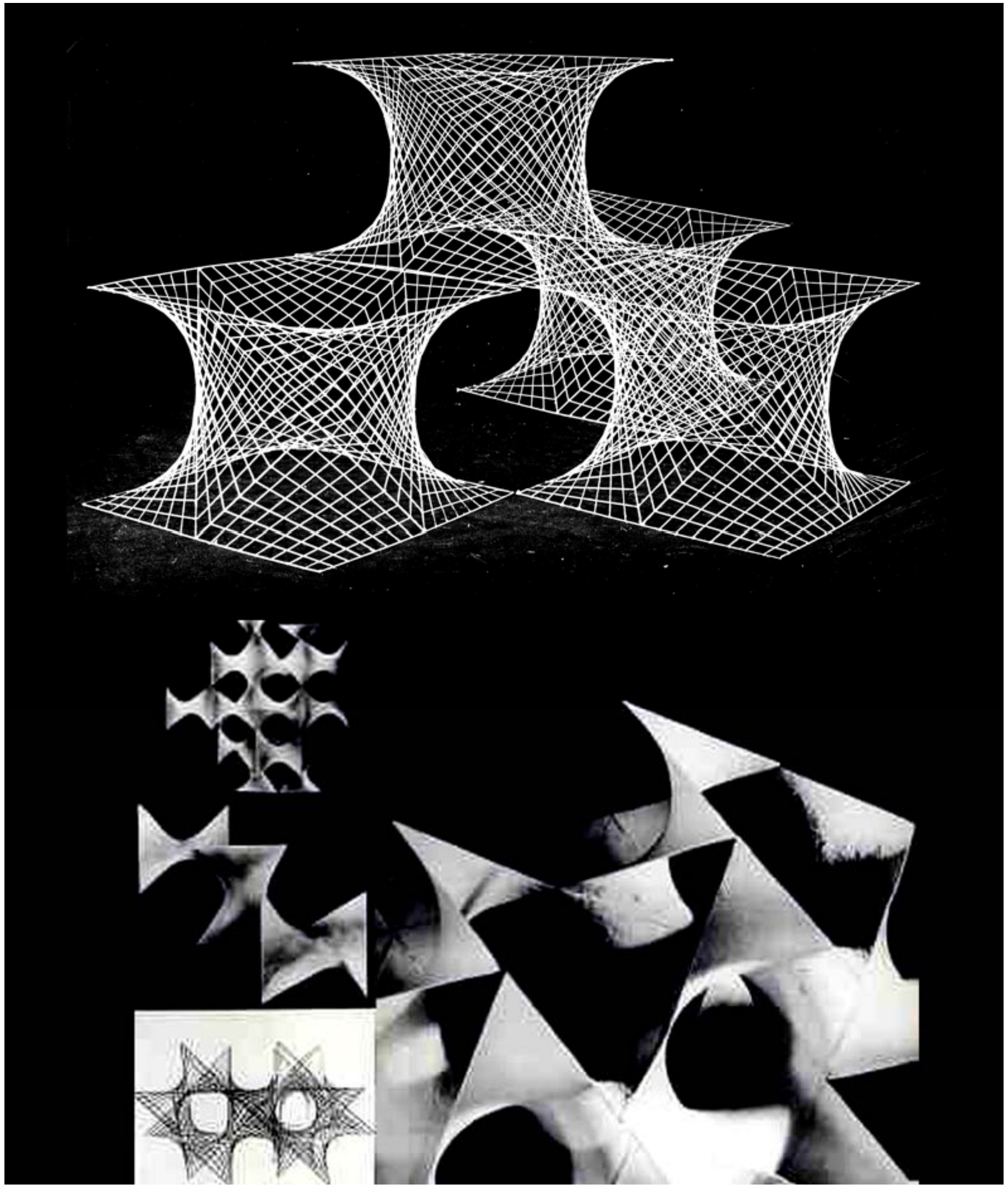


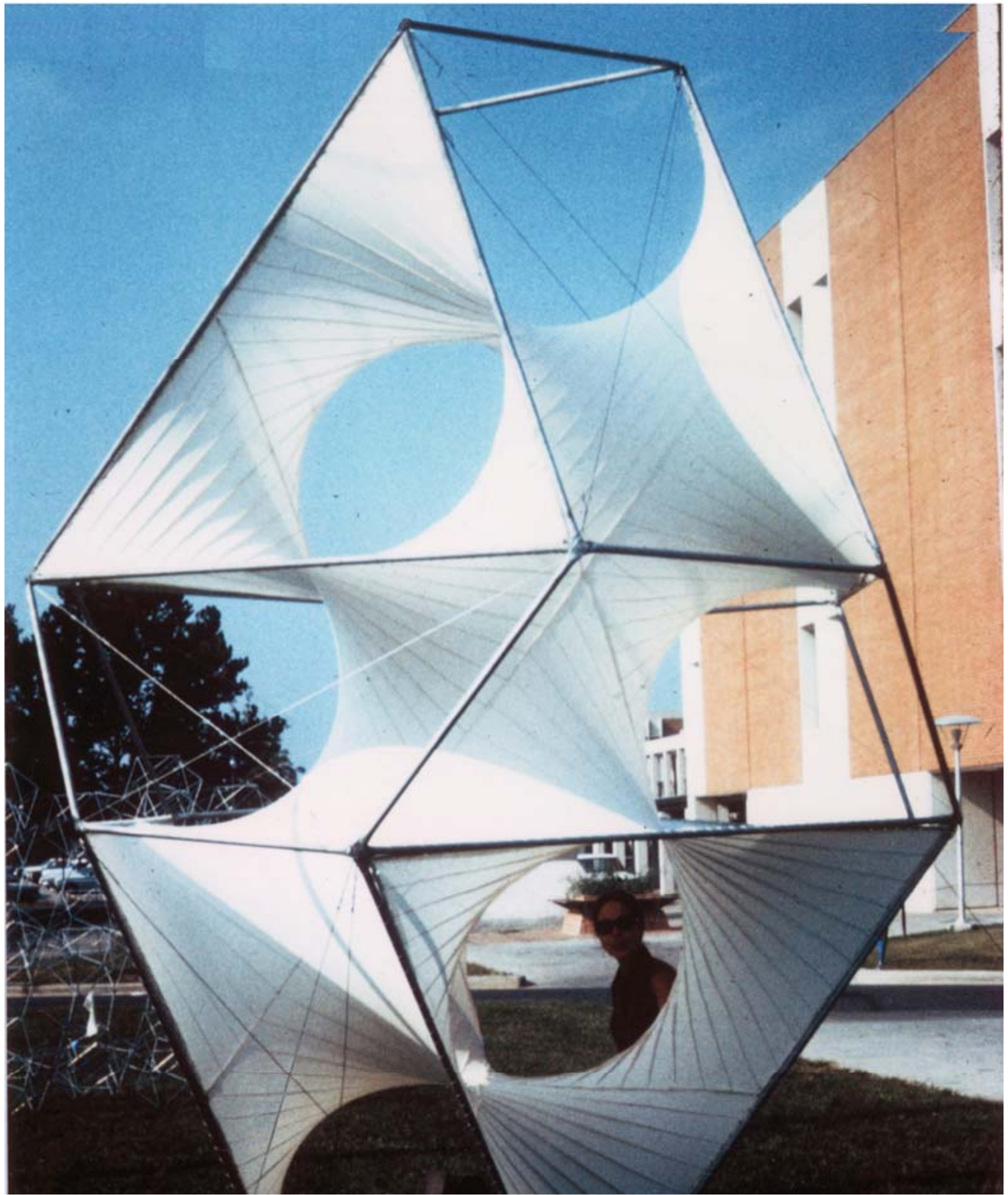






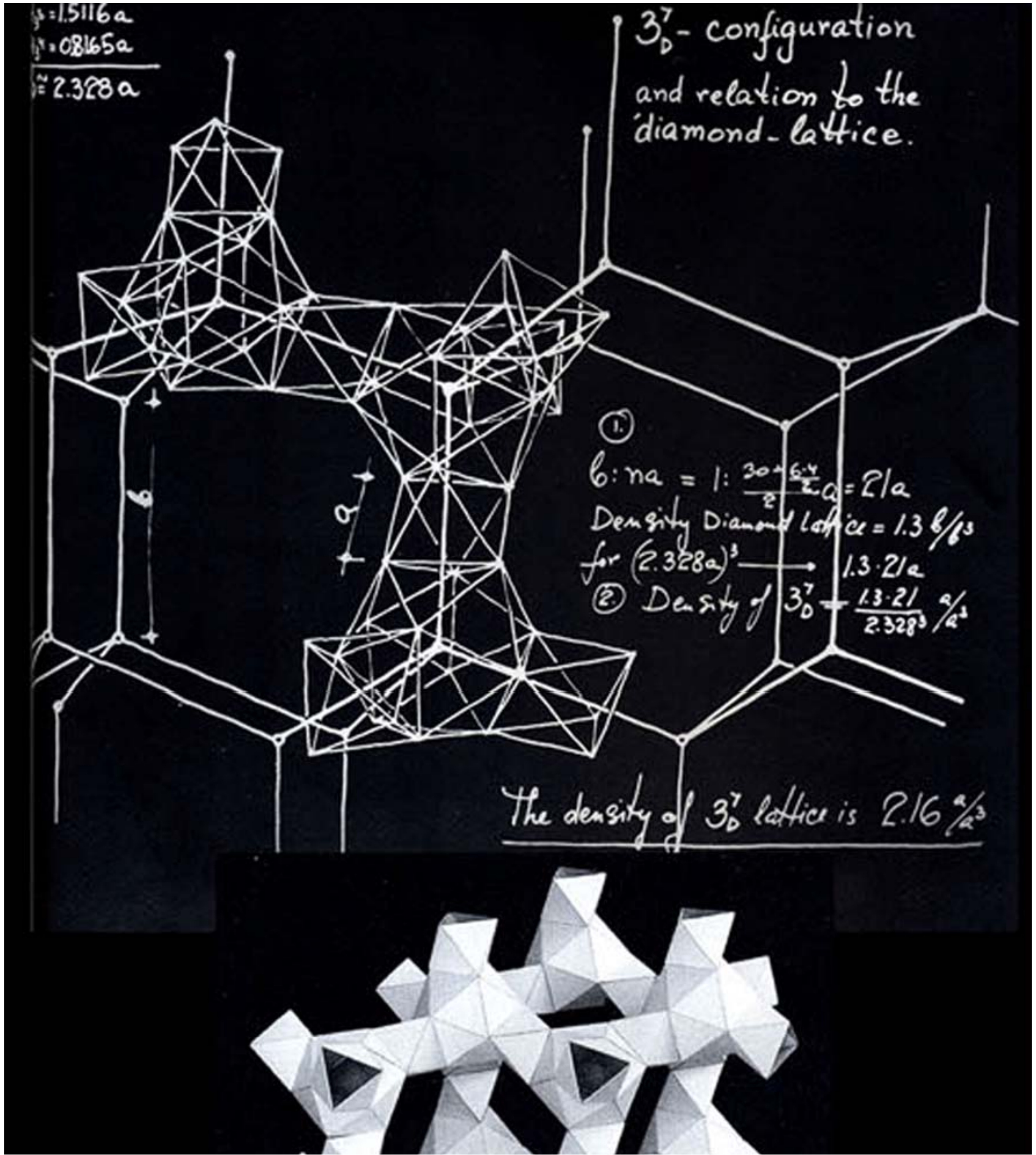


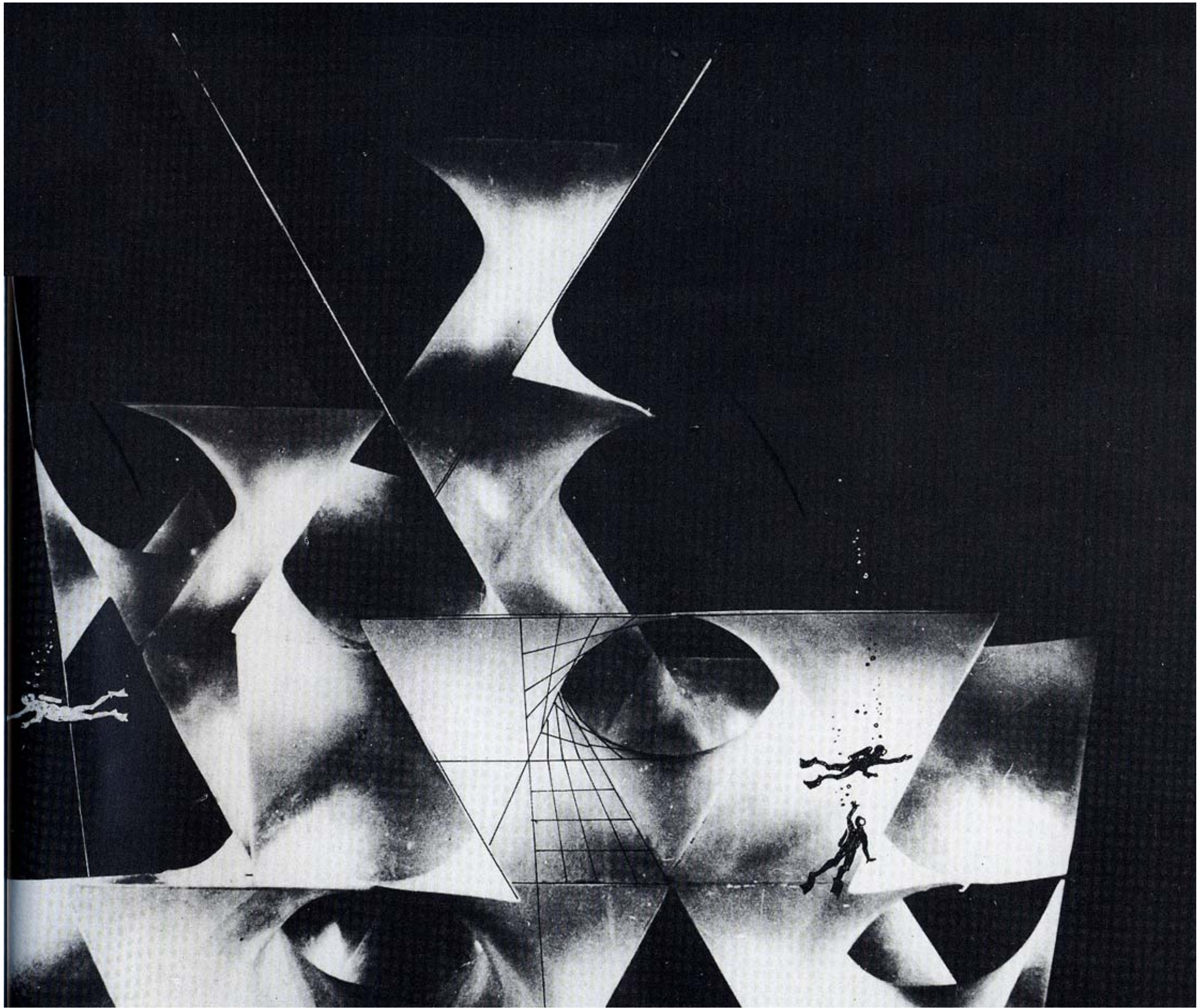


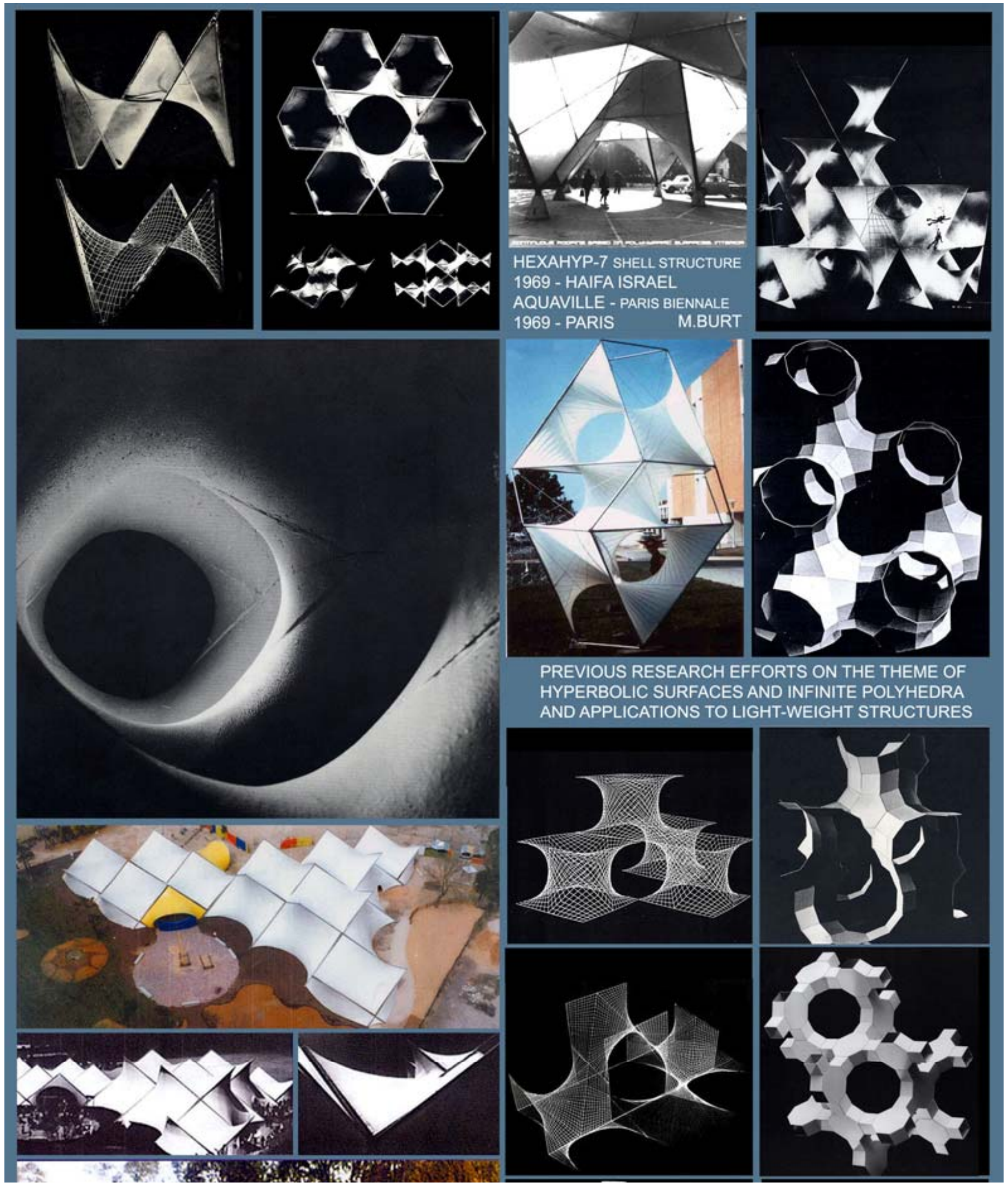


F

**By defining as 'morphic' those processes which display a movement toward greater 3-dimensional spatial order, symmetry or form** (Whyte-1969) and **morphology as the logical preoccupation with and manipulation of those processes,** than the **research into the nature of networks and the associated sponge surfaces may be classified as the essence of morphology.**


3⁷D- configuration
and relation to the
'diamond-lattice.
=1.5116a
=0.8165a
= 2.328 a
Density Diamond lattice = 1.3
for (2.328a)³ → 1.3·21a
② Density of 3⁷D = 1.3·21 / 2.328³
The density of 3⁷D lattice is 2.16




HEXAHYP-7 SHELL STRUCTURE
1969 - HAIFA ISRAEL
AQUAVILLE - PARIS BIENNALE
1969 - PARIS M.BURT
PREVIOUS RESEARCH EFFORTS ON THE THEME OF
HYPERBOLIC SURFACES AND INFINITE POLYHEDRA
AND APPLICATIONS TO LIGHT-WEIGHT STRUCTURES

G

the vertex figure characteristics (geometric-symmetrical and topological) of a given network, tightly correspond to the topological-symmetrical characteristics of the close-pack cells of it's dual. By proxy it may be stated that all constituents of a given 'trinity' (the dual networks pair and the associated sponge surface) act under the same topological – Symmetrical regime.

-Connectivity value (C) of the two continuous dual network graphs is one and the same for both, and is the same as genus-(g) value of the associated sponge surface:

**C=L-N+1=g** (with L&N as the number of Line edges and

H

Each of the sponge surfaces may be mapped with a grid, representing eventually a sponge polyhedron which conforms with the Euler's heorem and formula, stating that:

**/-E+F=2(1-g),** with g ≥ 2 (when V, E, F&g correspond to the number of /ertices, Edges, Faces and the genus value of the 2d-manifold, espectively).

I

e total average curvature value $\sum\alpha_{av.}$ of a vertex region of a
onge polyhedron may be expressed as :

$$\sum\alpha_{av.} = 2\pi\left[1-\frac{2(1-g_{T.U})}{V_{T.U.}}\right],$$ as derived from Descartes'

panded) theorem, (with $V_{T.U.}$ representing the number of
rtices in a translation unit, when the polyhedron is of a
riodic nature).

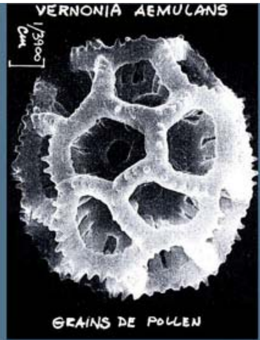
T-1

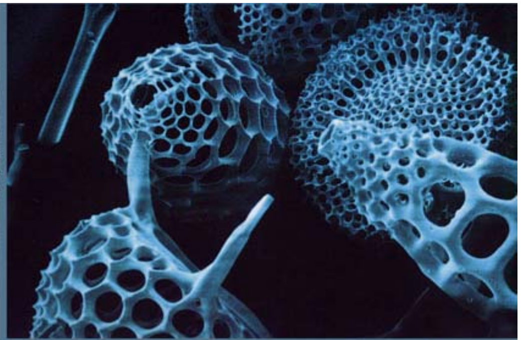
Nature is saturated with sponge structures on every possible scale of physical-biological reality. The term was first adopted in biology: "Sponge: any member of the phylum Porifera, sessile aquatic animals, with single cavity in the body, with numerous pores. The fibrous skeleton of such an animal, remarkable for its power of sucking up water". (Wordsworth dictionary).

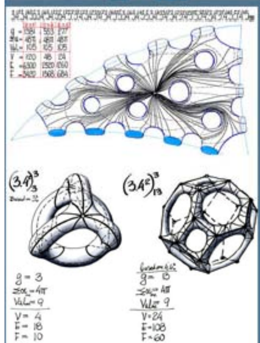
Of course the term applied to **'spherical sponges'.** It turns out that the key characteristic of porosity is attributable to a much wider morphological phenomenon.

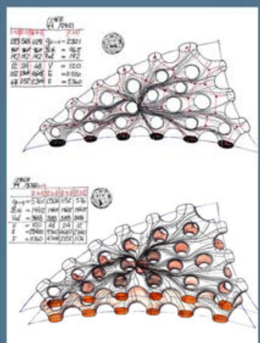
T-2

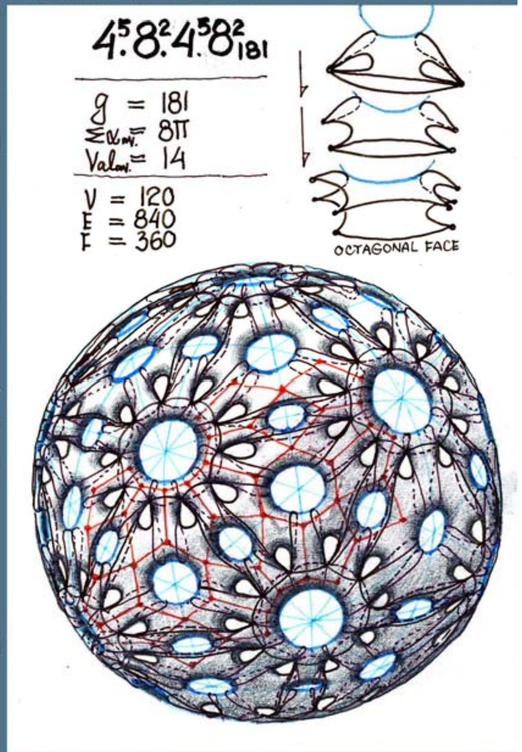
With some extrapolation of the perceiving mind it is right to claim that **the sponge phenomenon, with its porosity and permeability characteristics, is central to the physical morphological nature of the human habitat, and represents its defining imagery.**



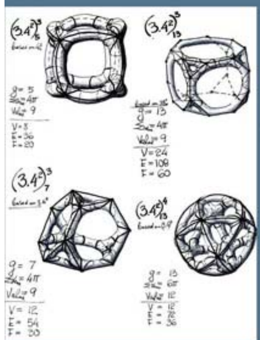



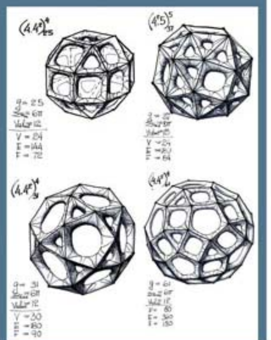


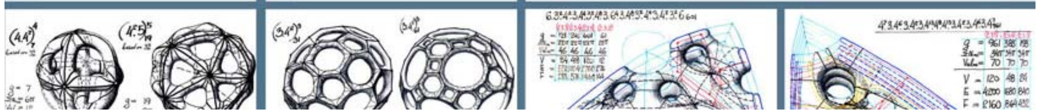


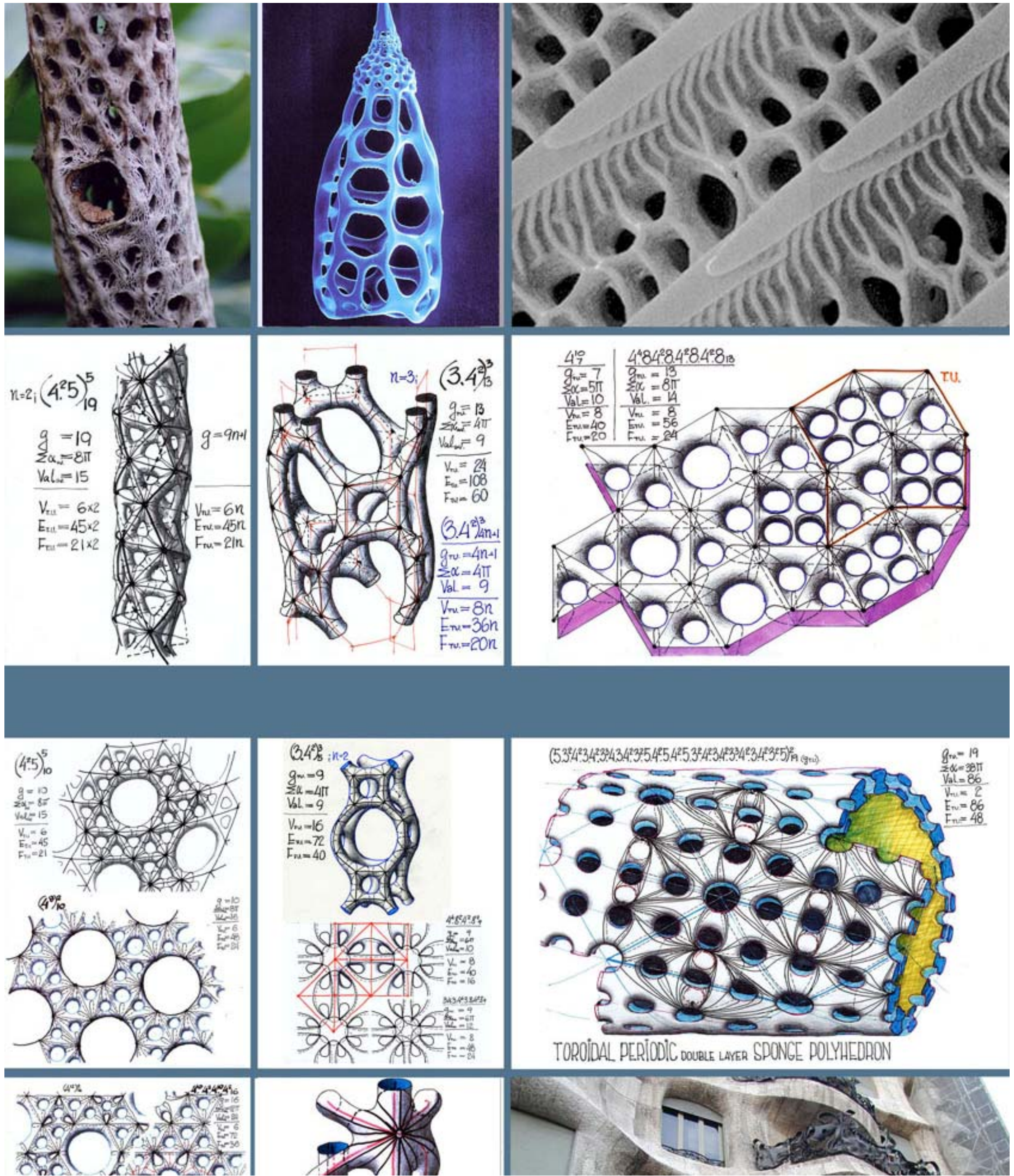


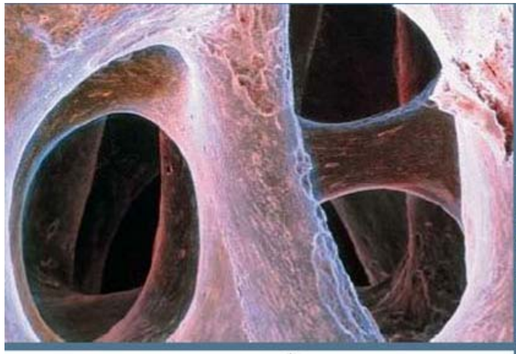
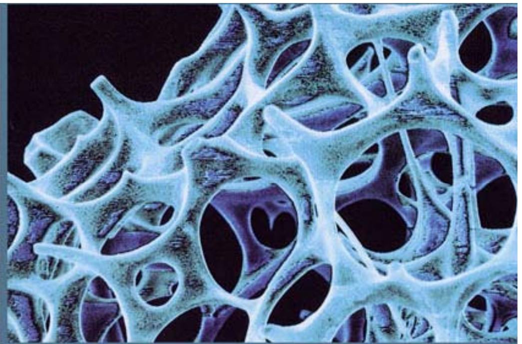
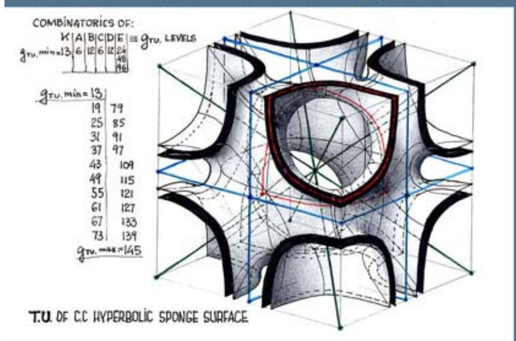
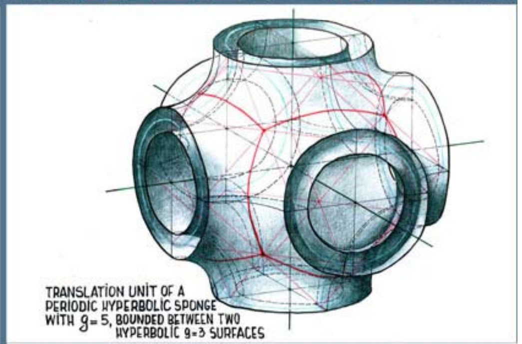
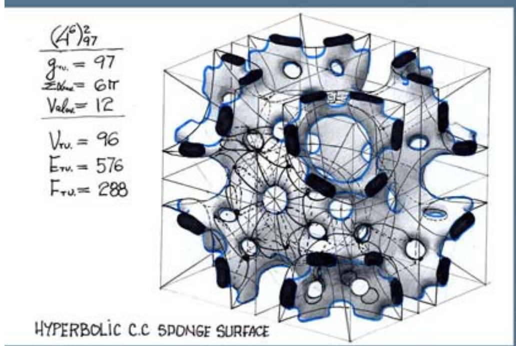
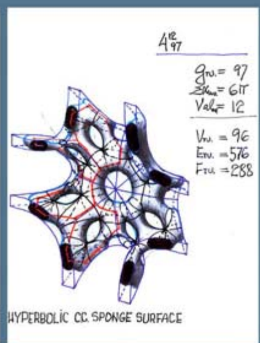
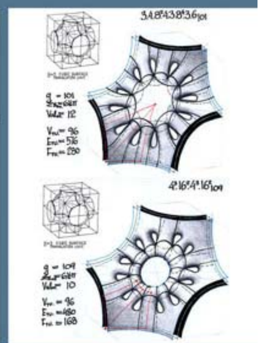
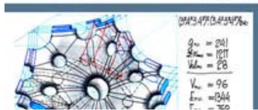
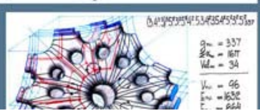

UNIFORM SPHERICAL SPONGE POLYHEDRA

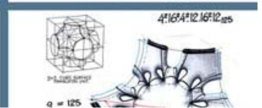


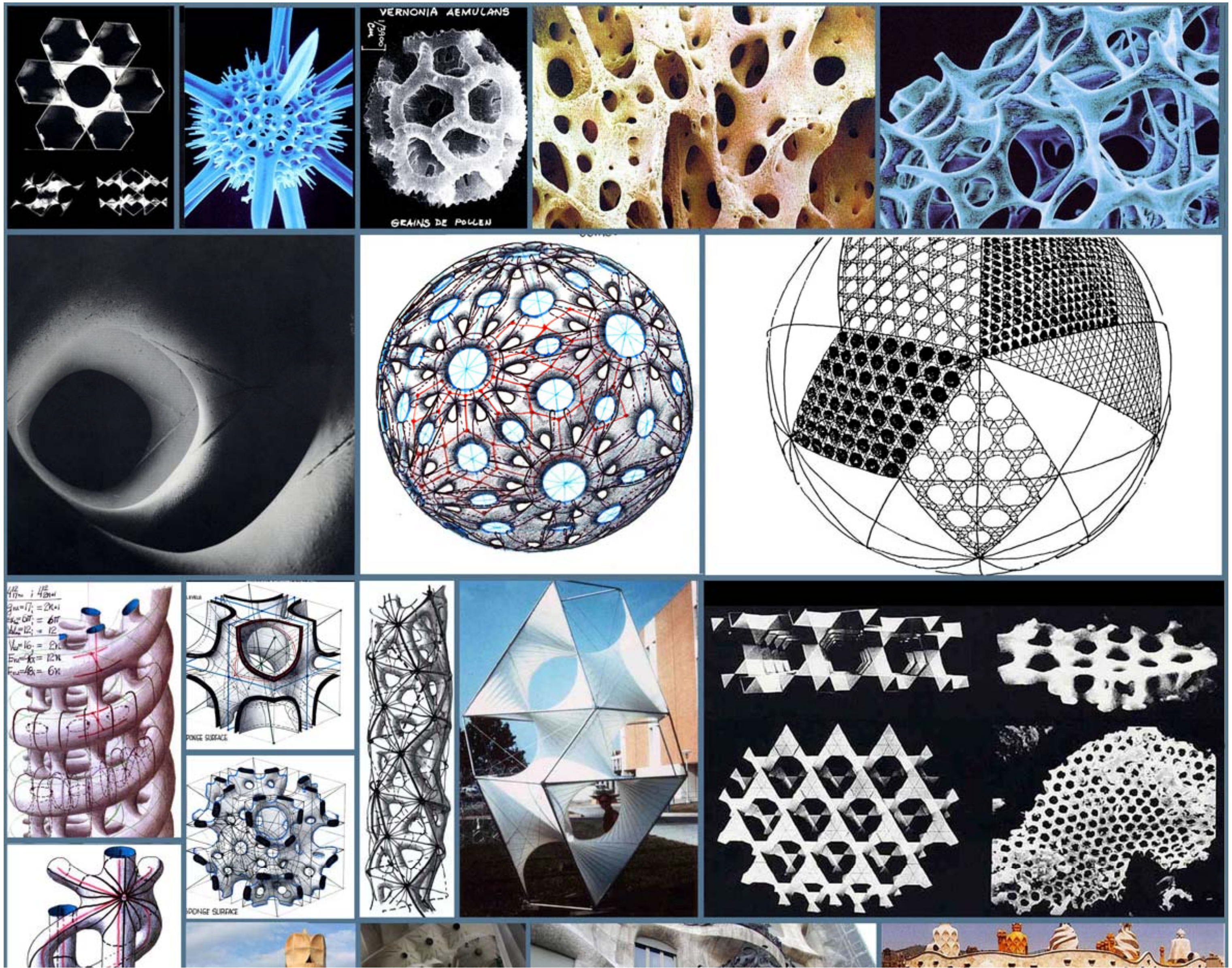


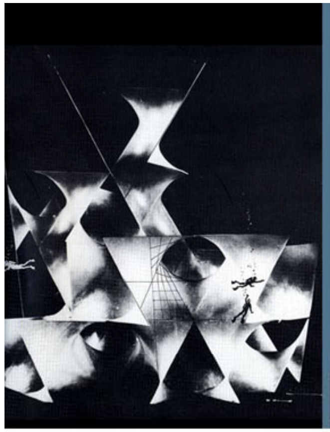


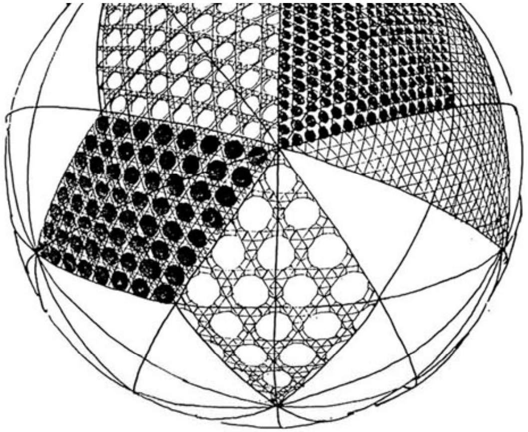

g = 9n+1
T.U.
TOROIDAL PERIODIC DOUBLE LAYER SPONGE POLYHEDRON

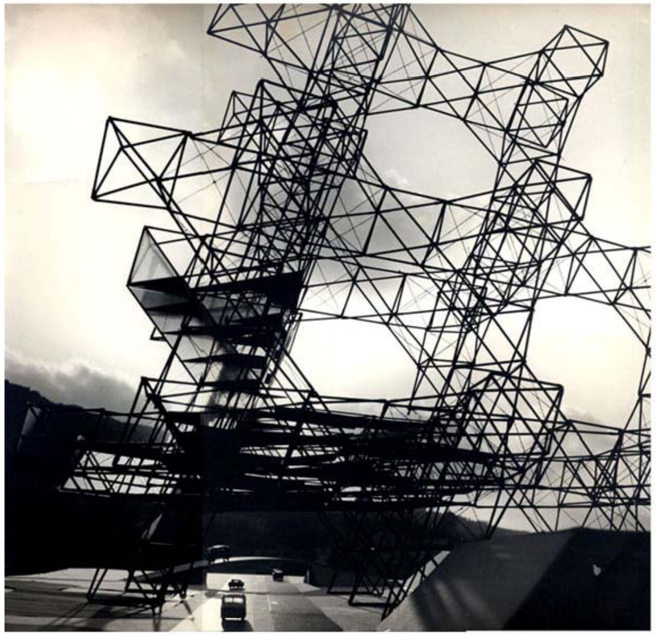



COMBINATORICS OF:
g TU. min = 13
19 79
25 85
31 91
37 97
43 109
49 115
55 121
61 127
67 133
73 139
g TU. max = 145
T.U. OF C.C HYPERBOLIC SPONGE SURFACE

TRANSLATION UNIT OF A
PERIODIC HYPERBOLIC SPONGE
WITH g = 5, BOUNDED BETWEEN TWO
HYPERBOLIC g = 3 SURFACES

g TU. = 97
Valnc = 12
V TU. = 96
E TU. = 576
F TU. = 288
HYPERBOLIC C.C SPONGE SURFACE

g TU. = 97
Valnc = 12
V TU. = 96
E TU. = 576
F TU. = 288
HYPERBOLIC CC SPONGE SURFACE




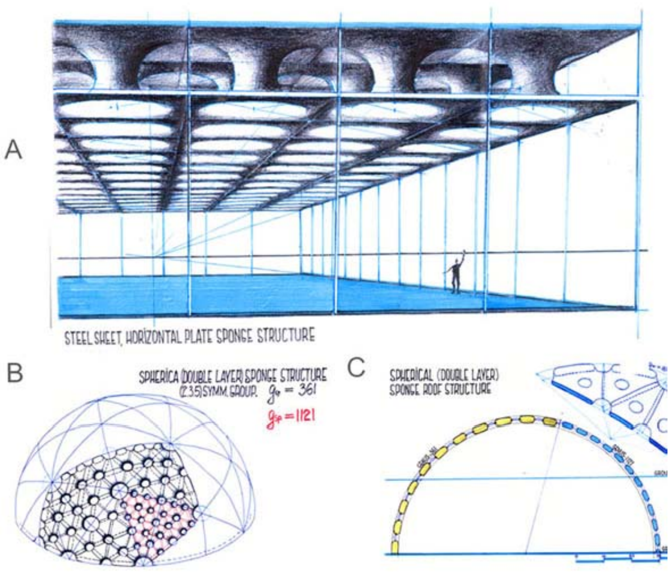


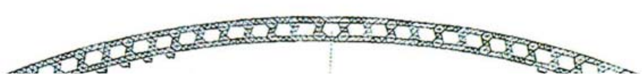

VERNONIA AEMULANS
GRAINS DE POLLEN








A
STEEL SHEET, HORIZONTAL PLATE SPONGE STRUCTURE
B
SPHERICA (DOUBLE LAYER) SPONGE STRUCTURE
(2,3,5) SYMM. GROUP. gp = 361
gp = 1121
C
SPHERICAL (DOUBLE LAYER)
SPONGE ROOF STRUCTURE



J

ignificant venture into **the field of periodic sponge surfaces and hedra** dictates a **systematic exploration of the uniform space lattice ain.**

ne as a shocking surprise to realize that in spite of the great efforts of the last centuries or so, in the exploration of the structure of matter and space allography included), **no systematic effort was committed to exhaustively re the network domain in the "abstract realm of the theoretically nable".**

K

Valency appears to be the most conspicuous and domineering characteristic of the 3-dimensional uniform networks, of the 'Trinity' type.

a) Anything less than Val.=3 does not lead to a construction of a space lattice, and –

b) There cannot be anything of a higher valency value than the dodecavalent Octet (close-packing of $3^3_0+3^4_0$) space lattice (?!).

L

**ial density** (in terms of a/a3) of the space lattice (the number of edges of ength of **<u>a</u>** per cubic volume of **a3**). As a referential basis we should have in that the spatial density of the tetravalent diamond lattice is ~ 1,299a/a3; of the hexavalent cubic lattice is 3,000 a/a3 and that of the dodecavalent lattice is ~ 8,485 a/a3 (By density we refer to the lowest possible value for tinct topology). **It is of great theoretical interest and probably even of tical importance how far down and up can the density values descend spire.**

= 6

= 2⅚ π

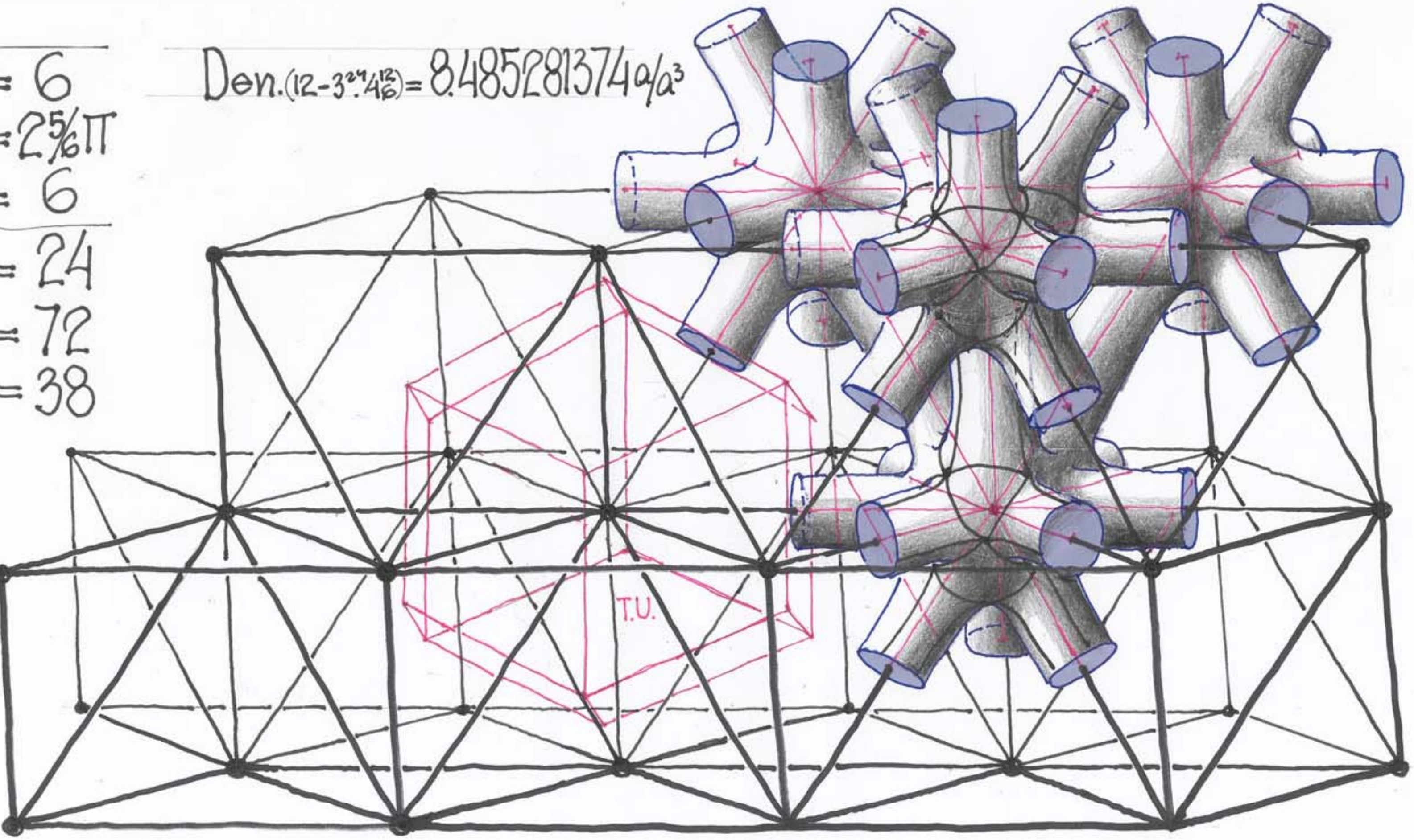
= 6

= 24

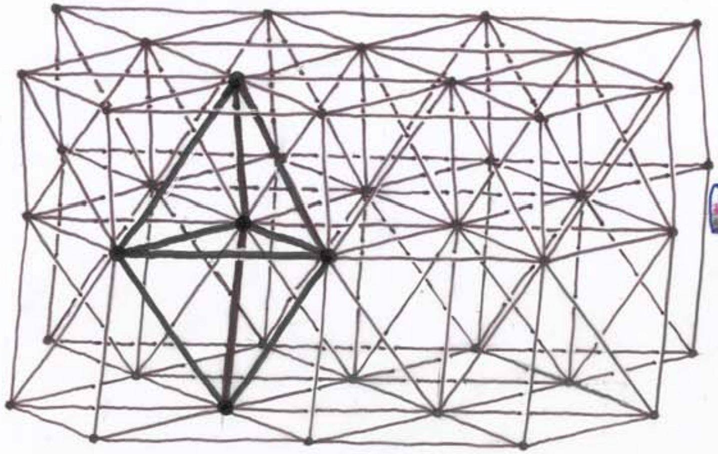
= 72

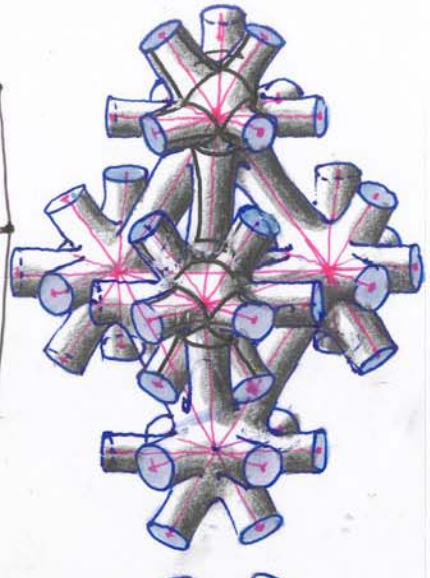
= 38

Den.(12-3$^{24}$.4$^{12}_{6}$) = 8.485281374 a/a$^3$



# VARIANTS A & B.





$$Den_{(12-3^{24}_{6})} = 8.4852281374\ a/a^3$$

$3.4^5_6$

$g_{TU} = 6$

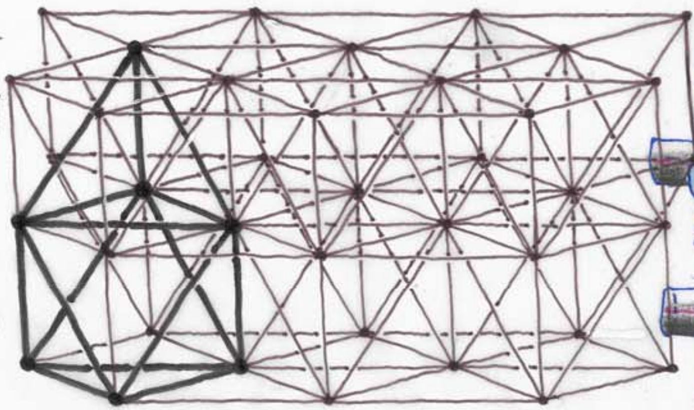
$\Sigma\alpha = 2\frac{5}{6}\pi$

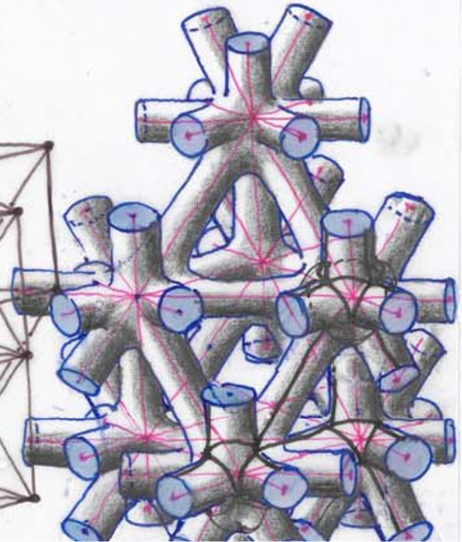
$Val = 6$

$V_{TU} = 24$

$E_{TU} = 72$

$F_{TU} = 38$





M

Although it is still too early to establish all the possible interrelations, it seems that the parameters - Val.; Den.; $\mathbf{C}_{T.U.}$ or $\mathbf{g}_{T.U.}$ and $\sum \alpha$, are capturing the essence of the related topological-geometrical phenomenon.

N

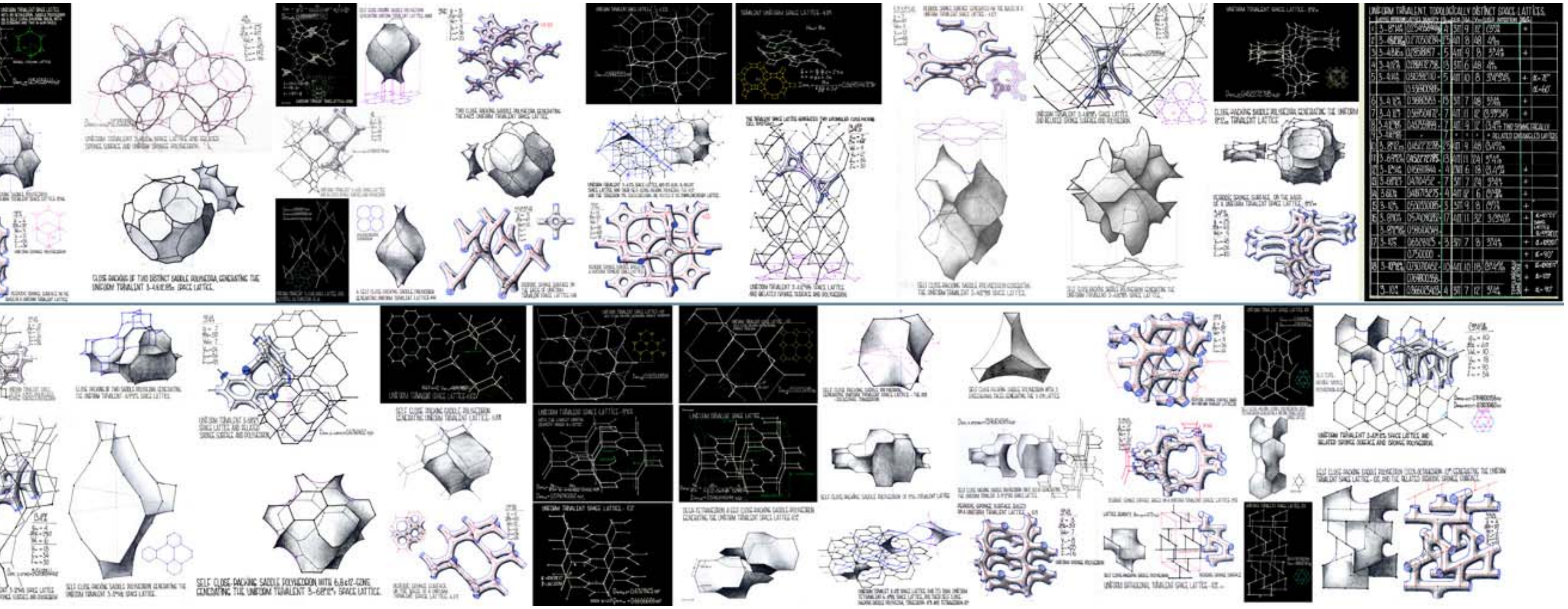
An assumption is formed that we are dealing with probably not more than few **hundreds of uniform space lattices in 3-D space and in view of the valency limiting values and symmetry constraints it seems that an exhaustive systematic search of these configurations is tenable.**

O

**Uniform Trivalent Space Lattices**

A.F. Wells in his monumental work on Structural Inorganic Chemistry (-1962) started that…."The theory of these nets does not appear to be known and in fact no attempt to derive them systematically seems to have been made until comparatively recently (P.100). As a result of these "recent" attempts he lists **three,** 3-D 3- connected nets... "in which all the smallest circuits are 10-gons"… One of these was announced again by A.F. Wells in 1977 and rediscovered by Toshikazu Sunada (Feb. notices of the American Mathematical society – 2008).

Looking into the issue (Jan-2008) it was quite surprising to


UNIFORM TRIVALENT TOPOLOGICALLY DISTINCT SPACE LATTICES

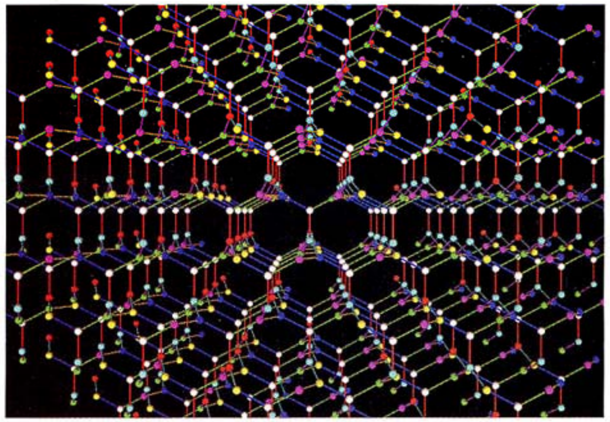
GEOMETRY

# Crystal Math

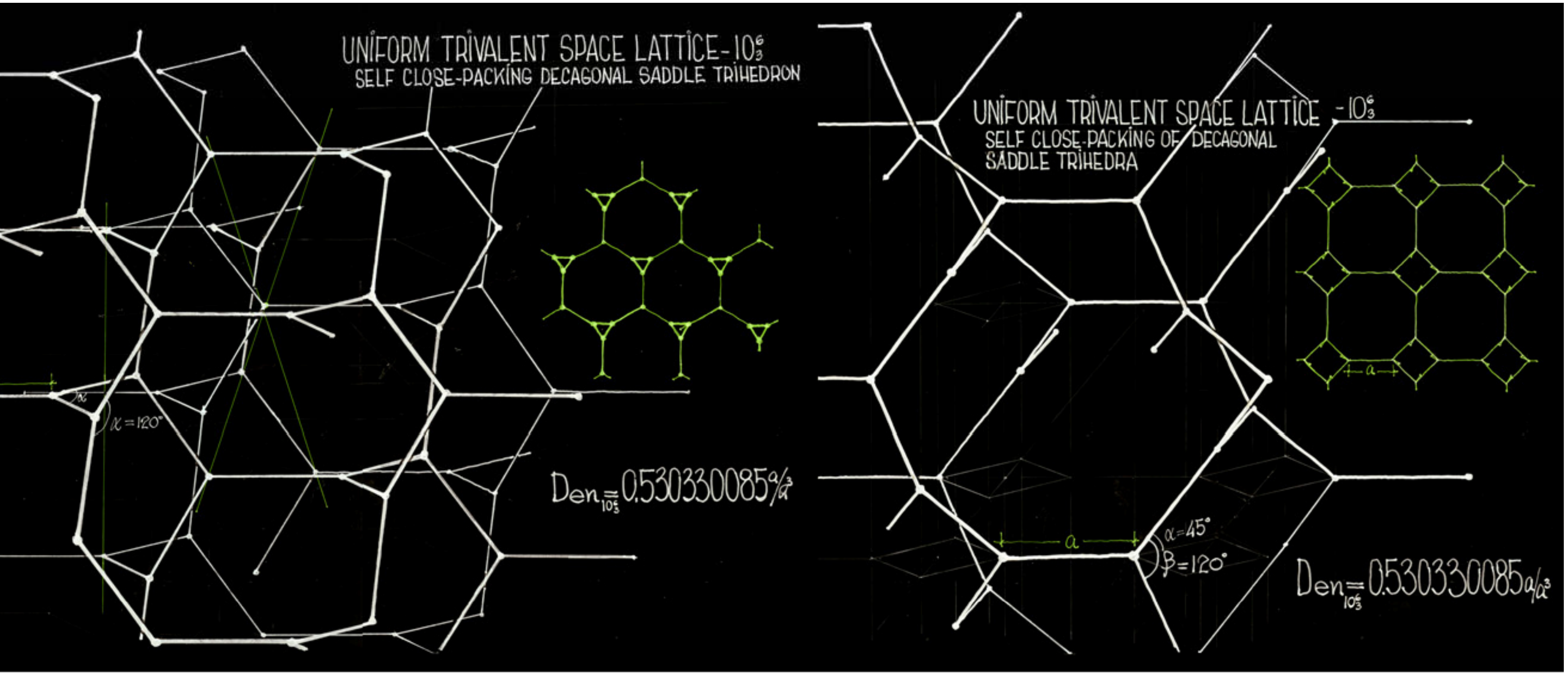
Diamonds are rarities not just on earth but also mathematically. The crystal structure of diamond has two key distinguishing properties, notes mathematician Toshikazu Sunada of Meiji University in Japan. It has maximal symmetry, which means that its components cannot be rearranged to make it any more symmetrical than it is, and a strong isotropic property, which means that it looks the same when viewed from the direction of any edge. In the February *Notices of the American Mathematical Society,* Sunada finds that out of an infinite universe of crystals that can exist mathematically, just one other shares these properties with diamond. Whereas diamond is a web of hexagonal rings, its cousin is made of 10-sided rings.

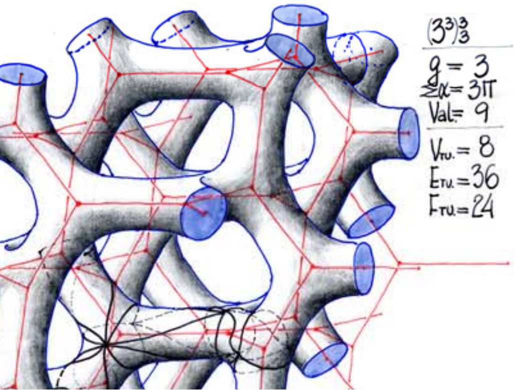
Sunada had originally thought that no one had described this object be-

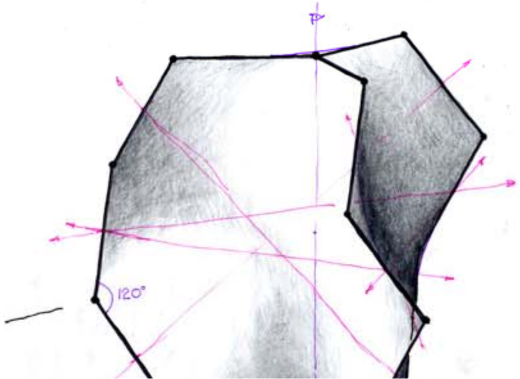
"I rediscovered the crystal structure mathematically in rather an accidental way" while working on another problem, Sunada says. After his paper was published, chemists and crystallographers informed him that they had long known about the crystal, which was called (10,3)-a by A. F. Wells in 1977. Diamond's mathematical twin can exist in a slightly distorted form as an arrangement of silicon atoms in strontium silicide. —*Charles Q. Choi*

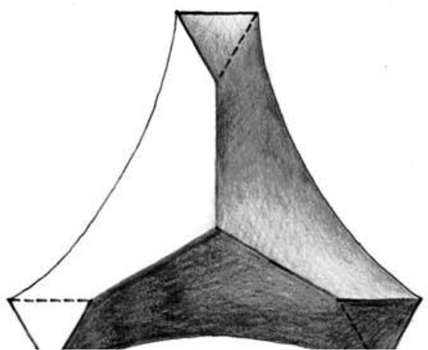


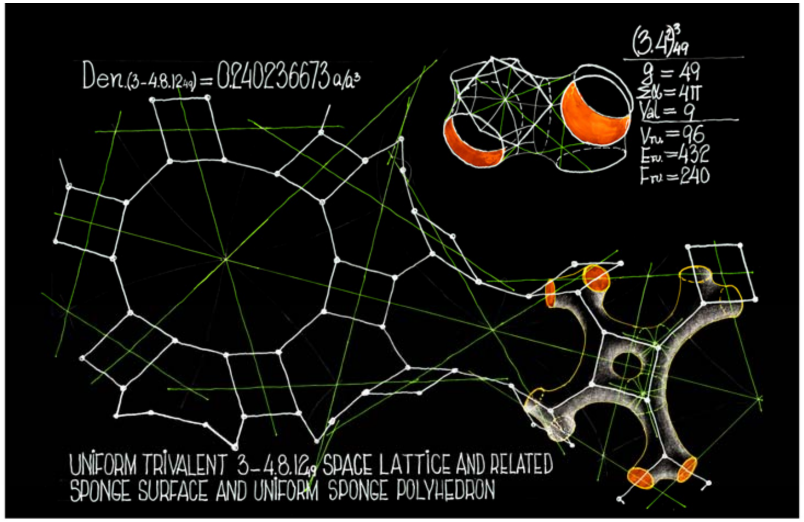
**TWO OUT OF INFINITY: Diamond and the K4, or (10,3)-a,**

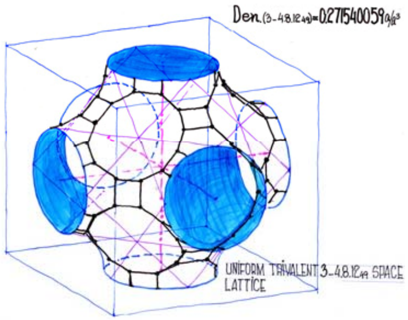

UNIFORM TRIVALENT SPACE LATTICE - $10^6_3$
SELF CLOSE-PACKING DECAGONAL SADDLE TRIHEDRON
$\alpha = 120°$
$Den_{10^6_3} = 0.530330085 a/a^3$
UNIFORM TRIVALENT SPACE LATTICE - $10^6_3$
SELF CLOSE-PACKING OF DECAGONAL SADDLE TRIHEDRA
a
$\alpha = 45°$
$\beta = 120°$
$Den_{10^6_3} = 0.530330085 a/a^3$

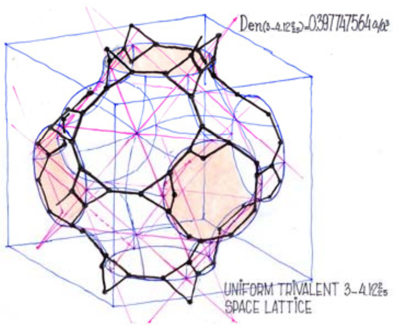

$(3^3)^3_3$
$g = 3$
$\Sigma\alpha = 3\pi$
$Val. = 9$
$V_{ru.} = 8$
$E_{ru.} = 36$
$F_{ru.} = 24$

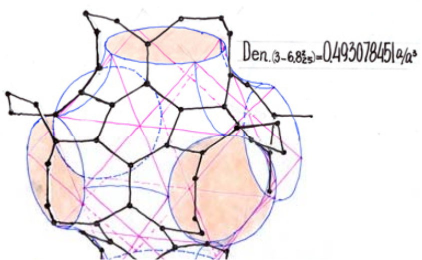

120°

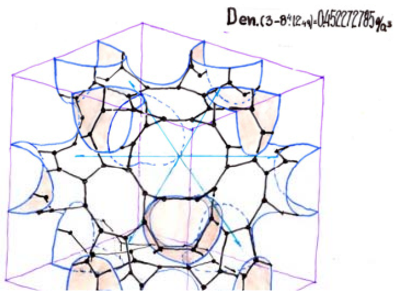



Den.(3-4.8.12₄₉) = 0.240236673 a/a³
(3.4⁹)³₄₉
g = 49
Σα = 4π
Val = 9
Vru = 96
Eru = 432
Fru = 240
UNIFORM TRIVALENT 3-4.8.12₄₉ SPACE LATTICE AND RELATED
SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON


Den.(3-4.8.12₄₉)=0.271540059 a/a³
UNIFORM TRIVALENT 3-4.8.12₄₉ SPACE
LATTICE


Den(3-4.12)=0.397747564 a/a³
UNIFORM TRIVALENT 3-4.12
SPACE LATTICE


Den.(3-6.8)=0.493078451 a/a³

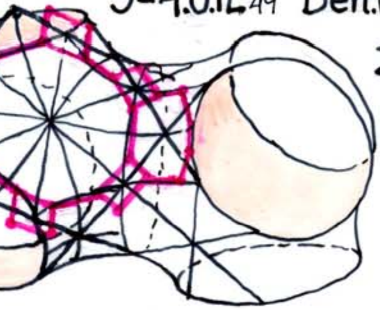

Den.(3-8.12)=0.452272785 a/a³

$3\text{-}6.8^2_{49}$ $\text{Den.}_{(3\text{-}6.8^2_{49})} = 0.47140452\ a/a^3$

$\Sigma\alpha = 390°$

$Con_{T.U.} \equiv (g_{T.U.}) = 49$

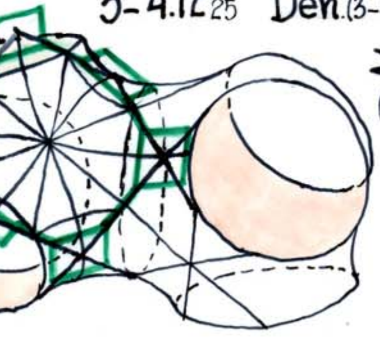
$3\text{-}4.8.12_{49}$ $\text{Den.}_{(3\text{-}4.8.12_{49})} = 0.240236673\ a/a^3$



$\Sigma\alpha = 375°$

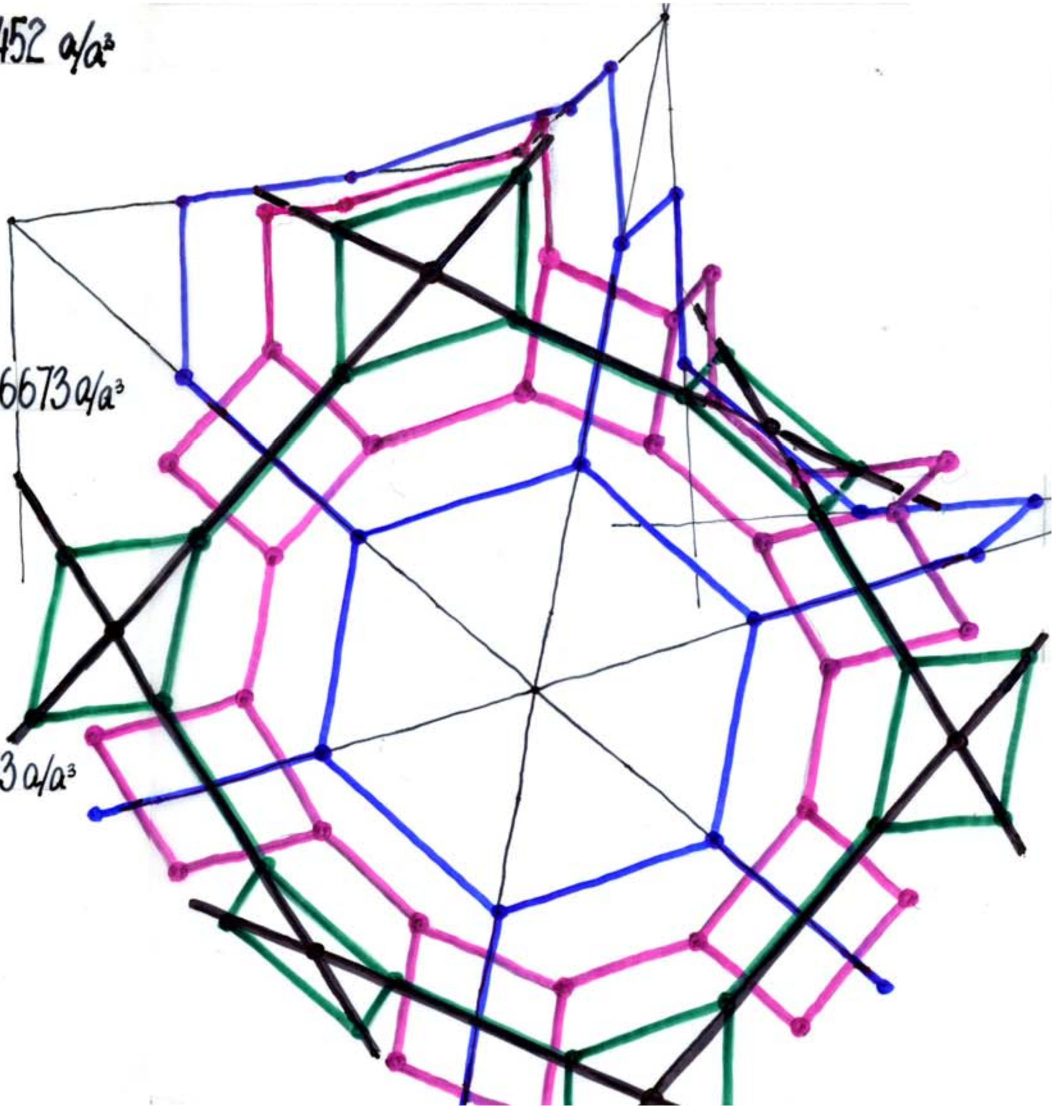
$Con_{T.U.} \equiv (g_{T.U.}) = 49$

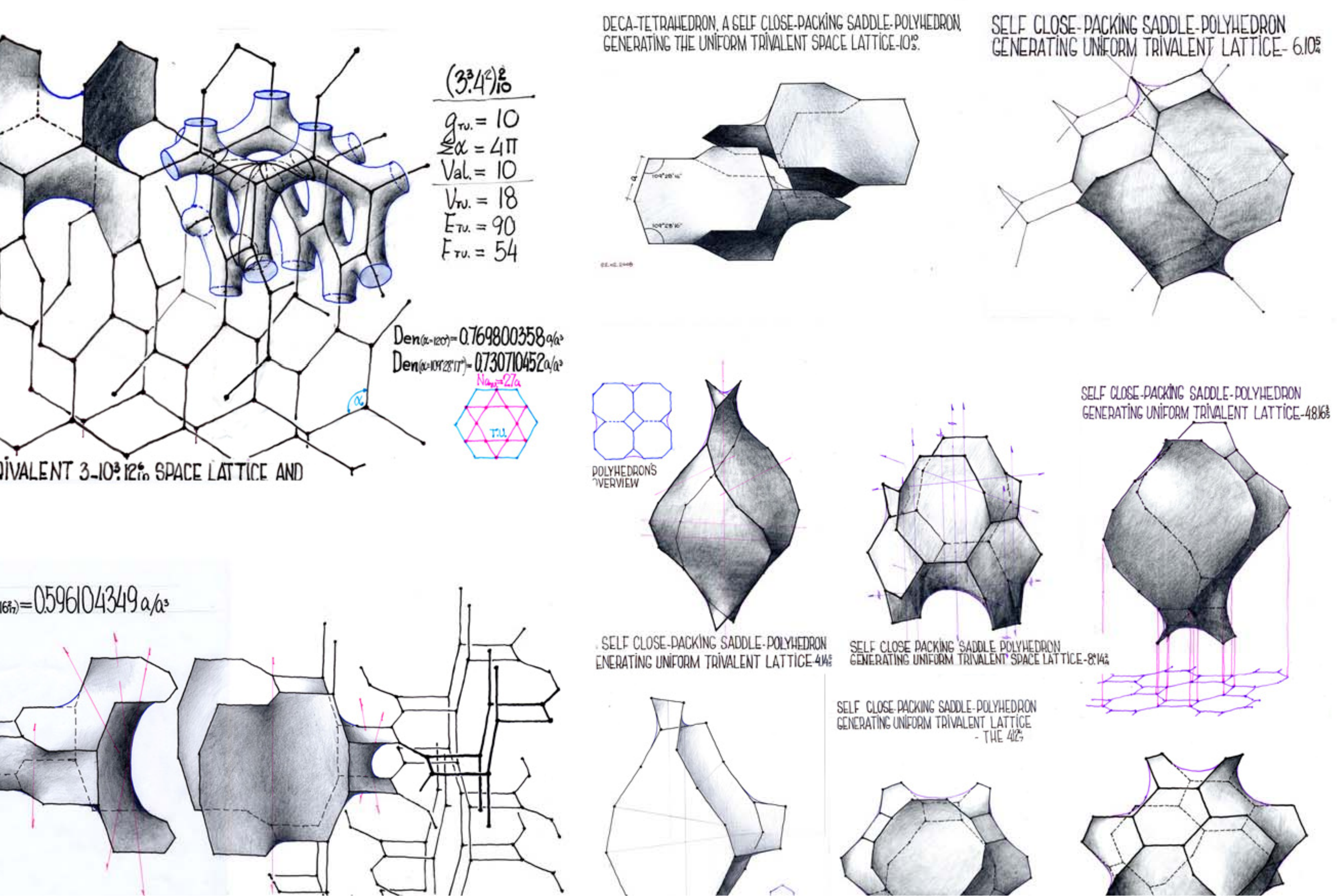
$3\text{-}4.12^2_{25}$ $\text{Den.}_{(3\text{-}4.12^2_{25})} = 0.319805153\ a/a^3$



$\Sigma\alpha = 390°$

$Con_{T.U.} \equiv (g_{T.U.}) = 25$

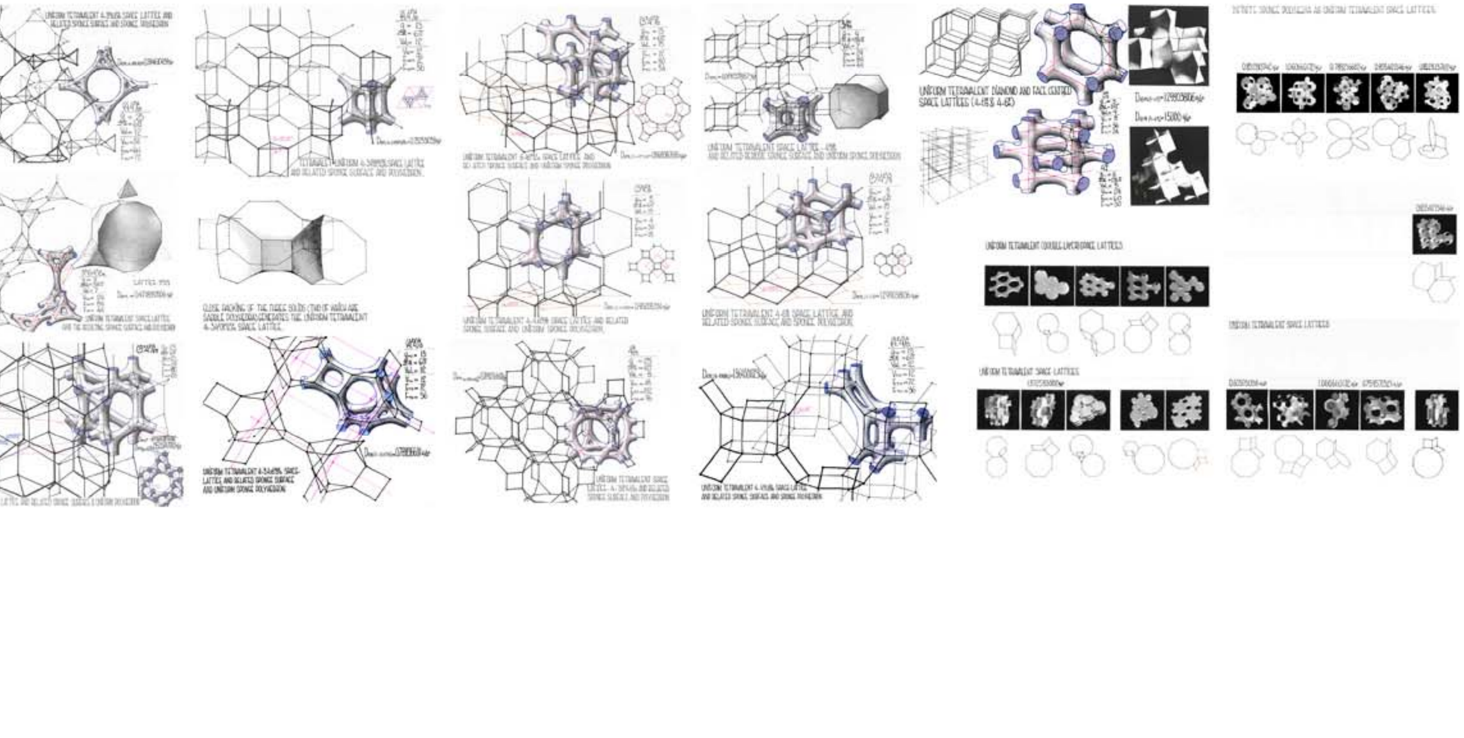


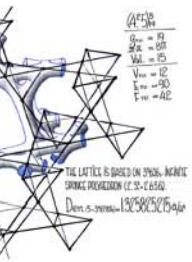


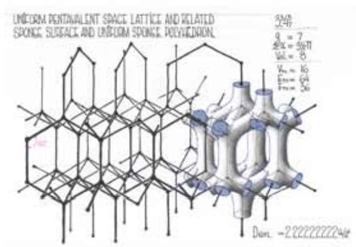
(3³4²)
g TU. = 10
Σα = 4π
Val. = 10
V TU. = 18
E TU. = 90
F TU. = 54
Den(α=120°) = 0.769800358 a/a³
Den(α=109°28'17") = 0.730710452 a/a³
Na = 27a
T.U.
α
TRIVALENT 3-10 SPACE LATTICE AND
= 0.596104349 a/a³
DECA-TETRAHEDRON, A SELF CLOSE-PACKING SADDLE-POLYHEDRON,
GENERATING THE UNIFORM TRIVALENT SPACE LATTICE-10
SELF CLOSE-PACKING SADDLE-POLYHEDRON
GENERATING UNIFORM TRIVALENT LATTICE- 6.10
POLYHEDRON'S
OVERVIEW
SELF CLOSE-PACKING SADDLE-POLYHEDRON
ENERATING UNIFORM TRIVALENT LATTICE-4.14
SELF CLOSE PACKING SADDLE POLYHEDRON
GENERATING UNIFORM TRIVALENT SPACE LATTICE-8.14
SELF CLOSE-PACKING SADDLE-POLYHEDRON
GENERATING UNIFORM TRIVALENT LATTICE-4.8.16
SELF CLOSE-PACKING SADDLE-POLYHEDRON
GENERATING UNIFORM TRIVALENT LATTICE
- THE 4.12

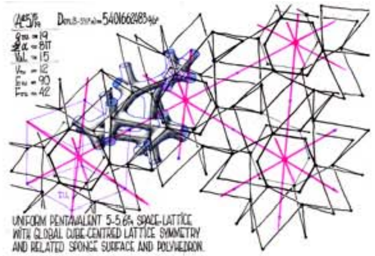
# UNIFORM TRIVALENT, TOPOLOGICALLY DISTINCT SPACE-LATTICES

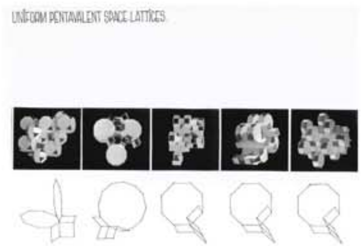
| | NOTATION | LATTICE CONNECTIVITY | | SPATIAL DENSITY | |
|---|---|---|---|---|---|
| 1 | $3-4.8.12_{49}$ | 49 | D.surf. | $0.240236673\,a/a^3$ | THE LOWEST DENSITY OF A UNIFORM SPACE LATTICE |
| | | | C.surf. | $0.271540059\,a/a^3$ | |
| 2 | $3-8^4.14^2_4$ | 4 | $(3.12^2)$ | $0.254558441\,a/a^3$ | |
| 3 | $3-4.8.12.18^2_{25}$ | 25 | C.C. | $0.270501284\,a/a^3$ | |
| 4 | $3-4.8.16_5$ | 5 | $(4.8^2)$ | $0.285181197\,a/a^3$ | |
| 5 | $3-4.12^5_{13}$ | 13 | F.C. | $0.288972738\,a/a^3$ | |
| 6 | $3-4.14^5_5$ | 5 | $(4.8^2)$ | $0.310392110\,a/a^3$ | |
| | | | $(4.8^2)$ | $0.336900926\,a/a^3$ | |
| 7 | $3-4.12^4_{13}$ | 13 | C.C. | $0.319805153\,a/a^3$ | |
| 8 | $3-4.12^4_7$ | 7 | Hex. | $0.369504172\,a/a^3$ | |
| 9 | $3-4.12^2_{25}$ | 25 | C. | $0.397747564\,a/a^3$ | |
| 10 | $3-4.12^3.18^2_7$ | 7 | Hex. | $0.437559199\,a/a^3$ | |
| 11 | $3-4.16^4.18^2_7$ | 7 | Hex. | $0.437559199\,a/a^3$ | |
| 12 | $3-8^4.12_{49}$ | 49 | F.C. surf. | $0.452272785\,a/a^3$ | |
| 13 | $3-6.9^3.12^2_{13}$ | 13 | Tet. | $0.452272785\,a/a^3$ | |
| 14 | $3-12^4.14^6_4$ | 4 | Hex. | $0.456911844\,a/a^3$ | |
| 15 | $3-6.8^2.12^2_7$ | 7 | Tet. | $0.4710452\,a/a^3$ | |
| 16 | $3-6.10^5_4$ | 4 | Hex. | $0.481713275\,a/a^3$ | |
| 17 | $3-6.8^2_{25}$ | 25 | C. | $0.493078451\,a/a^3$ | |
| 18 | $3-10^6_3$ | 3 | $(3.12^2)$ $(4.8^2)$ | $0.530330085\,a/a^3$ | |
| 19 | $3-8^3.10^2_{17}$ | 17 | Orth. | $0.574090262\,a/a^3$ | |
| 20 | $3-8^3.12^2.16^2_{17}$ | 17 | Orth. | $0.596104349\,a/a^3$ | |
| 21 | $3-10^{10}_3$ | 3 | Orth. | $0.6328125\,a/a^3$ | |
| | | | " | $0.7500000\,a/a^3$ | |

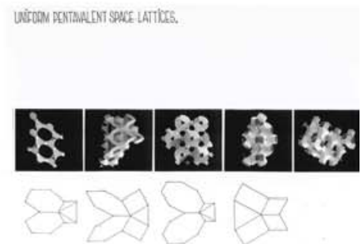

UNIFORM TETRAVALENT DIAMOND AND FACE CENTRED
UNIFORM TETRAVALENT SPACE LATTICES

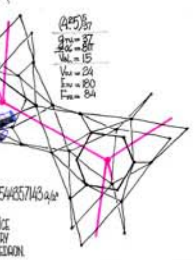


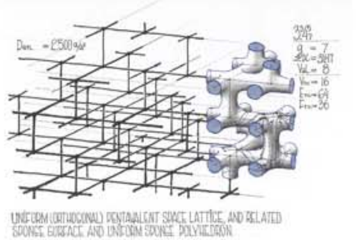

UNIFORM PENTAVALENT SPACE LATTICE AND RELATED
SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

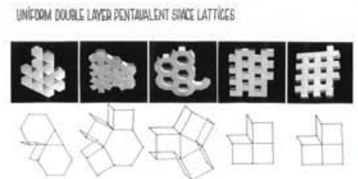

UNIFORM PENTAVALENT 5-5 6^4 SPACE-LATTICE
WITH GLOBAL CUBE-CENTRED LATTICE SYMMETRY
AND RELATED SPONGE SURFACE AND POLYHEDRON.

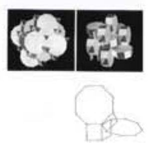

UNIFORM PENTAVALENT SPACE LATTICES.

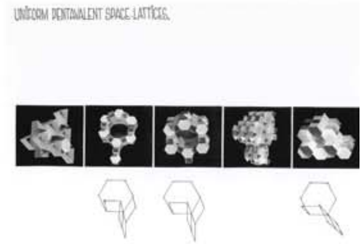

UNIFORM PENTAVALENT SPACE LATTICES.

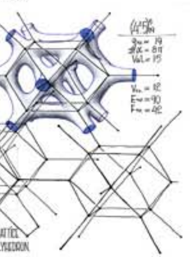


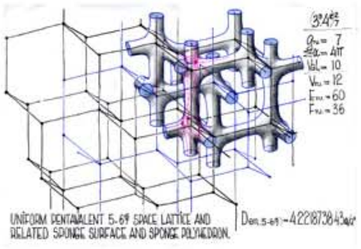

UNIFORM (ORTHOGONAL) PENTAVALENT SPACE LATTICE, AND RELATED
SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

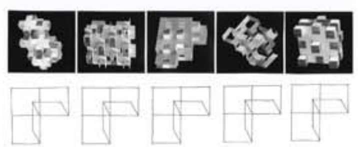

UNIFORM DOUBLE LAYER PENTAVALENT SPACE LATTICES

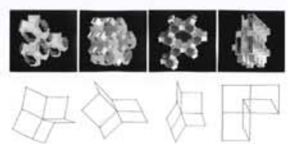


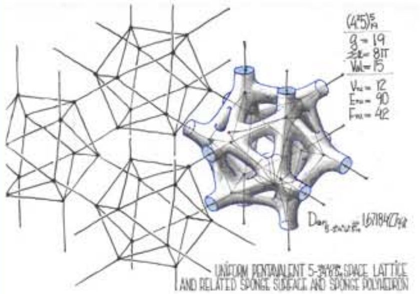

UNIFORM PENTAVALENT SPACE LATTICES.

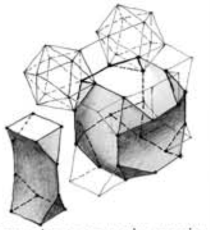



UNIFORM PENTAVALENT 5-6^9 SPACE LATTICE AND
RELATED SPONGE SURFACE AND SPONGE POLYHEDRON.

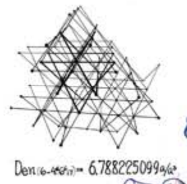




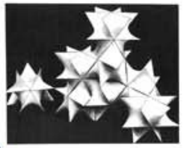


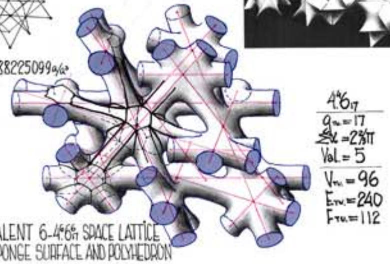



THREE SOLIDS, CLOSE PACKING OF WHICH GENERATES THE UNIFORM HEXAVALENT 6-... SPACE LATTICE.



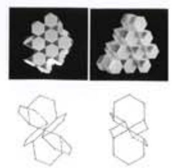

SELF CLOSE PACKING SADDLE POLYHEDRON



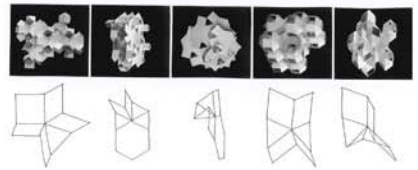



UNIFORM HEXAVALENT 6-4.6 SPACE LATTICE AND RELATED SPONGE SURFACE AND POLYHEDRON

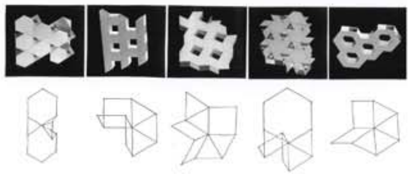
UNIFORM HEXAVALENT SPACE-LATTICES.



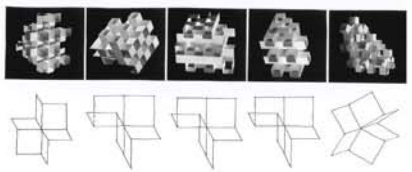
UNIFORM HEXAVALENT SPACE LATTICES.

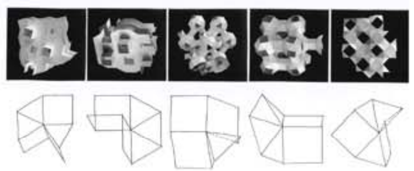


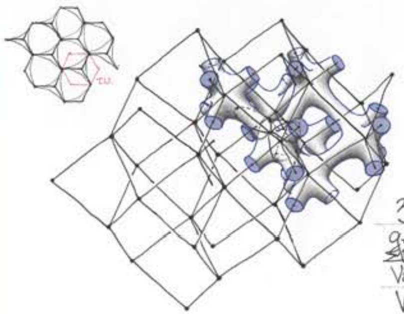
UNIFORM DOUBLE LAYER HEXAVALENT SPACE-LATTICES.

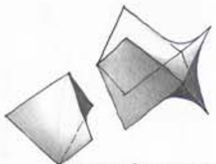


UNIFORM HEXAVALENT SPACE-LATTICES.



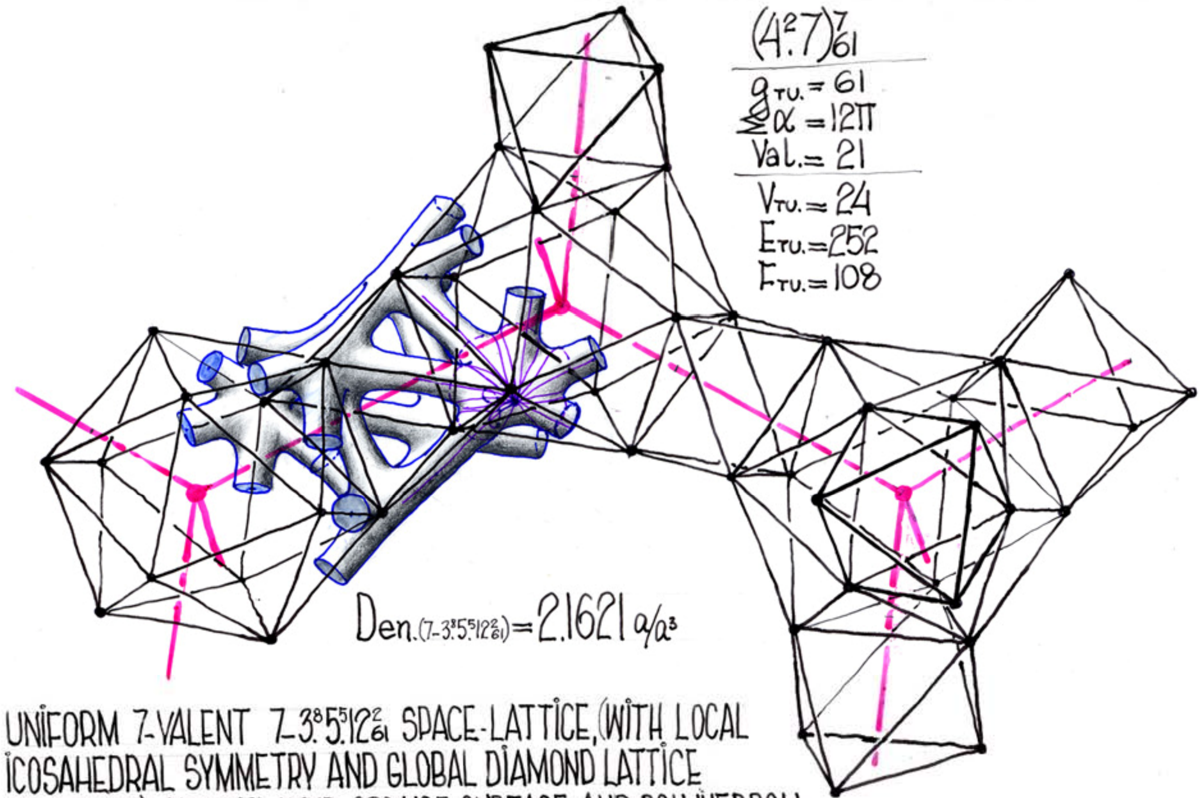




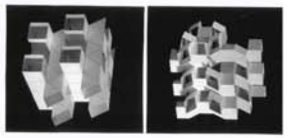



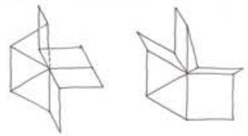
CLOSE PACKING SADDLE POLYHEDRA, GENERATING THE SPACE LATTICE.

UNIFORM HEXAVALENT 6-(4.6)^6_5 SPACE LATTICE AND RELATED PERIODIC SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

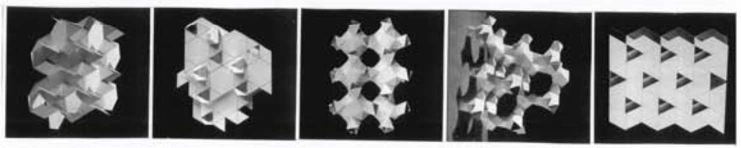


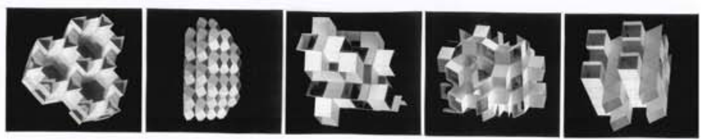

UNIFORM 7-VALENT $7\text{-}3^8.5^5.12^2_{61}$ SPACE-LATTICE, (WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL DIAMOND LATTICE SYMMETRY) AND RELATED SPONGE SURFACE AND POLYHEDRON.

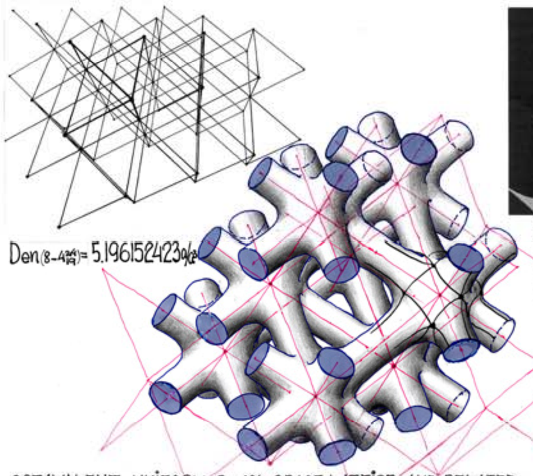


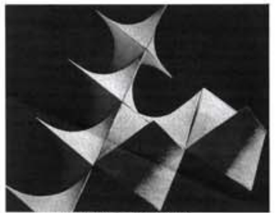


UNIFORM SEPTAVALENT SPACE-LATTICES.







SELF CLOSE PACKING
SADDLE POLYHEDRON,
GENERATING THE LATTICE.

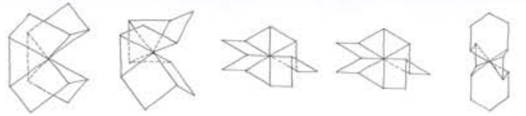
$4^{5}_{19}$

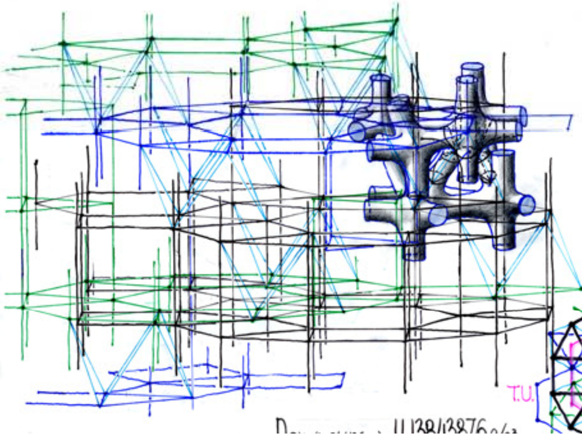
$g_{T.U.} = 19$
$\Sigma\alpha = 2\tfrac{1}{2}\pi$
Val. = 5

$V_{T.U.} = 144$
$E_{T.U.} = 360$
$F_{T.U.} = 180$

OCTAVALENT UNIFORM $8\text{-}4^{24}_{19}$ SPACE LATTICE AND RELATED
SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON

UNIFORM OCTAVALENT SPACE-LATTICES.

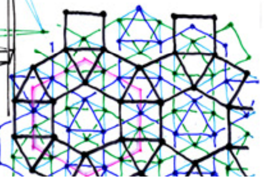




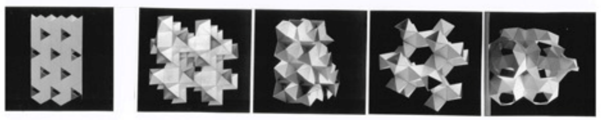
$4^{2}.3^{3}.4^{2}.(4^{2}.6)^{3}_{55}$

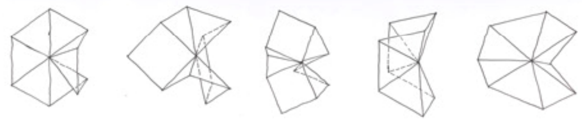
$g_{T.U.} = 55$
$\Sigma\alpha = 8\pi$
Val. = 16

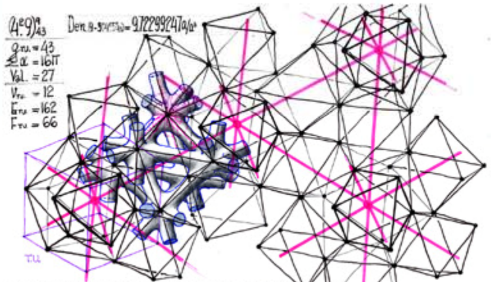
$V_{T.U.} = 36$
$E_{T.U.} = 288$
$F_{T.U.} = 144$



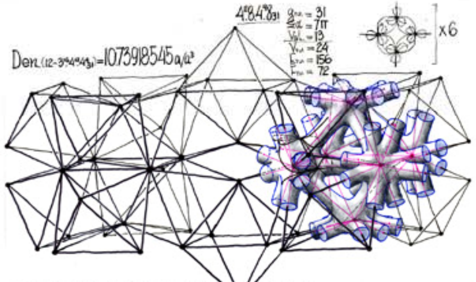

UNIFORM OCTAVALENT SPACE-LATTICES.



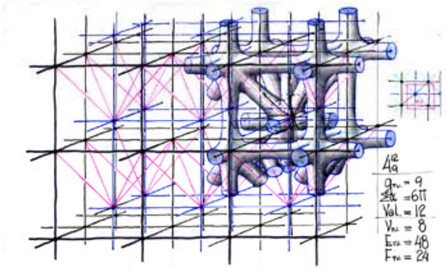




UNIFORM 9-VALENT 9-3…4…5… SPACE-LATTICE (WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL CUBE-CENTRED LATTICE SYMMETRY) AND REL.

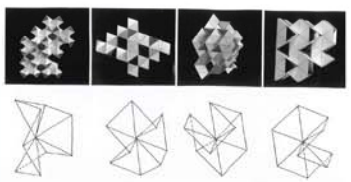


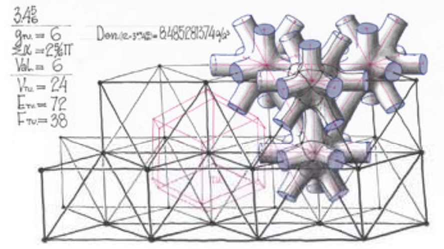
UNIFORM DODECAVALENT 12-3…4…4… SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.



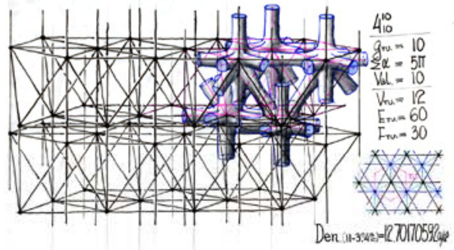
UNIFORM DECAVALENT 10-3…4… SPACE LATTICE AND RELATED SPONGE SURFACE AND SPONGE POLYHEDRA

UNIFORM 9-VALENT SPACE-LATTICES.



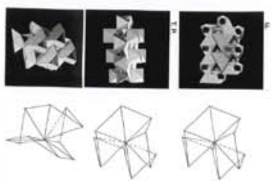


UNIFORM DODECAVALENT 12-3…4… SPACE LATTICE (OCTET LATTICE) AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON.

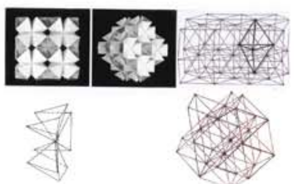


UNIFORM 11-VALENT 11-3…4… SPACE LATTICE AND RELATED SPONGE SURFACE AND UNIFORM SPONGE POLYHEDRON

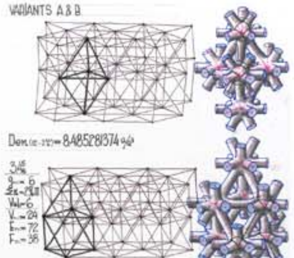
UNIFORM DECAVALENT SPACE-LATTICES.



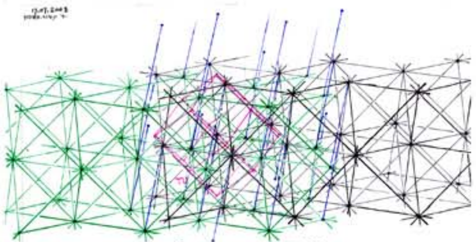
UNIFORM DODECAVALENT SPACE-LATTICES.



VARIANTS A & B.



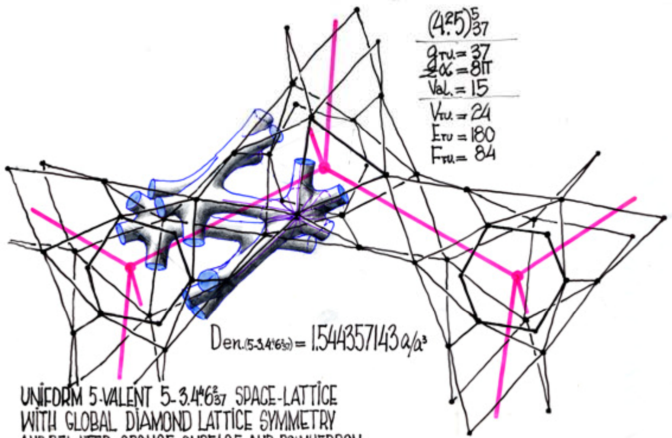
UNIFORM DODECAVALENT 12-3… SPACE LATTICE (OCTET) AND RELATED SPONGE SURFACES AND SPONGE POLYHEDRA.



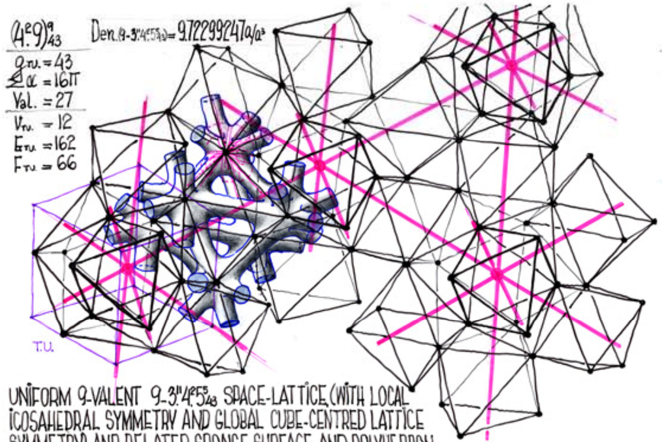
DENSITY = 18.38477631 $a/a^3$

TWO INTER-PENETRATING UNIFORM OCTET SPACE LATTICES WITH EDGE-a, WHEN JOINED TOGETHER WITH A SET OF PARALLEL a-EDGES, GENERATE A UNIFORM 13-VALENT SPACE LATTICE.

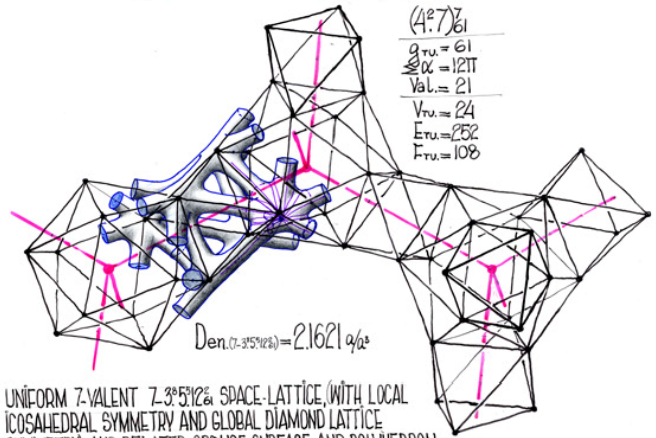
RM CRYSTALLINE SPACE LATTICES
CAL REPEATING ICOSAHEDRAL SYMMETRY



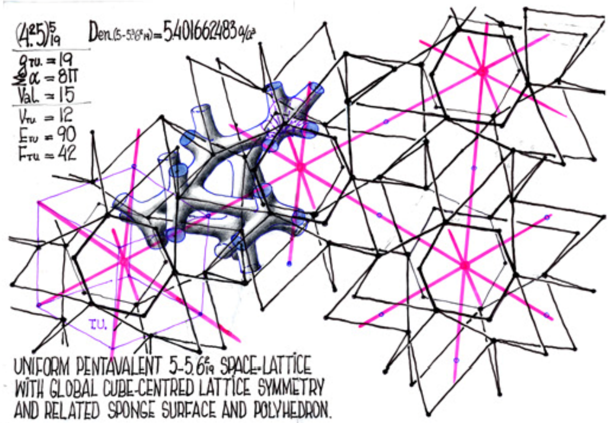

UNIFORM 5-VALENT $5\text{-}3.4^4 6^2_{37}$ SPACE-LATTICE WITH GLOBAL DIAMOND LATTICE SYMMETRY AND RELATED SPONGE SURFACE AND POLYHEDRON.



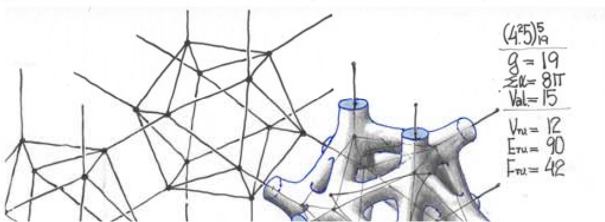

UNIFORM 9-VALENT $9\text{-}3^{11}4^6 5^5_{43}$ SPACE-LATTICE, (WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL CUBE-CENTRED LATTICE SYMMETRY), AND RELATED SPONGE SURFACE AND POLYHEDRON.

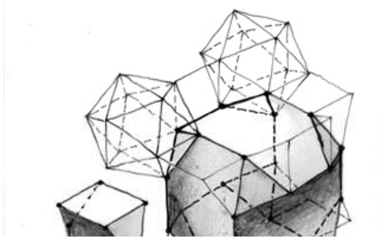



UNIFORM 7-VALENT $7\text{-}3^8 5^5 12^2_{61}$ SPACE-LATTICE, (WITH LOCAL ICOSAHEDRAL SYMMETRY AND GLOBAL DIAMOND LATTICE SYMMETRY) AND RELATED SPONGE SURFACE AND POLYHEDRON.




UNIFORM PENTAVALENT $5\text{-}5.6^2_{19}$ SPACE-LATTICE WITH GLOBAL CUBE-CENTRED LATTICE SYMMETRY AND RELATED SPONGE SURFACE AND POLYHEDRON.



P

**Jniform Dodecavalent and higher valency Space Lattices or: how far alency and spatial density values can go.**

Jniform dodecavalent 'octet' based space lattices exist in more than one opological version, but all come to same spatial density of **8,485281374 /a3.**

The infinite sponge polyhedron 3  gives rise to a uniform dodecavalent **12-34)** space lattice, the density of which **is 10,73918545a/a3 (!)**

Q

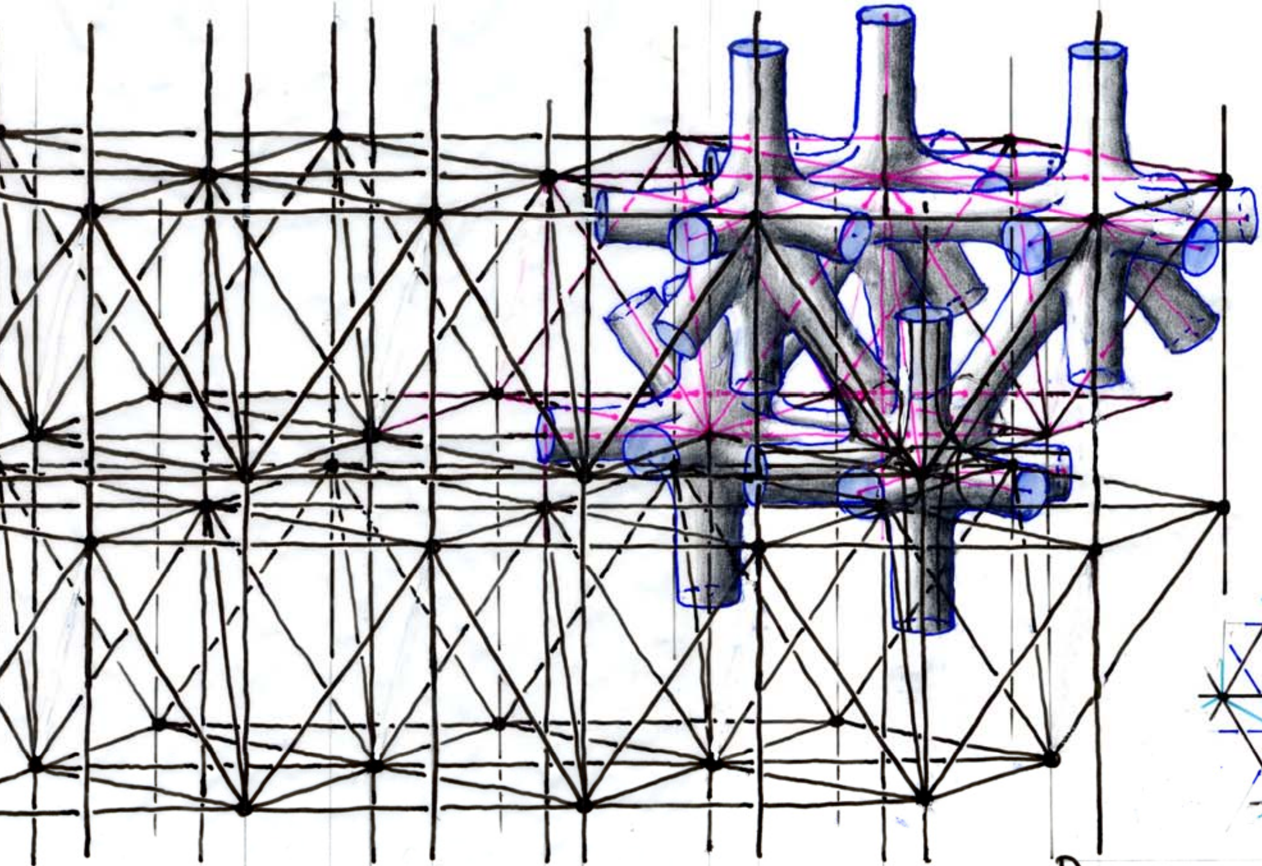
ne quest for higher density networks led to a spatial experiment as llows:

ets perform an **edge-length translation** of a given uniform n-valent ace lattice in an arbitrary direction from position A into position B. ne resulting network is a 4-dimensional feature, the 3-dimensional presentation of which displays a uniform(n+1)- valent lattice as ell.

ne spatial density of the resulting space lattice will be:

$$\mathbf{en.\ (n+1) = \frac{Den.(n)}{E_{T.U.}}(2E_{T.U.}+1),}$$

with $\mathbf{E_{T.U.}}$ as the number of edges thin the translation unit of the lattice.



$4^{10}_{10}$

$g_{TU.} = 10$

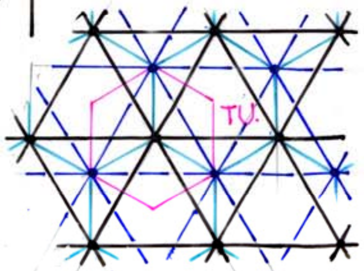
$\sum \alpha = 5\pi$

$Val. = 10$

$V_{TU.} = 12$

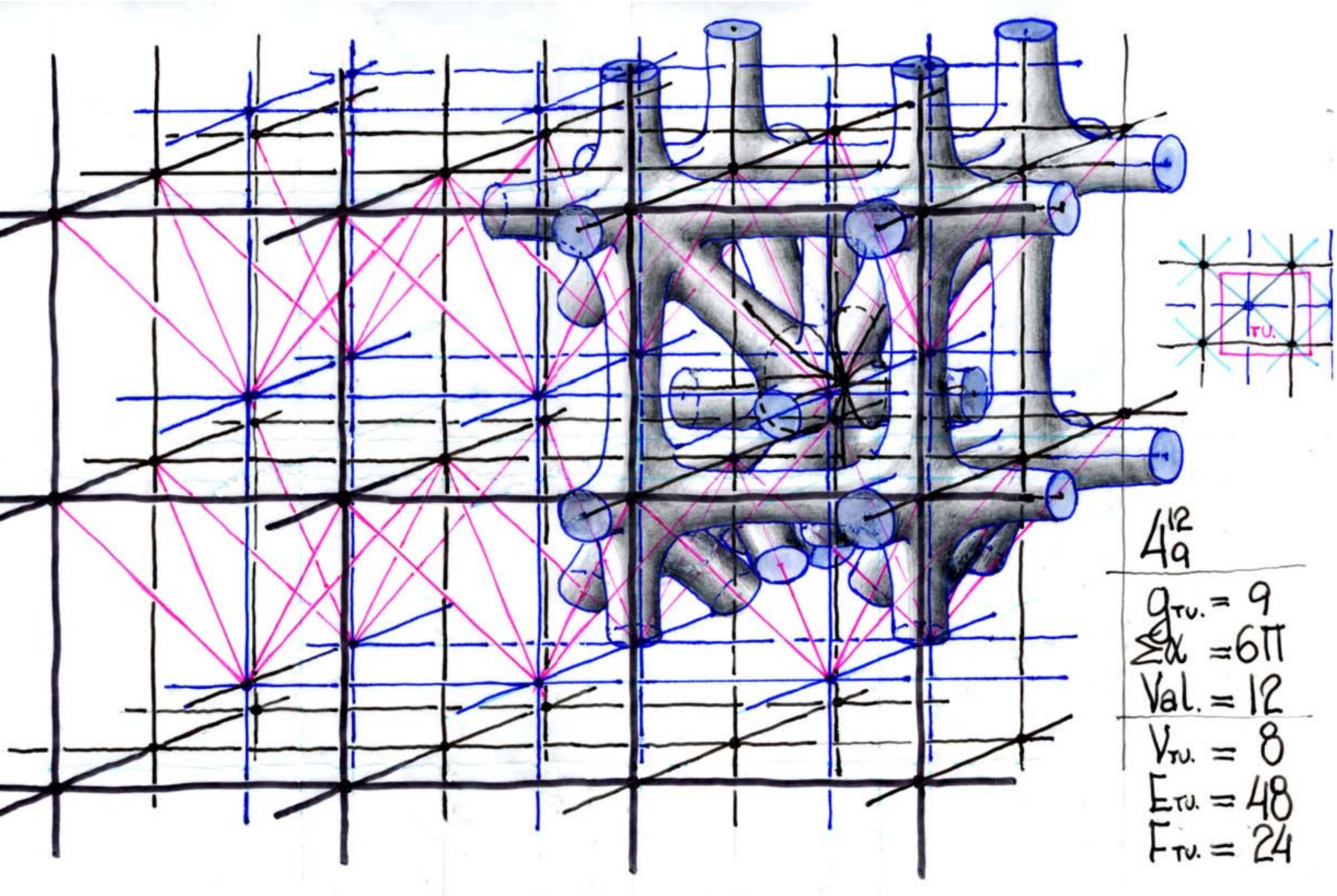
$E_{TU.} = 60$

$F_{TU.} = 30$



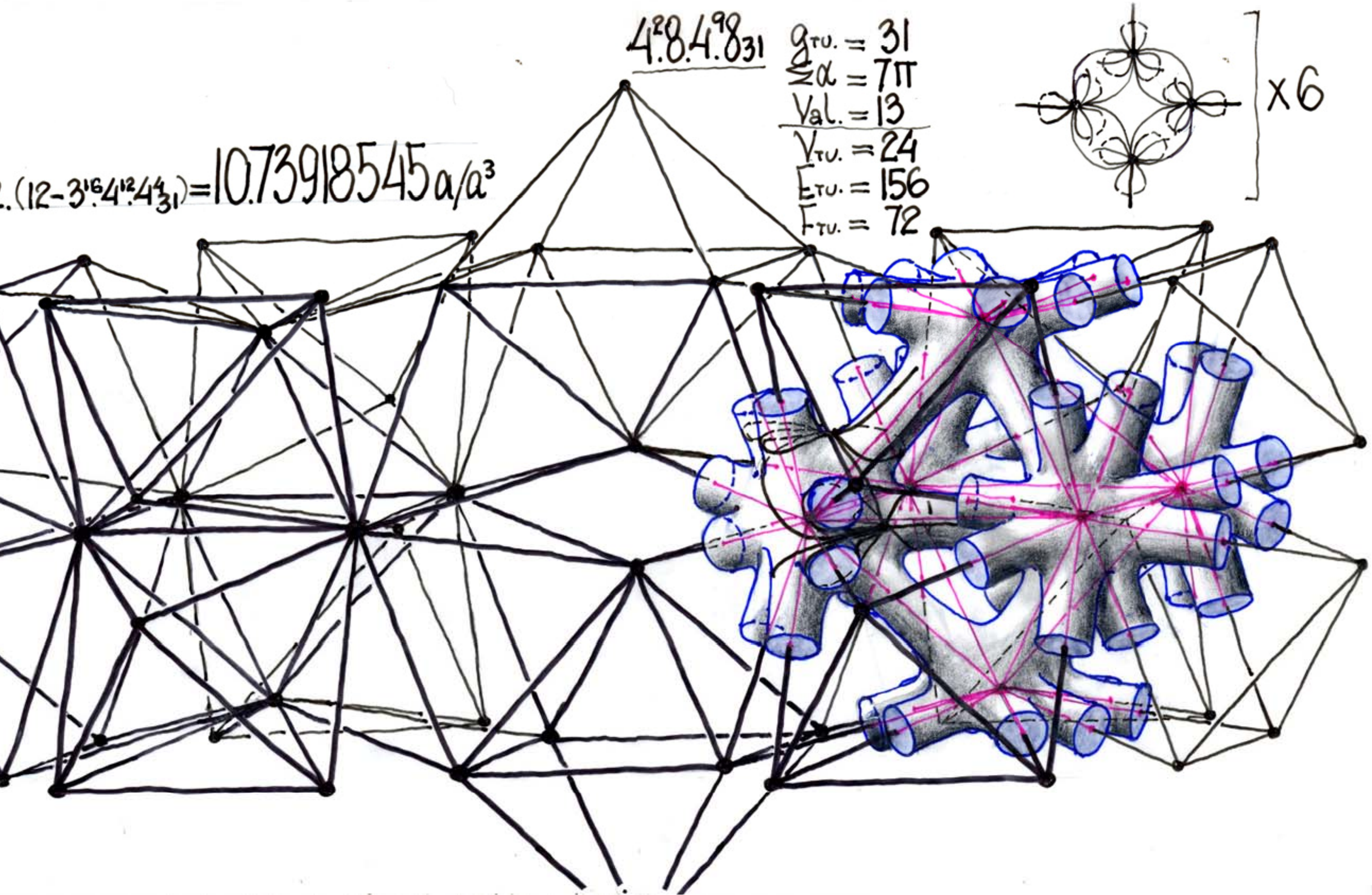

$Den._{(11\text{-}3^{15}4^{18}_{10})} = 12.70170592\, a/a^3$

RM 11-VALENT $11\text{-}3^{15}4^{18}_{10}$ SPACE LATTICE



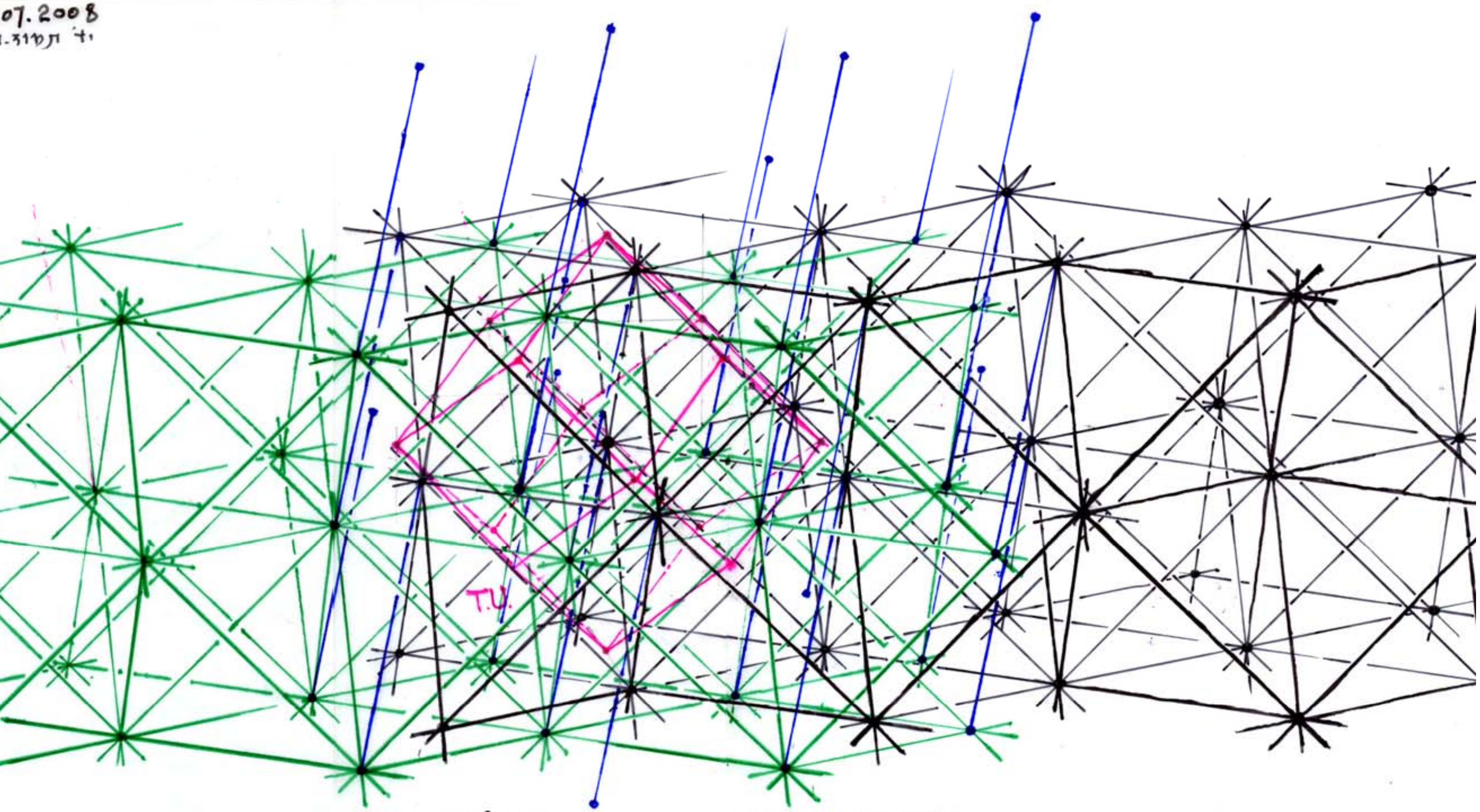

ORM DECAVALENT 10-$3^{12}4^{12}_{9}$ SPACE LATTICE

Den (10-$3^{12}4^{12}$) = 10,000 [illegible]



=10.73918545 a/a³
g TU. = 31
Σα = 7π
Val. = 13
V TU. = 24
E TU. = 156
F TU. = 72
x6

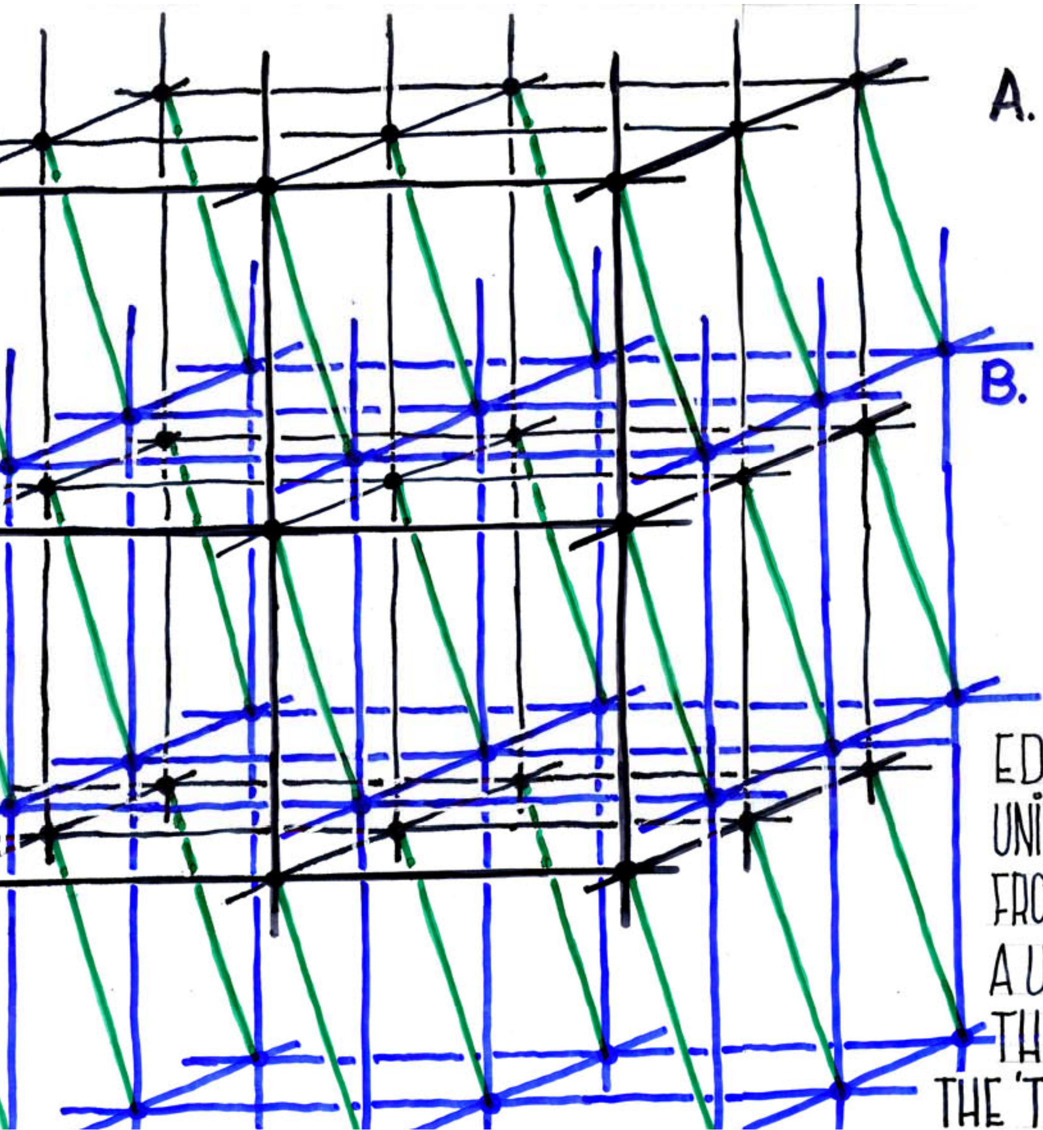
07.2008




DENSITY$_{(13-3^{24}.4^{12}{}_{12})}$ = 18.38477631 $a/a^3$

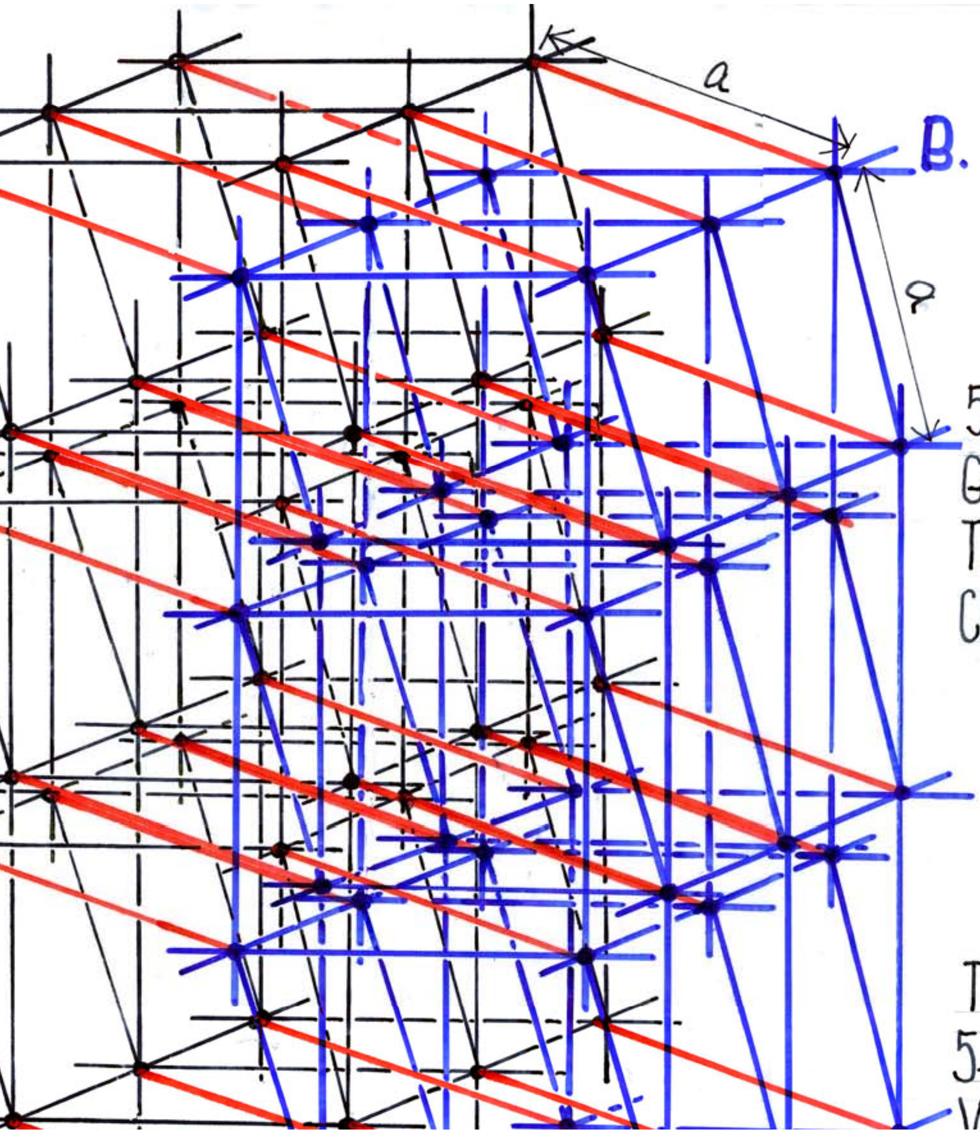
TWO INTER-PENETRATING UNIFORM OCTET SPACE LATTICES WITH EDGE-$a$,

[illegible]




# ENTANGLED NETWORKS

EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT (CUBIC) LATTICE FROM A-TO B-POSITION, RESULTING IN A UNIFORM SEPTAVALENT LATTICE, THE DENSITY OF WHICH IS 7.00 $a/a^3$. THE 'TRANSLATION LATTICE' IS A 3-D




# ENTANGLED NETWORKS

5-DIMENSIONAL OCTAVALENT NETWORK, GENERATED BY A DOUBLE EDGE-LENGTH TRANSLATION OF A UNIFORM HEXAVALENT CUBIC SPACE-LATTICE.

THE 3-D REPRESENTASION OF THE 5-D NETWORK IS A UNIFORM OCTA-VALENT SPACE LATTICE WITH A

R

The edge length translation could be performed **m** times, leading to a uniform **(n+m)**-valent space lattice, the spatial density of which will amount to:

$$\mathbf{Den.(n+m)} = \frac{\mathbf{Den.(n)}}{\mathbf{E_{T.U.}}} \left[ 2^{m} \cdot \mathbf{E_{T.U.}} + \frac{(1+m)m}{2} \right]$$

**In fact m and the spatial density values can reach to infinity (!)** and that, at least theoretically, without causing any edge intersections. These lattices represent a novel class, that of the **Uniform Entangled Space Lattices.**

S

**onclusion**, the networks in 3-D space represent two basically different otypes:
orks which could be characterized as consisting of **a trinity of spatially ciated features of a dual pair of space lattices and the reciprocal erbolic surface partition**, subdividing the space between the two, entangled networks, in themselves representing few classes, the nature of duals and the associated partition surfaces are still to be explored.

cerning the 'Trinity Networks' (1), on the basis of their symmetry constraints d space and the resulting valency range in a vertex (Val.=3÷12), a lusion was drawn that their domain of all 'theoretically imaginable' uniform e lattices is limited in scope to the extent that enables their exhaustive ematic identification and enumeration. So far nearly 180 such uniform e lattices were identified with their density ranging from 0,24023 a/a3 (for ivalent) to 10,7392 a/a3 (for the dodecavalent)